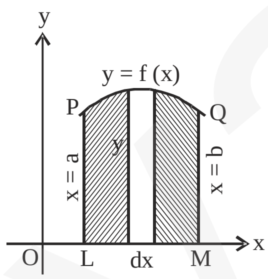
8 APPLICATION OF THE INTEGRALS

KEY CONCEPT INVOLVED

Area Under Simple Curves

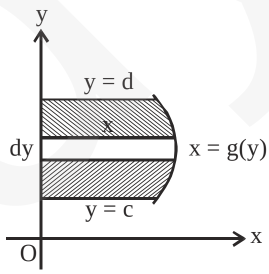
1. Let us find the area bounded by the curve $y = f(x)$, x-axis and the ordinated $x = a$ and $x = b$. Consider the area under the curve as composed of large number of thin vertical strips let there be an arbitary strip of hieght y and width dx.Area of elementary strip dA= ydx, where $y = f(x)$. Total Area A of the region between x-axis.ordinated $x = a$, $x = b$ and the curve $y = f(x)$ = Sum of areas of elementry thin strips across the region PQML



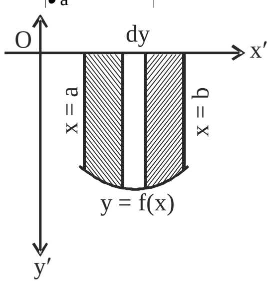
$$A = \int_a^b dA = \int_a^b y dx = \int_a^b f(x) dx$$

2. The area A of the region bounded by the curve $x = g(y)$, y-axis and the lines $y = c$ and $y = d$ is given by

$$A = \int_c^d x dy$$

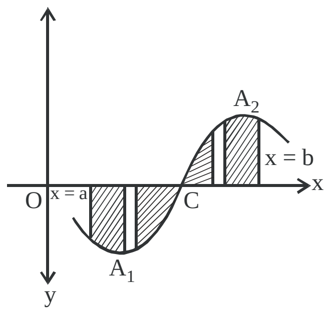


3. If the curve under consideration lies below x-axis, then $f(x) < 0$ from $x = a$ to $x = b$, the area bounded by the curve $y = f(x)$, and the ordinates $x = a$, $x = b$ and x-axis is negative. But the numerical value of the area is to be taken into consideration.Then Area= $\left| \int_a^b f(x) dx \right|$



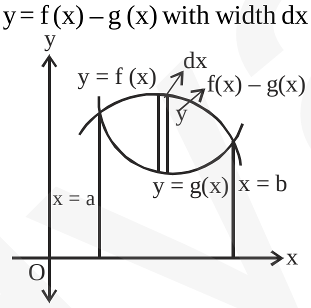
4. Let some portion of the curve is above x-axis and some portion is below x-axis. Let A_1 be the area below x-axis and A_2 be the area above of x-axis. Therefore Area bounded by the curve y = f (x), x-axis and the ordinates x = a and x = b.

$$A = |A_1| + A_2$$



Area between Two curves

5. Let the two curves be y = f (x) and y = g (x). Suppose these curves intersect at x = a and x = b. Consider the elementary strip of height y where y = f (x) – g (x) with width dx

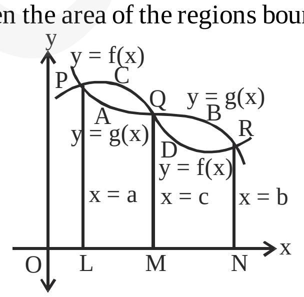


$\therefore$ da = ydx

$\Rightarrow$ $A = \int_a^b \left(f(x) - g(x)\right) dx = \int_a^b f(x)\,dx - \int_a^b g(x)\,dx$

i.e. A= Area bounded by the curve y = f (x) – Area bounded by the curve y = g (x)

6. If the two curves y = f (x) and y = g (x) intersects at x = a, x = c and x = b such that a < c < b. If f (x) > g (x) in [a, c] and f (x) < g (x) in [c, b], Then the area of the regions bounded by curve.



= Area of the region PAQCP + Area of the region QDRBQ = $\int_a^c \left(f(x) - g(x)\right) dx + \int_c^b \left(g(x) - f(x)\right) dx$