CBSE Class 12 Maths Notes Chapter 13 Probability

Event: A subset of the sample space associated with a random experiment is called an event or a case.
e.g. In tossing a coin, getting either head or tail is an event.

Equally Likely Events: The given events are said to be equally likely if none of them is expected to occur in preference to the other.
e.g. In throwing an unbiased die, all the six faces are equally likely to come.

Mutually Exclusive Events: A set of events is said to be mutually exclusive, if the happening of one excludes the happening of the other, i.e. if A and B are mutually exclusive, then (A ∩ B) = Φ
e.g. In throwing a die, all the 6 faces numbered 1 to 6 are mutually exclusive, since if any one of these faces comes, then the possibility of others in the same trial is ruled out.

Exhaustive Events: A set of events is said to be exhaustive if the performance of the experiment always results in the occurrence of at least one of them.
If $E_1, E_2, ..., E_n$ are exhaustive events, then $E_1 ∪ E_2 ∪ ...... ∪ E_n$ = S.
e.g. In throwing of two dice, the exhaustive number of cases is 6^2 = 36. Since any of the numbers 1 to 6 on the first die can be associated with any of the 6 numbers on the other die.

Complement of an Event: Let A be an event in a sample space S, then the complement of A is the set of all sample points of the space other than the sample point in A and it is denoted by A'or $\bar{A}$.
i.e. A' = {n : n   S, n   A]

Note:
(i) An operation which results in some well-defined outcomes is called an experiment.
(ii) An experiment in which the outcomes may not be the same even if the experiment is performed in an identical condition is called a random experiment.

Probability of an Event

If a trial result is n exhaustive, mutually exclusive and equally likely cases and m of them are favourable to the happening of an event A, then the probability of happening of A is given by

$$P(A) = \frac{\text{Number of favourable cases to } A}{\text{Number of exhaustive cases}} = \frac{n(A)}{n(S)} = \frac{m}{n}$$

Note:

(i) $0 \le P(A) \le 1$

(ii) Probability of an impossible event is zero.

(iii) Probability of certain event (possible event) is 1.

(iv) $P(A \cup A') = P(S)$

(v) $P(A \cap A') = P(\Phi)$

(vi) $P(A')' = P(A)$

(vii) $P(A \cup B) = P(A) + P(B) - P(A \cap S)$

Conditional Probability: Let E and F be two events associated with the same sample space of a random experiment. Then, probability of occurrence of event E, when the event F has already occurred, is called a conditional probability of event E over F and is denoted by P(E/F).

Similarly, conditional probability of event F over E is given as

$$P(F/E) = \frac{P(F \cap E)}{P(E)}, \text{ where } P(E) \ne 0$$

Properties of Conditional Probability: If E and E are two events of sample space S and G is an event of S which has already occurred such that $P(G) \ne 0$, then

(i) $P[(E \cup F)/G] = P(F/G) + P(F/G) - P[(F \cap F)/G], P(G) \ne 0$

(ii) $P[(E \cup F)/G] = P(F/G) + P(F/G)$, if E and F are disjoint events.

(iii) $P(F'/G) = 1 - P(F/G)$

(iv) $P(S/E) = P(E/E) = 1$

Multiplication Theorem: If E and F are two events associated with a sample space S, then the probability of simultaneous occurrence of the events E and F is

$P(E \cap F) = P(E) \cdot P(F/E)$, where $P(F) \ne 0$

or

$P(E \cap F) = P(F) \cdot P(F/F)$, where $P(F) \ne 0$

This result is known as multiplication rule of probability.

Multiplication Theorem for More than Two Events: If F, F and G are three events of sample space, then

$$P(E \cap F \cap G) = P(E) \cdot P\left(\frac{F}{E}\right) \cdot P\left(\frac{G}{E \cap F}\right)$$

Independent Events: Two events E and F are said to be independent, if probability of occurrence or non-occurrence of one of the events is not affected by that of the other. For any two independent events E and F, we have the relation

(i) P(E ∩ F) = P(F) . P(F)

(ii) P(F/F) = P(F), P(F) ≠ 0

(iii) P(F/F) = P(F), P(F) ≠ 0

Also, their complements are independent events,

i.e. $P(\bar{E} \cap \bar{F}) = P(\bar{E}) . P(\bar{F})$

Note: If E and F are dependent events, then P(E ∩ F) ≠ P(F) . P(F).

Three events E, F and G are said to be mutually independent, if

(i) P(E ∩ F) = P(E) . P(F)

(ii) P(F ∩ G) = P(F) . P(G)

(iii) P(E ∩ G) = P(E) . P(G)

(iv) P(E ∩ F ∩ G) = P(E) . P(F) . P(G)

If atleast one of the above is not true for three given events, then we say that the events are not independent.

Note: Independent and mutually exclusive events do not have the same meaning.

Baye's Theorem and Probability Distributions

Partition of Sample Space: A set of events E_1, E_2,...,E_n is said to represent a partition of the sample space S, if it satisfies the following conditions:

(i) $E_i \cap E_j = \Phi$; i ≠ j; i, j = 1, 2, ... n

(ii) $E_1 \cup E_2 \cup ...... \cup E_n = S$

(iii) $P(E_i) > 0$,   i = 1, 2,..., n

Theorem of Total Probability: Let events E_1, E_2, ..., E_n form a partition of the sample space S of an experiment. If A is any event associated with sample space S, then

$$P(A) = P(E_1) \cdot P(A/E_1) + P(E_2) \cdot P(A/E_2) + ... + P(E_n) \cdot P(A/E_n) = \sum_{j=1}^{n} P(E_j) \cdot P(A/E_j)$$

Baye's Theorem: If E_1, E_2,...,E_n are n non-empty events which constitute a partition of sample space S, i.e. E_1, E_2,..., E_n are pairwise disjoint $E_1 \cup E_2 \cup .... \cup E_n = S$ and $P(E_i) > 0$, for all i = 1, 2, ... n Also, let A be any non-zero event, the probability

$$P(E_i/A) = \frac{P(E_i) \cdot P(A/E_i)}{\sum_{i=1}^{n} P(E_i) \cdot P(A/E_i)} , \forall \ i = 1, 2, 3, ..., n$$

Random Variable: A random variable is a real-valued function, whose domain is the sample space of a random experiment. Generally, it is denoted by capital letter X.

Note: More than one random variables can be defined in the same sample space.

Probability Distributions: The system in which the values of a random variable are given along with their corresponding probabilities is called probability distribution.

Let X be a random variable which can take n values $x_1, x_2,..., x_n$.

Let $p_1, p_2,..., p_n$ be the respective probabilities.

Then, a probability distribution table is given as follows:

X	x_1	x_2	x_3	...	x_n
P	p_1	p_2	p_3	...	p_n

such that $P_1 + p_2 + P_3 +... + p_n = 1$

Note: If x_i is one of the possible values of a random variable X, then statement $X = x_i$ is true only at some point(s) of the sample space. Hence ,the probability that X takes value x, is always non-zero, i.e. $P(X = x_i) \neq 0$

Mean and Variance of a Probability Distribution: Mean of a probability distribution is

$\sum_{i=1}^{n} x_i \cdot p_i$. It is also called expectation of X, i.e. $E(X) = \mu = \sum_{i=1}^{n} x_i p_i$.

Variance is given by $V(X)$ or σ_x^2 and defined as $\sigma^2 = \sum_{i=1}^{n} x_i^2 \cdot p(x_i) - \left(\sum_{i=1}^{n} x_i \cdot p(x_i) \right)^2$

or $\qquad\qquad\qquad\qquad E(X^2) - [E(X)]^2$

or $\qquad\qquad$ Equivalently $V(X) = E(X - \mu)^2$ or $\sum_{i=1}^{n} (x_i - \mu)^2 p(x_i)$

Standard deviation is given by, $\sigma_x = \sqrt{v(x)} = \sqrt{\sum_{i=1}^{n} (x_i - \mu)^2 p(x_i)}$

Bernoulli Trial: Trials of a random experiment are called Bernoulli trials if they satisfy the following conditions:

(i) There should be a finite number of trials.

(ii) The trials should be independent.

(iii) Each trial has exactly two outcomes, success or failure.

(iv) The probability of success remains the same in each trial.

Binomial Distribution: The probability distribution of numbers of successes in an experiment consisting of n Bernoulli trials obtained by the binomial expansion (p + q)n, is called binomial distribution.

Let X be a random variable which can take n values x_1, x_2,..., x_n. Then, by binomial distribution, we have $P(X = r) = {}^nC_r p^r q^{n-r}$

where,

n = Total number of trials in an experiment

p = Probability of success in one trial

q = Probability of failure in one trial

r = Number of success trial in an experiment

Also, p + q = 1

Binomial distribution of the number of successes X can be represented as

X	0	1	2	... r...	n
$P(X)$	${}^nC_0 p^0 q^n$	${}^nC_1 p^1 q^{n-1}$	${}^nC_2 p^2 q^{n-2}$	${}^nC_r p^r q^{n-r}$	${}^nC_n p^n$

Mean and Variance of Binomial Distribution

(i) Mean(μ) = $\Sigma x_i p_i$ = np

(ii) Variance(σ^2) = $\Sigma x_i^2 p_i - \mu^2$ = npq

(iii) Standard deviation (σ) = $\sqrt{\text{Variance}}$ = $\sqrt{npq}$

Note: Mean > Variance