CBSE Class 7 Maths Notes Chapter 2 Fractions and Decimals

The number of the $\frac{a}{b}$, where a and b are natural numbers is known as fraction.

e.g. $\frac{3}{5}$ is a fraction, where 3 is numerator and 5 is denominator.

A fraction whose numerator is less than its denominator is called a proper fraction.

e.g. $\frac{2}{9}, \frac{4}{5}, \frac{6}{11}$ etc.

A fraction whose numerator is more than or equal to its denominator is called an improper fraction.

e.g. $\frac{11}{3}, \frac{25}{12}, \frac{42}{23}, \frac{16}{16}$ etc.

A number which can be expressed as the sum of a natural number and a proper fraction is called a mixed fraction.

e.g. $1\frac{3}{4}, 4\frac{5}{7}, 7\frac{9}{13}, 12\frac{6}{25}$ etc.

Fractions having the same denominator but different numerators are called like fractions.

e.g. $\frac{5}{13}, \frac{6}{13}, \frac{7}{13}, \frac{8}{13}$, etc.

Fractions having different denominator are called, unlike fractions.

e.g. $\frac{3}{7}, \frac{5}{8}, \frac{11}{14}$, etc.

To multiply a whole number with a proper or an improper fraction, we multiply the whole number with the numerator of the fraction, keeping the denominator same.

e.g. $2 \times \frac{5}{3} = \frac{2 \times 5}{3} = \frac{10}{3}$

To multiply a mixed fraction to whole numbers, first convert the mixed fraction to an improper fraction and then multiply.

$$e.g. \ 3 \times 2\frac{5}{7} = 3 \times \frac{19}{7} = \frac{57}{7} = 8\frac{1}{7}$$

In $\frac{1}{2}$ of 2, $\frac{1}{2}$ of 3, $\frac{1}{5}$ of 20, etc, 'of' represents multiplication.

$$\text{Multiplication of two fractions} = \frac{\text{Product of Numerators}}{\text{Product of Denominators}}$$

$$e.g. \ \frac{3}{5} \times \frac{1}{7} = \frac{3 \times 1}{5 \times 7} = \frac{3}{35}$$

The value of the product of two proper fractions is smaller than each of the two fractions.

The value of the product of two improper fractions is more than each of the two fractions.

To divide a whole number by any fraction, multiply that whole number by the reciprocal of that fraction.

$$e.g. \ 5 \div \frac{2}{3} = 5 \times \frac{3}{2} = \frac{15}{2}$$

To divide two fractions, multiply the first fraction by the reciprocal of the second fraction.

$$e.g. \ \frac{2}{3} \div \frac{4}{6} = \frac{2}{3} \times \frac{6}{4} = \frac{12}{12} = 1$$

The non − zero numbers whose product with each other is 1, are called the reciprocals of that number.
e.g. Reciprocal of $\frac{5}{9}$ is $\frac{9}{5}$

While dividing a whole number by a mixed fraction, first convert the mixed fraction into an improper fraction and then solve.

$$e.g. \ 4 \div 2\frac{2}{5} = 4 \div \frac{12}{5} = 4 \times \frac{5}{12} = \frac{20}{12}$$

Place − value expansion of decimal number.
e.g. 253.417

Hundreds	Tens	Ones	Tenths	Hundredths	Thousandths	Number
2	5	3	4	1	7	253.417

When a decimal number is multiplied by 10, 100 or 1000, the digits in the product are same as in the decimal number but the decimal point in the product is shifted to the right by as, many of places as there are zeroes over one.

e.g. 0.07 × 10 = 0.7

0.07 × 100 = 7

0.07 × 1000 = 70

To multiply two decimal numbers, first, multiply them as whole numbers. Count the number of digits to the right of the decimal point in both the numbers. Add the number of digits counted. Put the decimal point in the product by counting the number of digits equal to the sum obtained from its rightmost place.

e.g. 1.2 × 2.43 = 2.916.

While dividing a number by 10,100 or 1000, the digits of the number and the quotient are same but the decimal point in the quotient shifts to the loft by as many places as there are zeroes over 1.

e.g. 2.38 ÷ 10 = 0.238

2.38 ÷ 100 = 0.0238

2.38 ÷ 1000 = 0.00238

While dividing one decimal number by another, first shift the decimal points to the right by equal number of places in both, to convert the divisor to a natural number and then divide.

e.g. $\frac{1.44}{1.2} = \frac{14.4}{12} = 1.2$

To divide a decimal number by a whole number, we first divide them as whole numbers. Then place the decimal point in the quotient as in the decimal number.

e.g. $\frac{8.4}{4} = 2.1$

Proper Fraction: A proper fraction is a number that represents a part of a whole.

For example: $\frac{4}{7}$ is a proper fraction. Here, 4 is the numerator and 7 is the denominator.

In a proper fraction, the denominator is bigger than the numerator.

More Examples: $\frac{2}{3}, \frac{3}{4}, \frac{5}{9}, \frac{6}{13}, \frac{7}{19}$.

Improper Fraction: An improper fraction is a combination of whole and a proper fraction.

For example: $\frac{7}{4}$ is an improper fraction. Here, 7 is the numerator and 4 is the denominator.

In an improper fraction, the numerator is bigger than the denominator.

More Examples: $\frac{11}{3}, \frac{19}{7}, \frac{21}{8}, \frac{35}{17}, \frac{41}{19}$.

Mixed Fraction: An improper fraction can be expressed as a mixed fraction by dividing the numerator by denominator to obtain the quotient and the remainder. Then, the mixed fraction will be written as

Quotient $\frac{\text{Remainder}}{\text{Divisor}}$. For example: The improper fraction $\frac{7}{4}$ can be converted to the mixed fraction $1\frac{3}{4}$.

More Examples: $1\frac{2}{3}, 3\frac{4}{7}, 5\frac{6}{11}, 7\frac{3}{4}, 2\frac{1}{2}$.

Multiplication of Fractions

There arise many situations when we have to multiply two fractions. For example: to calculate the area of a rectangle with given length and breadth.

Multiplication of a Fraction by a Whole Number

To multiply a whole number with a proper or an improper fraction, we multiply the whole number with the numerator of the fraction, keeping the denominator same.

Multiplication of a mixed fraction to a whole number: To multiply a mixed fraction by a whole number, we first convert the mixed fraction to an improper fraction and then multiply.

Fraction as an operator 'of' indicates multiplication.

Product of two whole numbers: The product of two whole numbers is bigger in size than each of the two whole numbers. For example, 3 x 4 = 12 and 12 > 4, 12 > 3.

Product of two proper fractions: When two proper fractions are multiplied, the product is less than both the fractions or, we say the size of the product of two proper fractions is smaller than the size of each of the two fractions.

Examples: We have,

$\frac{2}{3} \times \frac{4}{5} = \frac{8}{15}$	$\frac{8}{15} < \frac{2}{3}, \quad \frac{8}{15} < \frac{4}{5}$	Product is less than both the fractions
$\frac{1}{5} \times \frac{2}{7} = \frac{2}{35}$	$\frac{2}{35} < \frac{1}{5}, \quad \frac{2}{35} < \frac{2}{7}$	Product is less than both the fractions
$\frac{3}{5} \times \frac{7}{8} = \frac{21}{40}$	$\frac{21}{40} < \frac{3}{5}, \quad \frac{21}{40} < \frac{7}{8}$	Product is less than both the fractions
$\frac{2}{5} \times \frac{4}{9} = \frac{8}{45}$	$\frac{8}{45} < \frac{2}{5}, \quad \frac{8}{45} < \frac{4}{9}$	Product is less than both the fractions

Product of two Improper Fractions: The product of two improper fractions is greater than each of the two fractions. In this case, the size of the product is more than the size of each of the two fractions.

Examples: We have,

$\frac{7}{3} \times \frac{5}{2} = \frac{35}{6}$	$\frac{35}{6} > \frac{7}{3}, \quad \frac{35}{6} > \frac{5}{2}$	Product is greater than both the fractions
$\frac{6}{5} \times \frac{4}{3} = \frac{24}{15}$	$\frac{24}{15} > \frac{6}{5}, \quad \frac{24}{15} > \frac{4}{3}$	Product is greater than both the fractions
$\frac{9}{2} \times \frac{7}{4} = \frac{63}{8}$	$\frac{63}{8} > \frac{9}{2}, \quad \frac{63}{8} > \frac{7}{4}$	Product is greater than both the fractions
$\frac{3}{2} \times \frac{8}{7} = \frac{24}{14}$	$\frac{24}{14} > \frac{3}{2}, \quad \frac{24}{14} > \frac{8}{7}$	Product is greater than both the fractions
$\frac{7}{4} \times \frac{5}{3} = \frac{35}{12}$	$\frac{35}{12} > \frac{7}{4}, \quad \frac{35}{12} > \frac{5}{3}$	Product is greater than both the fractions
$\frac{6}{5} \times \frac{7}{3} = \frac{42}{15}$	$\frac{42}{15} > \frac{6}{5}, \quad \frac{42}{15} > \frac{7}{3}$	Product is greater than both the fractions
$\frac{9}{5} \times \frac{7}{4} = \frac{63}{20}$	$\frac{63}{20} > \frac{9}{5}, \quad \frac{63}{20} > \frac{7}{4}$	Product is greater than both the fractions
$\frac{3}{2} \times \frac{9}{5} = \frac{27}{10}$	$\frac{27}{10} > \frac{3}{2}, \quad \frac{27}{10} > \frac{9}{5}$	Product is greater than both the fractions
$\frac{5}{4} \times \frac{9}{2} = \frac{45}{8}$	$\frac{45}{8} > \frac{5}{4}, \quad \frac{45}{8} > \frac{9}{2}$	Product is greater than both the fractions

Division of Fractions

There arise situations when we have to divide a whole number by a fraction or a fraction by another fraction. For example: to cut a paper strip of given length into smaller strips of a certain length.

Division of the Whole Number by a Fraction

Reciprocal of a fraction

Observe these products:

$$7 \times \frac{1}{7} = 1 \qquad\qquad \frac{5}{4} \times \frac{4}{5} = 1$$

$$\frac{1}{9} \times 9 = 1 \qquad\qquad \frac{2}{7} \times \frac{7}{2} = 1$$

$$\frac{2}{3} \times \frac{3}{2} = \frac{2 \times 3}{3 \times 2} = \frac{6}{6} = 1 \qquad\qquad \frac{9}{5} \times \frac{5}{9} = 1$$

To divide a whole number by a fraction

To divide a whole number by any fraction, we multiply that whole number by the reciprocal of that fraction.

Thus,

$$1 \div \frac{1}{2} = 1 \times \frac{2}{1} = 1 \times \text{reciprocal of } \frac{1}{2}$$

$$3 \div \frac{1}{2} = 3 \times 2 = 3 \times \textbf{reciprocal of } \frac{1}{2}$$

So, $\qquad 2 \div \frac{3}{4} = 2 \times \text{reciprocal of } \frac{3}{4} = 2 \times \frac{4}{3}$

To divide a whole number by a mixed fraction

To divide a whole number by a mixed fraction, we first convert the mixed fraction into an improper fraction and then perform the division. Thus,

$$4 \div 2\frac{2}{5} = 4 \div \frac{12}{5} = 4 \times \frac{5}{12} = \frac{4 \times 5}{12} = \frac{20}{12} = \frac{20 \div 4}{12 \div 4} = \frac{5}{3}.$$

Division of a Fraction by a Whole Number

To divide a fraction by a whole number, we multiply that fraction by the reciprocal of that whole number. Thus,

$$\frac{3}{4} \div 3 = \frac{3}{4} \div \frac{3}{1} = \frac{3}{4} \times \frac{1}{3} = \frac{3 \times 1}{4 \times 3}$$

$$= \frac{3}{12} = \frac{3 \div 3}{12 \div 3} = \frac{1}{4}$$

$$\frac{2}{3} \div 7 = \frac{2}{3} \div \frac{7}{1} = \frac{2}{3} \times \frac{1}{7}$$

$$= \frac{2 \times 1}{3 \times 7} = \frac{2}{21}$$

$$\frac{5}{7} \div 6 = \frac{5}{7} \div \frac{6}{1} = \frac{5}{7} \times \frac{1}{6}$$

$$= \frac{5 \times 1}{7 \times 6} = \frac{5}{42}$$

$$\frac{2}{7} \div 8 = \frac{2}{7} \div \frac{8}{1} = \frac{2}{7} \times \frac{1}{8} = \frac{2 \times 1}{7 \times 8}$$

$$= \frac{2}{56} = \frac{2 \div 2}{56 \div 2} = \frac{1}{28}$$

To divide a mixed fraction by a whole number

While dividing mixed fractions by whole numbers, we convert the mixed fractions into improper fractions.

Thus,

$$2\frac{3}{2} \div 5 = \frac{7}{2} \div 5 = \frac{7}{2} \div \frac{5}{1}$$

$$= \frac{7}{2} \times \frac{1}{5} = \frac{7 \times 1}{2 \times 5} = \frac{7}{10}$$

$$4\frac{2}{5} \div 3 = \frac{22}{5} \div 3 = \frac{22}{5} \div \frac{3}{1} = \frac{22}{5} \times \frac{1}{3}$$

$$= \frac{22 \times 1}{5 \times 3} = \frac{22}{15} = 1\frac{7}{15}$$

$$2\frac{3}{5} \div 2 = \frac{13}{5} \div 2 = \frac{13}{5} \div \frac{2}{1} = \frac{13}{5} \times \frac{1}{2}$$

$$= \frac{13 \times 1}{5 \times 2} = \frac{13}{10} = 1\frac{3}{10}$$

Division of a Fraction by Another Fraction

$$\frac{1}{3} \div \frac{5}{6} = \frac{1}{3} \times \text{reciprocal of } \frac{5}{6} = \frac{1}{3} \times \frac{6}{5} = \frac{1 \times 6}{3 \times 5} = \frac{6}{15} = \frac{6 \div 3}{15 \div 3} = \frac{2}{5}$$

$$\frac{1}{2} \div \frac{3}{4} = \frac{1}{2} \times \text{reciprocal of } \frac{3}{4} = \frac{1}{2} \times \frac{4}{3} = \frac{1 \times 4}{2 \times 3} = \frac{4}{6} = \frac{4 \div 2}{6 \div 2} = \frac{2}{3}$$

So, to divide a fraction (dividend) by a fraction (divisor), we multiply the fraction (dividend) by the reciprocal of the fraction (divisor).

Decimal number and its expanded form

Hundreds (100)	Tens (10)	Ones (1)	Tenths $\left(\frac{1}{10}\right)$	Hundredths $\left(\frac{1}{100}\right)$	Thousandths $\left(\frac{1}{1000}\right)$	Number
2	5	3	1	4	7	253.147
6	2	9	3	2	1	**629.321**
0	4	3	1	9	2	**43.192**
5	1	4	2	5	1	514.251
2	3	6	5	1	2	236.512
7	2	4	5	0	3	724.503
6	1	4	3	2	6	614.326
0	1	0	5	3	0	**10.53**

Using this table, we can write a decimal number in its expanded form also. For example:

$$253.147 = 2 \times 100 + 5 \times 10 + 3 \times 1 + 1 \times \left(\frac{1}{10}\right) + 4 \times \left(\frac{1}{100}\right) + 7 \times \left(\frac{1}{1000}\right)$$

$$629.321 = 6 \times 100 + 2 \times 10 + 9 \times 1 + 3 \times \left(\frac{1}{10}\right) + 2 \times \left(\frac{1}{100}\right) + 1 \times \left(\frac{1}{1000}\right)$$

$$43.192 = 4 \times 10 + 3 \times 1 + 1 \times \left(\frac{1}{10}\right) + 9 \times \left(\frac{1}{100}\right) + 2 \times \left(\frac{1}{1000}\right)$$

$$514.251 = 5 \times 100 + 1 \times 10 + 4 \times 1 + 2 \times \left(\frac{1}{10}\right) + 5 \times \left(\frac{1}{100}\right) + 1 \times \left(\frac{1}{1000}\right)$$

$$236.512 = 2 \times 100 + 3 \times 10 + 6 \times 1 + 5 \times \left(\frac{1}{10}\right) + 1 \times \left(\frac{1}{100}\right) + 2 \times \left(\frac{1}{1000}\right)$$

$$724.503 = 7 \times 100 + 2 \times 10 + 4 \times 1 + 5 \times \left(\frac{1}{10}\right) + 0 \times \left(\frac{1}{100}\right) + 3 \times \left(\frac{1}{1000}\right)$$

$$614.326 = 6 \times 100 + 1 \times 10 + 4 \times 1 + 3 \times \left(\frac{1}{10}\right) + 2 \times \left(\frac{1}{100}\right) + 6 \times \left(\frac{1}{1000}\right)$$

$$10.53 = 1 \times 10 + 0 \times 1 + 5 \times \left(\frac{1}{10}\right) + 3 \times \left(\frac{1}{100}\right)$$

So, a decimal number and its expanded form can be written if the place values of the digits are known to us.

Comparison of decimal numbers: To compare the decimal numbers, we first compare the digits on the left of the decimal point, starting from the leftmost digit.
If all the digits on the left of the decimal point are exactly the same, then we compare the digits on the right of the decimal point starting from the tenths place.
If the digits at the tenths place are also the same, then we compare the digits at the hundredths place and so on.
Example:
John has ₹ 15.50 and Salma has ₹ 15.75.
To find who has more money, we shall compare the two decimal numbers 15.50 and 15.75.
Here, both the digits 1 and 5, on the left of the decimal point are the same.
Also, comparing digits at the tenths place, we find that 5 < 7. So, Salma has more money than John.

Need of decimals: While converting lower units of money, length, and weight, to their higher units, we are required to use decimals. For example,
3 paise = ₹ 0.03
5 g = 0.005 kg
7 cm = 0.07 m.

Addition and subtraction of decimals: We are already familiar with how to add and subtract decimals.

Thus, 21.36 + 37.35 is 58.71

Multiplication of Decimal Numbers by 10, 100 and 1000

(i) Decimal numbers as a fraction with denominator 10 or 100 or 1000, etc.

We observe that,

2.3 = $\frac{23}{10}$ whereas 2.35 = $\frac{235}{100}$ Thus, depending upon the position of the decimal point, the decimal number can be converted to a fraction with denominator 10 or 100 or 1000, etc.

(ii) Multiplication of a decimal by 10 or 100 or 1000

Let us look at the following table:

1.7 × 10 = 17.0 or 17

1.7 × 100 = 170.0 or 170

1.7 × 1000 = 1700.0 or 1700

2.35 × 10 = 23.5

2.35 × 100 = 235.0 or 235

2.35 × 1000 = 2350.0 or 2350

We observe that in 1.7 × 10 = 17.

The digits in the number 1.7 and product 17 are same,

i.e., 1 and 7 and 0 is decimal in 17,

i.e., 17.0 is placed one digit to the right as compared to its placement in 17.

Note that 10 has one zero.

The observation is the same in the other products involving 10, in the table.

In 1.7 × 100 = 170, the digits in 1.7 (i.e., 1.70) and 170 are same,

i.e., 1, 7 and 0.

The decimal in the product 170 (i.e., 170.0) is placed two digits to the right as compared so its position in 1.70.

Note that 100 has two zeros.

The observation is the same in the other products involving 100 in the table.

In 1.7 × 1000 = 1700, the digits in 1.7 (i.e., 1.700) and 1700 are same,

i.e., 1, 7 and 0.

The decimal in the product 1700 (i.e., 1700.0) is placed three digits to the right as compared to its position in 1.700.

Note that 1000 has three zeros.

The observation is the same in the other products involving 1000 in the table.

Result: When we multiply a number by 10 or 100 or 1000, we get a product in which the digits are the same as in the number itself, but the decimal is shifted to the right by one digit (for multiplication by 10), by two digits (for multiplication by 100) and by three digits (for multiplication by 1000).

Thus, 1.92 × 10 = 19.2

1.92 × 100 = 192

1.92 × 1000 − 1.920 × 1000 = 1920.