

# CBSE Class 7 Maths Notes Chapter 7 Congruence of Triangles

If two figures have exactly the same shape and size, then they are said to be congruent.

For congruence, we use the symbol '='

Two plane figures are congruent, if each when superposed on the other covers it exactly.

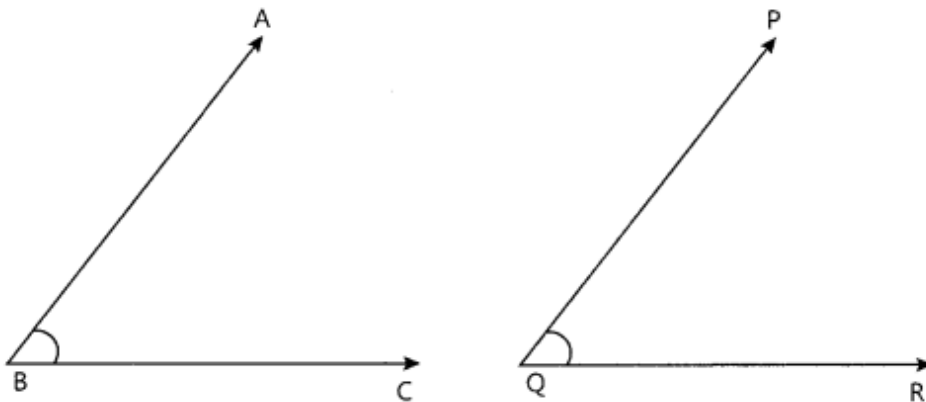
e.g.  $F_1$  and  $F_2$  are congruent if the trace copy of  $F_1$  fits exactly on that  $F_2$ . We can write this as  $F_1 = F_2$

Two line segments,  $\overline{AB}$  and  $\overline{CD}$  are congruent if they have equal lengths. We can write this as  $\overline{AB} = \overline{CD}$ .

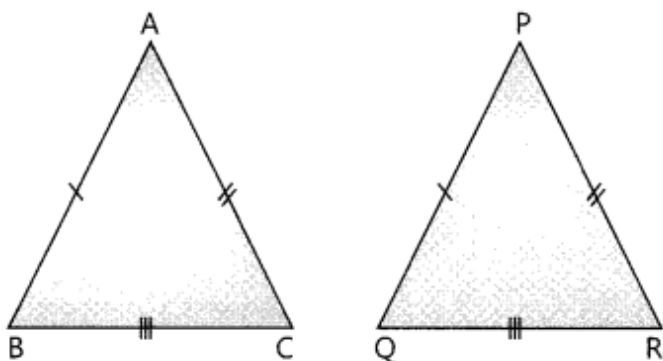
However, it is common to write it as  $\overline{AB} \cong \overline{CD}$ .



Two angles  $\angle ABC$  and  $\angle PQR$ , are congruent if their measures are equal. We can write this as  $\angle ABC = \angle PQR$  or  $m\angle ABC = m\angle PQR$ . Also, it is common to write it as  $\angle ABC \cong \angle PQR$ .

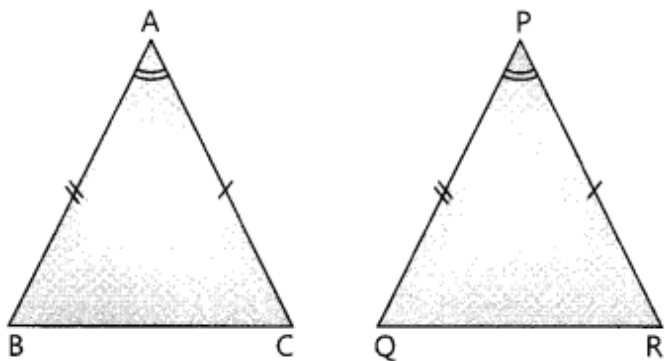


**SSS Congruence of two triangles:** Two triangles are congruent if the three sides of the one are equal to the three corresponding sides of the other.



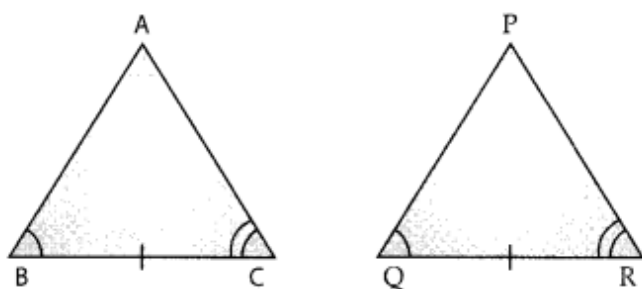
where,  $AB = PQ$ ,  $AC = PR$ ,  $BC = QR$

**SAS Congruence of two triangles:** Two triangles are congruent if two sides and the angle included between them in one of the triangles are equal to the corresponding sides and the angle included between them of the other triangle.



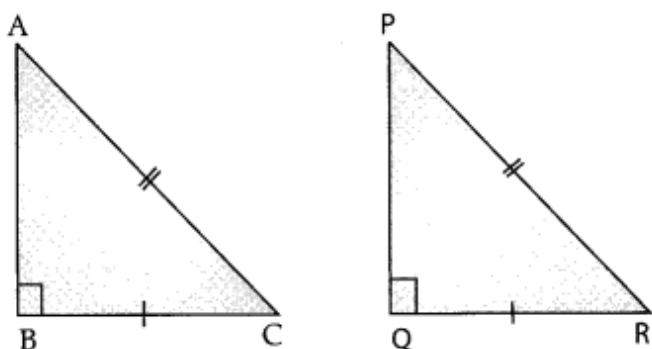
where,  $AB = PQ$ ,  $AC = PR$ ,  $\angle BAC = \angle QPR$ .

**ASA congruence of two triangles:** Two triangles are congruent if two angles and the side included between them in one of the triangles are equal to the corresponding angles and the side included between them of the other triangle.



where,  $BC = QR$ ,  $\angle ABC = \angle PQR$ ,  $\angle ACB = \angle PRQ$ .

**RHS Congruence of two right-angled triangles:** Two right-angled triangles are congruent if the hypotenuse and a leg of one of the triangles are equal to the hypotenuse and the corresponding leg of the other triangle.



where,  $AC = PR$ ,  $BC = QR$ ,  $\angle ABC = \angle PQR = 90^\circ$ .

## Congruence of Plane Figures

Two figures  $F_1$  and  $F_2$  are said to be congruent if they cover each other completely.

In this case, we write  $F_1 = F_2$ .

## Congruence Among Line Segments

If two line segments have the same (i.e., equal) length, they are congruent. Conversely, if two line segments are congruent, they have the same length.

Line segments are congruent  $\Leftrightarrow$  their lengths are the same

If line segment  $AB$  is congruent to line segment  $CD$ , then we write  $AB = CD$

Sometimes we also write  $AB = CD$

and simply say that the line segments  $AB$  and  $CD$  are equal.



## Congruence of Angles

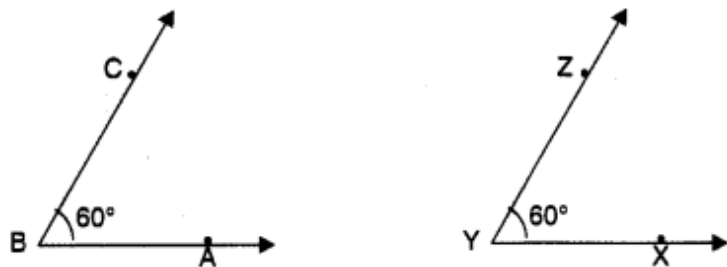
If two angles have the same measure, they are congruent.

Conversely, if two angles are congruent, their measures are the same.

Angles congruent  $\Leftrightarrow$  Angle measures same

If  $\angle ABC$  is congruent to  $\angle XYZ$ , then we write  $\angle ABC = \angle XYZ$  or  $m\angle ABC = m\angle XYZ$

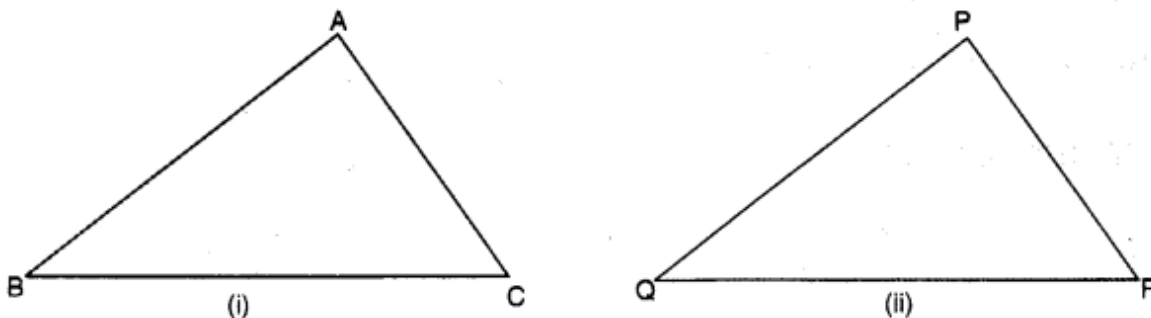
where  $m$  stands for the measure.



In the above case, the measure is  $60^\circ$ .

## Congruence of Triangles

“Two triangles are congruent if they are copies of each other and when superposed, they cover each other exactly.”



$\triangle ABC$  and  $\triangle PQR$  have the same size and shape. They are congruent. So we would express this as  $\triangle ABC = \triangle PQR$ .

This means that, when we place  $\triangle PQR$  on  $\triangle ABC$ , P falls on A, Q falls on B and R falls on C, also  $\overline{PQ}$  falls along  $\overline{AB}$ ,  $\overline{QR}$  falls along  $\overline{BC}$  and  $\overline{PR}$  falls along  $\overline{AC}$ .

If under a given correspondence, two triangles are congruent, then their corresponding parts (i.e. angles and sides) that match one another are equal. Thus in these two congruent triangles, we have:

Corresponding vertices: A and P, B and Q; C and R.

Corresponding sides:  $\overline{AB}$  and  $\overline{PQ}$ ,  $\overline{BC}$  and  $\overline{QR}$ ;  $\overline{AC}$  and  $\overline{PR}$ .

Corresponding angles:  $\angle A$  and  $\angle P$ ,  $\angle B$  and  $\angle Q$ ;  $\angle C$  and  $\angle R$ .

While talking about the congruence of triangles, not only the measures of angles and lengths of sides matter, but also the matching of vertices. In the above case, the correspondence is  $A \leftrightarrow P$ ,  $B \leftrightarrow Q$ ,  $C \leftrightarrow R$ . We may write this as  $ABC \leftrightarrow PQR$ .

## Criteria for Congruence of Triangles

### SSS Congruence Criterion

If under a given correspondence, the three sides of one triangle are respectively equal to the three sides of another triangle, then the triangles are congruent.

### SAS Congruence Criterion

If under a corresponding two sides and the angle included between them of a triangle are respectively equal to two sides and the angle included between them of another triangle, then the triangles are congruent.

### ASA Congruence Criterion

If under a correspondence two angles and the included side of a triangle are equal to two angles and the included side of another triangle, then the triangles are congruent.

Note: If two angles of a triangle are given, we can always find the third angle of the triangle. So, if, two angles and one side of one triangle are equal to the two angles and the corresponding side of another triangle, then the third angle of the two triangles shall also be equal (as the sum of the three angles of a triangle is  $180^\circ$ ), so we may convert it into "two angles and the included side" form of congruence and then apply the ASA congruence rule.

## **Congruence Among Right-Angled Triangles**

### **RHS Congruence Criterion**

If under a correspondence, the hypotenuse and one side of a right-angled triangle are respectively equal to the hypotenuse and one side of another right-angled triangle, then the triangles are congruent.

We call this "RHS" congruence because R stands for Right-angle, H stands for Hypotenuse and S stands for Side.