CBSE Class 12 Maths Notes Chapter 9 Differential Equations

Differential Equation: An equation involving independent variable, dependent variable, derivatives of dependent variable with respect to independent variable and constant is called a differential equation.

Ordinary Differential Equation: An equation involving derivatives of the dependent variable with respect to only one independent variable is called an ordinary differential equation.
e.g.

$$\frac{dy}{dx}+\frac{d^2y}{dx^2}-2=0.$$

From any given relationship between the dependent and independent variables, a differential equation can be formed by differentiating it with respect to the independent variable and eliminating arbitrary constants involved.

Order of a Differential Equation: Order of a differential equation is defined as the order of the highest order derivative of the dependent variable with respect to the independent variable involved in the given differential equation.

Note: Order of the differential equation, cannot be more than the number of arbitrary constants in the equation.

Degree of a Differential Equation: The highest exponent of the highest order derivative is called the degree of a differential equation provided exponent of each derivative and the unknown variable appearing in the differential equation is a non-negative integer.
Note
(i) Order and degree (if defined) of a differential equation are always positive integers.
(ii) The differential equation is a polynomial equation in derivatives.
(iii) If the given differential equation is not a polynomial equation in its derivatives, then its degree is not defined.

Formation of a Differential Equation: To form a differential equation from a given relation, we use the following steps:
Step I: Write the given equation and see the number of arbitrary constants it has.
Step II: Differentiate the given equation with respect to the dependent variable n times, where n is the number of arbitrary constants in the given equation.
Step III: Eliminate all arbitrary constants from the equations formed after differentiating in step (II) and the given equation.
Step IV: The equation obtained without the arbitrary constants is the required differential equation.

Solution of the Differential Equation
A function of the form y = Φ(x) + C, which satisfies given differential equation, is called the solution of the differential equation.
General solution: The solution which contains as many arbitrary constants as the order of the differential equation, is called the general solution of the differential equation, i.e. if the solution of a differential equation of order n contains n arbitrary constants, then it is the general solution.

Particular solution: A solution obtained by giving particular values to arbitrary constants in the general solution of a differential equation, is called the particular solution.

Methods of Solving First Order and First Degree Differential Equation
Variable separable form: Suppose a differential equation is $\frac{dy}{dx}$ = F(x, y). Here, we separate the variables and then integrate both sides to get the general solution, i.e. above equation may be written as $\frac{dy}{dx}$ = h(x) . k(y)
Then, by separating the variables, we get $\frac{dy}{k(y)}$ = h(x) dx.
Now, integrate above equation and get the general solution as K(y) = H(x) + C
Here, K(y) and H(x) are the anti-derivatives of $\frac{1}{K(y)}$ and h(x), respectively and C is the arbitrary constant.

Homogeneous differential equation: A differential equation $\frac{dy}{dx} = \frac{f(x,y)}{g(x,y)}$ is said to be homogeneous, if f(x, y) and g(x, y) are homogeneous functions of same degree, i.e. it may be written as

$$\frac{dy}{dx} = \frac{x^n f\left(\frac{y}{x}\right)}{x^n g\left(\frac{y}{x}\right)} = \frac{f(y/x)}{g(y/x)} = F(y/x) \quad \text{.......(i)}$$

To check that given differential equation is homogeneous or not, we write differential equation as $\frac{dy}{dx}$ = F(x, y) or $\frac{dx}{dy}$ = F(x, y) and replace x by λx, y by λy to write F(x, y) = λ F(x, y).
Here, if power of λ is zero, then differential equation is homogeneous, otherwise not.

Solution of homogeneous differential equation: To solve homogeneous differential equation, we put
y = vx
$\Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$
in Eq. (i) to reduce it into variable separable form. Then, solve it and lastly put v = $\frac{y}{x}$ to get required solution.

Note: If the homogeneous differential equation is in the form of $\frac{dy}{dx}$ = F(x, y), where F(x, y) is homogeneous function of degree zero, then we make substitution $\frac{x}{y}$ = v, i.e. x = vy and we proceed further to find the general solution as mentioned above.

Linear differential equation: General form of linear differential equation is
$\frac{dy}{dx} + Py = Q$ ...(i)
where, P and Q are functions of x or constants.
or $\frac{dx}{dy} + P'x = Q'$ ...(ii)
where, P' and Q' are functions of y or constants.
Then, solution of Eq. (i) is given by the equation
$y \times IF = \int (Q \times IF)\,dx + C$
where, IF = Integrating factor and IF = $e^{\int P dx}$
Also, solution of Eq. (ii) is given by the equation
$x \times IF = \int (Q' \times IF)\,dy + C$
where, IF = Integrating factor and IF = $e^{\int P' dy}$