eVidyarthi
FREE Education

3 MATRICES

KEY CONCEPT INVOLVED

1. **Matrices -** A system of mn numbers (real or complex) arranged in a rectangular array of m rows and n columns is called a matrix of order m × n. An m × n matrix (to be read as 'm by n' matrix)
An m × n matrix is written as

$$A = \begin{bmatrix} a_{11} & a_{12} & \dots\dots & a_{1n} \\ a_{21} & a_{22} & \dots\dots & a_{2n} \\ \vdots & \vdots & \dots\dots & \vdots \\ \vdots & \vdots & \dots\dots & \vdots \\ a_{m1} & a_{m2} & \dots\dots & a_{mn} \end{bmatrix}$$

The numbers a_{11}, a_{12} etc are called the elements or entries of the matrix. If A is a matrix of order m × n, then we shall write $A = [a_{ij}]_{m \times n}$ where, a_{ij} represent the number in the *i*-th row and *j*-th column.

2. **Row Matrix -** A single row matrix is called a row matrix or a row vector. e.g. the matrix $[a_{11}, a_{12}, \dots\dots a_{1n}]$ is a row matrix.

3. **Column Matrix -** A single column matrix is called a column matrix or a column vector. e.g. the matrix $\begin{bmatrix} a_{11} \\ a_{21} \\ \vdots \\ a_{m1} \end{bmatrix}$ is a m × 1 column matrix.

4. **Order of a Matrix -** A matrix having m rows and n columns is of the order m × n. i.e. consisting of m rows and n columns is denoted by $A = [a_{ij}]_{m \times n}$.

5. **Square Matrix -** If m = n, i.e. if the number of rows and columns of a matrix are equal, say n, then it is called a square matrix of order n.

6. **Null or Zero Matrix -** If all the elements of a matrix are equal to zero, then it is called a null matrix and is denoted by $O_{m \times n}$ or 0.

7. **Diagonal Matrix -** A square matrix, in which all its elements are zero except those in the leading diagonal is called a diagonal matrix, thus in a diagonal matrix, $a_{ij} = 0$, if $i \neq j$, e.g. the diagonal matrices of order 2 and 3 are $\begin{bmatrix} K_1 & 0 \\ 0 & K_2 \end{bmatrix}, \begin{bmatrix} K_1 & 0 & 0 \\ 0 & K_2 & 0 \\ 0 & 0 & K_3 \end{bmatrix}$

8. **Scalar Matrix -** A square matrix in which all the diagonal element are equal and all other elements equal to zero is called a scalar matrix.
i.e. in a scalar matrix $a_{ij} = k$ for $i = j$ and $a_{ij} = 0$ for $i \neq j$. Thus $\begin{bmatrix} K & 0 & 0 \\ 0 & K & 0 \\ 0 & 0 & K \end{bmatrix}$ is a scalar matrix.

9. **Unit Matrix or Identity Matrix -** A square matrix in which all its diagonal elements are equal to 1 and all other elements equal to zero is called a unit matrix or identity matrix.

e.g. a unit or identity matrix of order 2 and 3 are $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ respectively.

10. **Upper triangular Matrix -** A square matrix A whose elements $a_{ij} = 0$ for $i > j$ is called an upper triangular matrix.
11. **Lower triangular Matrix -** A square matrix A whose elements $a_{ij} = 0$ for $i < j$ is called a lower triangular matrix.
12. **Equal Matrices -** Two matrices A and B are said to be equal, written as A = B if
    (i) they are of the same order i.e. have the same number of rows and columns, and
    (ii) the elements in the corresponding places of the two matrices are the same.
13. **Transpose of a matrix -** Let A be a m × n matrix then the matrix of order n × m obtained by changing its rows into columns and columns into rows is called the transpose of A and is denoted by A' or A^T.
14. **Negative of Matrix -** Let $A = [a_{ij}]_{m \times n}$ be a matrix. Then the negative of the matrix A is defined as the matrix $[-a_{ij}]_{m \times n}$ and is denoted by –A.
15. **Symmetric Matrix -** a square matrix A is said to be symmetric if $A' = A$
    Thus a square matrix $A = [a_{ij}]$ is symmetric if $A = [a_{ij}]$ is symmetric if $a_{ij} = -a_{ji}$ for all values of i and j.
16. **Skew-Symmetric Matrix -** A square matrix A is said to be skew-symmetric if $A' = -A$ Thus a square matrix $A = [a_{ij}]$ is skew-symmetric if $a_{ij} = -a_{ji}$ for all values of i and j.
    In particular $a_{ii} = -a_{ii} \Rightarrow 2a_{ii} = 0 \Rightarrow a_{ii} = 0$ i.e. all diagonal elements of a skew-symmetric matrix are o.
17. For any square matrix A with real number entries, $A + A'$ is a symetric matrix and $A - A'$ is a skew symetric matrix.
18. Any square matrix can be expressed as the sum of a symetric and a skew symetric matrix.

    If A be a square matrix, then we can write $A = \frac{1}{2}(A + A') + \frac{1}{2}(A - A')$, here $\frac{1}{2}(A + A')$ is symetric matrix

    and $\frac{1}{2}(A \quad A \ )$ is skew symetric matrix.
19. **Addition of Matrices -** Let there be two matrices A and B of the same order m × n. then the sum denoted by A + B is defined to be the matrix of order m × n obtained by adding the corresponding elements of A and B.
    Thus if $A = [a_{ij}]_{m \times n}$ and $B = [b_{ij}]_{m \times n}$ then $A + B = [a_{ij} + b_{ij}]_{m \times n}$
20. **Scalar Multiplication of a Matrix -** Let $A = [a_{ij}]_{m \times n}$ be a matrix and K is a scalar. Then the matrix obtained by multiplying each element of matrix A by K is called the scalar multiplication of matrix A by K and is denoted by KA or AK.
21. **Multiplication of Matrices -** Product of two matrices exists only if number of column of first matrix is equal to the number of rows of the second. Let A be m × n and B be n × p matrices. Then the product of matrices A and B denated by A.B is the matrix of order m × p whose (i, j)th element is obtained by adding the products of corresponding elements of *i*th row of A and *j*th column of B.
22. **Elementary Row Operations -** The operations known as elementary row operations on a matrix are-
    (i) The interchange of any two rows of a matrix. (The notations $R_i \leftrightarrow R_j$ is used for the interchange of the *i*-th and *j*-th rows.)
    (ii) The multiplication of every element of a row by a non-zero element (constant).
    (The notations $K.R_i$ is used for the multiplication of every element of *i*-th row by a constant K.
    (iii) The addition of the elements of a row, the product of the corresponding elements of any other row by any non-zero constant. (The notation $R_i + K.R_j$ is generally used for addition to the elements of *i*-th row to the element of *j*-th row multiplied by the constant K ($K \neq 0$))
23. **Invertible matrices -** If A is a square matrix of order m, and if there exists another square matrix B of the same order m, such that AB = BA = I, then B is called the Inverse matrix of A and it is denoted by A^{-1}. In that care A is said to be invertible.

24. If A and B are invertible matrices of the same order, then $(AB)^{-1} = B^{-1}. A^{-1}$.

25. **Inverse of a matrix by elementry operations -** Let X, A and B be matrices of, the same order such that X = AB. In order to apply a sequence of elementry row operations on the matrix equation X = AB, we will apply these row operations simultaneously on X and on the first matrix A of the product AB on RHS.
Similarly, in order to apply a sequence of elementry column operations on the matrix equation X = AB, we will apply, these operations simultaneously on X and on the second matrix B of the product AB on RHS.
In view of the above discussion, we conclude that if A is a matrix such that A^{-1} exists, then to find A^{-1} using elementry row operations, write A = IA and apply a sequence of row operation on A = IA till we get, I =BA. The matrix B will be the inverse of A. Similarly, if we with to find A^{-1} using column operations, then, write A = AI on A = IA till we get, I = BA. The matrix and apply a sequence of column operations on A = AI till we get, I = AB.
Remark - In case, after applying one or more elementry row (column) operations on A = IA (A = AI). If we obtain all zero in one or more rows of the matrix A on L.H.S., that A^{-1} does not exist.

CONNECTING CONCEPTS

1. The elements a_{ij} of a matrix for which i = j are called the diagonal elements of a matrix and the line along which all these elements lie is called the principal diagonal or the diagonal of the matrix.

2. **Properties of transpose of the matrices-**
(i) $(A + B)' = A' + B'$
(ii) $(KA)' = KA'$, where K is constant
(iii) $(AB)' = B'A'$
(iv) $(A')' = A$

3. **Properties of Matrix addition-**
(i) **Matrix Addition is Commutative -** If A and B be two m × n matrices, then A + B = B + A
(ii) **Matrix Addition is Associative -** If A, B and C be three m × n matrices, then
$$(A + B) + C = A + (B + C)$$

4. **Properties of Multiplication of a Matrix by a Scalar-**
(i) If K_1 and K_2 are scalars and A be a matrix, then $(K_1 + K_2) A = K_1 A + K_2 A$.
(ii) If K_1 and K_2 are scalars and A be a matrix, then $K_1 (K_2 A) = (K_1 K_2) A$.
(iii) If A and B are two matrices of the same order and K, a scalar, then K (A + B) = KA + KB.
(iv) If K_1 and K_2 are two scalars and A is any matrix then $(K_1 + K_2) A = K_1 A + K_2 A$.
(v) If A is any matrix and K be a scalar.
then (–K) A = – (KA) = K (–A).

5. **Properties of Matrix Multiplication -**
(i) **Associative law for Multiplication -** If A, B and C be three matrices of order m × n and n × p and p × q, respectively, then (AB) C = A (BC).
(ii) **Distributive Law -** If A, B, C be three matrices of order m × n, n × p and n × q respectively.
then $A \cdot (B + C) = A \cdot B + A \cdot C$
(iii) Matrix Multiplication is not commutative.
i.e. $A \cdot B \neq B \cdot A$
(iv) **The existence of multiplicative Identity :** For every square matrix A, there exists an identity matrix of same order such that IA = AI = A.

6. If A be any n × n square matrix, then
$$A \cdot (\text{Adj } A) = (\text{Adj } A) \cdot A = |A|. I_n$$
where I_n is an n × n unit matrix

7. (i) Only square matrix can have inverse
(ii) The matrix $B = A^{-1}$, will also be a square matrix of same order A.
(iii) The square matrix A is said to be invertible if A^{-1} exists.

8. Every invertible matrix possesses a unique inverse.