# 9 DIFFERENTIAL EQUATIONS

## KEY CONCEPT INVOLVED

1. **Differential Equation** – An equation containing an independent variable dependent variable and differential coefficient of dependent variable with respect to independent variable is called a differential equation.

   e.g. $\frac{dy}{dx} + 2xy = x^3$ and $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = x^2$

2. **Order of a differential Equation** – The order of a differential equation is the order of the highest order derivative appearing in the equation.
3. **Degree of a differential Equation** – The degree of a differential equation is the degree of the highest order derivative when differential coefficients are made free from radicals and fractions.
4. **Solution of a differential Equation** – The solution of a differential equation is a relation between the variables involved, not involving the differential coefficients, such that this relation and derivatives obtained form it satisfy the given differential equation.
5. **General Solution** – The solution which contains as many as orbirary constants as the order of the differential equation is called the general solution of the differential equation.
6. **Particular Solution** – Solution obtained by giving particular values to the arbitrary constants in the general solution of a differential equation is called a particular solution.
7. **Equations in variable separable form** – If the differential equation can be reduced to the form f (x) dx = g (y) dy we say that the variables have been separated on integrating both sides of this reduced form, we get the general solution of the differential equation.

   $$\int f(x)\, dx = \int g(y)\, dy + c$$

8. **Equations Reducible to variable separable form** – Differential equations of the form $\frac{dy}{dx} = f(ax + by + c)$ can be reduced to variable separable form by the substitution ax + by + c = v
9. **Homogeneous Differential Equation** – A function f(x,y) is called a homogeneous function of degree n if $F(\lambda x, \lambda y) = \lambda^n F(x, y)$ for any non zero constant $\lambda$.

   A differential equation of the form $\frac{dy}{dx} = F(x, y)$ is said to be homogeneous if F (x, y) is a homogeneous function of degree zero. To solve such ... a homogenous differential equation of the type

   $$\frac{dy}{dx} = F(x) = g\left(\frac{y}{x}\right) \qquad \text{...(i)}$$

   (i) Put $y = vx$ and $\frac{dy}{dx} = v + x\frac{dv}{dx}$ in equation (i), we get reduces to the form $v + x\frac{dv}{dx} = g(v)$

   $$\Rightarrow \qquad x \times \frac{dy}{dx} = g(v) - v$$

   Now, on separating the variables, we get

   $$\frac{dv}{g(x) - v} = \frac{dx}{x}$$

   Integrate both sides to obtain the solution in terms of v and x.

   Replace v by $\frac{y}{x}$ in the solution obtained to obtain the solution in terms of x and y.

If the homogeneous differential equation is in the form $\frac{dy}{dx}$ = F (x, y), where F (x, y) is homogeneous function of degree, then we make substitution $\frac{x}{y}$ = v i.e., x = vy and the proceed further to find the general solution as discussed above by writting $\frac{dx}{dy} = F(x, y) = h\left(\frac{x}{y}\right)$

**10. Linear differential Equations** – A differential equation is known as first order linear differential equation, if the dependent variable y and its derivative are related as $\frac{dy}{dx} + Py = Q$, where P and Q are constant or functions of x.

Steps involved to solve first order linear differential equation:

(i) Write the given differential equation in the form $\frac{dy}{dx} + Py = Q$ and obtain P and Q.

(ii) Find integrating factor, I.F. = $e^{\int p dx}$

(iii) Multiply both sides of equation in (i) by I.F.

(iv) Integrate both sides of the equation obtained in (iii) w.r.t. x to obtain

$$y(\text{I.F.}) = \int Q.(\text{I.F.})\, dx + C$$

This gives the required solution.

In case, the first order linear differential equation is in the form $\frac{dx}{dy} + P_1 x = Q_1$, where , $P_1$ and $Q_1$ are constants or functions of y only. Then I.F. = $e^{\int P_1 dy}$ and the solution of the differential equation is given by $x . (\text{I. F}) = \int (Q_1 \cdot \text{I.F.})\, dy + C$

## CONNECTING CONCEPTS

**1. Formation of Differential Equations** – Formation of a differential from a given equation representing a family of curves means finding a differential equation whose solution is the given equation. If an equation representing a family of curves, contains n arbitrary constants, then we differentiable the given equation n times to obtain n more equations. Using all these equations, we eliminate the constants. The equation so obtained is the differential equation of order n for the family of given curves.

**2. Methods of solving a differential equation of the type $\frac{dy}{dx} = f(x)$** – To solve this type of differential equations, first we write the differential equation as dy = f (x) dx

Then integrate boht sides with respect t x to obtain the solution

$$\int dy = \int f(x)\, dx + C$$

or

$$y = \int f(x)\, dx + C$$

**3. Differential Equations of the type $\frac{dy}{dx} = f(y)$** – To solve this type of differential equations, first we write in the form of $dx = \frac{1}{f(y)} dy$ them integrate both sides to obtain the general solution

$$\Rightarrow \int dx = \int \frac{1}{f(y)} dy + c \text{ or } x = \int \frac{1}{f(y)} dy + c$$

**4.** Differential Equations of the type $\frac{d^2y}{dx^2} = f(x)$

(i) Integrate both sides of the differential equation in (i) with respect to x to obtain a first order first degree differential equation.

(ii) Integrate both sides of the first order differential equation obtained in (ii) with respect to x.