2 INVERSE TRIGONOMETRIC FUNCTIONS

KEY CONCEPT INVOLVED

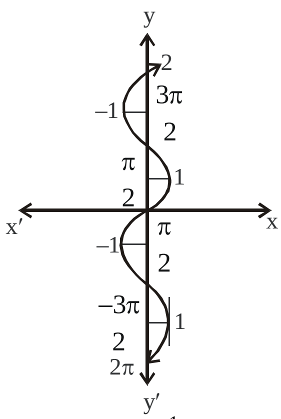
1.

	Functions	Domain	Range
(i)	sin	R	[–1, 1]
(ii)	cos	R	[–1, 1]
(iii)	tan	$R - \{x : x = (2n+1)\frac{\pi}{2}, n \in z\}$	R
(iv)	cot	$R - \{x : x = n\pi, n \in z\}$	R
(v)	sec	$R - \{ x : x = (2n+1)\frac{\pi}{2} \} n \in z\}$	R – [–1, 1]
(vi)	cosec	$R - \{x : x = n\pi, n \in z\}$	R– [–1, 1]

2. **Inverse Function -** If $f : X \to Y$ such that $y = f(x)$ is one-one and onto, then we define another function $g : Y \to X$ such that $x = g(y)$, where $x \in X$ and $y \in Y$ which is also one-one and onto. In such a case domain of g = Range of f and Range of g = domain of f
g is called inverse of f or $g = f^{-1}$
Inverse of $g = g^{-1} = (f^{-1})^{-1} = f$.

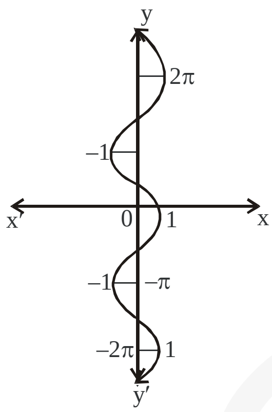
3. **Principal value Branch of function $\sin^{-1}$ -** It may be noted that for the domain [-1, 1] the range sould be any one of the intervals $\left[-\frac{3\pi}{2}, \frac{-\pi}{2}\right], \left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$ or $\left[\frac{\pi}{2}, \frac{3\pi}{2}\right]$ corresponding to each interval we get a branch of the function $\sin^{-1}$ the branch with range $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$ is called the principal value branch.

Thus $\sin^{-1} : [-1, 1] \to \left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$



$y = \sin^{-1} x$

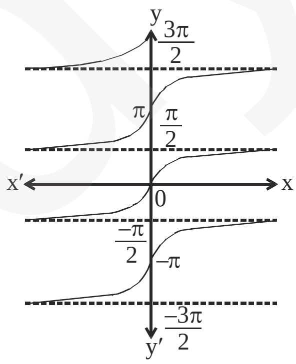
4. **Principal Value branch of function $\cos^{-1}$ -** Domain of the function $\cos^{-1}$ is $[-1, 1]$.
Its range is one of the intervals $(-\pi, 0)$, $(0, \pi)$, $(\pi, 2\pi)$. etc. The branch with range $(0, \pi)$ is called the principal value branch of the function $\cos^{-1}$ thus $\cos^{-1} : [-1, 1] \to [0, \pi]$



$y = \cos^{-1} x$

5. **Principal Value branch of function $\tan^{-1}$ -** The function $\tan^{-1}$ is defined whose domain is set of real numbers and range is one of the intervals $\left(\frac{-3\pi}{2}, \frac{-\pi}{2}\right), \left(\frac{-\pi}{2}, \frac{\pi}{2}\right), \left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$ etc.

Graph of the function is as shown in the adjoining figure the branch with range $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$ is called the pricnipal value branch of function $\tan^{-1}$. Thus $\tan^{-1} : R \to \left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$.

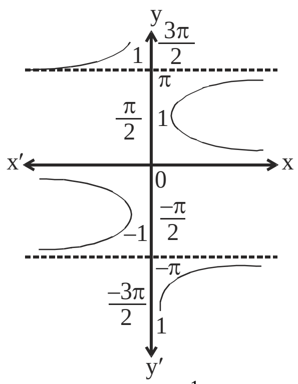


$y = \tan^{-1} x$

6. **Principal Value branch of function cosec^{-1} -** The function cosec^{-1} is defined on a function whose domain is $R - (-1, 1)$ and the range is anyone of the interval $\left[\frac{-3\pi}{2}, \frac{-\pi}{2}\right] - \{\pi\}, \left[\frac{-\pi}{2}, \frac{\pi}{2}\right] - \{0\}, \left[\frac{\pi}{2}, \frac{3\pi}{2}\right] - \{\pi\}, ...$

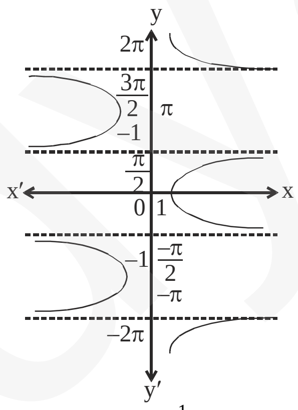
The function corresponding to the range $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right] - \{0\}$ is called the principal value branch of cosec^{-1}

Thus, $\text{cosec}^{-1} : R - (-1, 1) \to \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] - \{0\}$



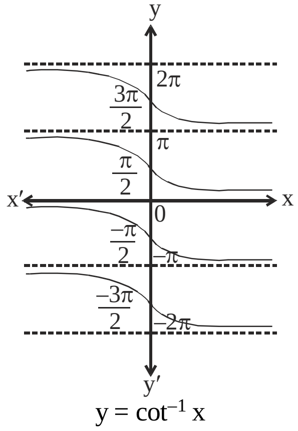
$y = \text{cosec}^{-1} x$

7. Principal value branch of function $\sec^{-1}$ - The $\sec^{-1}$ is defined as a function whose domain is $R - (-1, 1)$ and the range could be any of the intervals is ...., $[-p, 0] - \left\{\frac{-\pi}{2}\right\}$, $[0, p] - \left\{\frac{\pi}{2}\right\}$, $[\pi, 2\pi] - \left\{\frac{3\pi}{2}\right\}$..... etc. The branch corresponding to range $[0, \pi] - \left\{\frac{\pi}{2}\right\}$ is known as the principal value branch of $\sec^{-1}$. Thus $\sec^{-1} : R - (-1, 1) \to [0, \pi] - \left\{\frac{\pi}{2}\right\}$.



$y = \sec^{-1} x$

8. Principal Value branch of function $\cot^{-1}$ - The $\cot^{-1}$ function is defined as the function whose domain is R and the range is any of the intervals......... $(-\pi, 0)$ $(0, \pi)$, $(\pi, 2\pi)$ etc. The branch corresponding to $(0, \pi)$ is called the principal value branch of the function $\cot^{-1}$, then $\cot^{-1} : R \to (0, \pi)$



$y = \cot^{-1} x$

9.

Inverse function	Domain	Principal Value branch
$\sin^{-1}$	$[-1, 1]$	$\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$
$\cos^{-1}$	$[-1, 1]$	$[0, \pi]$
cosec^{-1}	$R - (-1, 1)$	$\left[\frac{-\pi}{2}, \frac{\pi}{2}\right] - \{0\}$
$\sec^{-1}$	$R - (-1, 1)$	$[0, \pi] - \left\{\frac{\pi}{2}\right\}$
$\tan^{-1}$	R	$\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$
$\cot^{-1}$	R	$(0, \pi)$

CONNECTING CONCEPTS

1. (i) $\sin^{-1} 1/x = \text{cosec}^{-1} x,\ x \geq 1,\ x \leq -1$
   (ii) $\cos^{-1} 1/x = \sec^{-1} x,\ x \geq 1,\ x \leq -1$
   (iii) $\tan^{-1} 1/x = \cot^{-1} x,\ x > 0$
   (iv) $\text{cosec}^{-1} 1/x = \sin^{-1} x,\ x \in [-1, 1]$
   (v) $\sec^{-1} 1/x = \cos^{-1} x,\ x \in [-1, 1]$
   (vi) $\cot^{-1} 1/x = \tan^{-1} x,\ x > 0$
2. (i) $\sin^{-1}(-x) = -\sin^{-1} x,\ x \in [-1, 1]$
   (ii) $\tan^{-1}(-x) = -\tan^{-1} x,\ x \in R$
   (iii) $\text{cosec}^{-1}(-x) = -\text{cosec}^{-1} x,\ |x| \geq 1$
   (iv) $\cos^{-1}(-x) = \pi - \cos^{-1} x,\ x \in [-1, 1]$
   (v) $\sec^{-1}(-x) = \pi - \sec^{-1} x,\ |x| \geq 1$
   (vi) $\cot^{-1}(-x) = \pi - \cot^{-1} x,\ x \in R$
3. (i) $\sin^{-1} x + \cos^{-1} x = \pi/2,\ x \in [-1, 1]$
   (ii) $\tan^{-1} x + \cot^{-1} x = \pi/2,\ x \in R$
   (iii) $\text{cosec}^{-1} x + \sec^{-1} x = \pi/2,\ |x| \geq 1$
4. (i) $\tan^{-1} x + \tan^{-1} y = \tan^{-1} \frac{x+y}{1-xy},\ xy < 1$
   (ii) $\tan^{-1} x - \tan^{-1} y = \tan^{-1} \frac{x-y}{1+xy},\ xy > -1$
   (iii) $2\sin^{-1} x = \sin^{-1}\left(2x\sqrt{1-x^2}\right),\ -\frac{1}{\sqrt{2}} \leq x \leq \frac{1}{\sqrt{2}}$
   (iv) $2\cos^{-1} x = \cos^{-1}(2x^2 - 1),\ -\frac{1}{\sqrt{2}} \leq x \leq 1$
   (v) $2\tan^{-1} x = \tan^{-1}\frac{2x}{1-x^2},\ -1 < x < 1 = \sin^{-1}\frac{2x}{1+x^2},\ |x| \leq 1 = \cos^{-1}\frac{1-x^2}{1+x^2},\ x \geq 0$