

CBSE Class 8 Maths Notes Chapter 1 Rational Numbers

Rational Number: A number is called rational if we can write the number in the form of $\frac{p}{q}$, where p and q are integers and $q \neq 0$

i.e., $1 = \frac{1}{1}$, $2 = \frac{2}{1}$, $0 = \frac{0}{1}$ and $\frac{5}{8}$, $\frac{-3}{14}$, $\frac{7}{-15}$ are all rational numbers.

Between two rational numbers x and y, there exists a rational number $\frac{x+y}{2}$

We can find countless **rational numbers** between two rational numbers.

$\frac{-x}{y}$ is called the additive inverse of $\frac{x}{y}$ and vice-versa.

$\frac{y}{x}$ is called the multiplicative inverse or reciprocal of $\frac{x}{y}$.

Rational number 0 is the additive identity for all rational numbers because a number does not change when 0 is added to it.

Rational number 1 is the multiplicative identity for all rational numbers because on multiplying a rational number with 1, its value does not change.

Rational numbers can be represented on a number line.

Properties on Rational Numbers

(i) Closure Property

Rational numbers are closed under :

Addition

$$\text{eg. } \frac{3}{5} + \frac{(-4)}{9} = \frac{27 - 20}{45} = \frac{7}{45}$$

which is a rational number.

Subtraction

$$\frac{5}{8} - \frac{3}{7} = \frac{35 - 24}{56} = \frac{11}{56}$$

$$\frac{2}{5} - \frac{3}{4} = \frac{8 - 15}{20} = \frac{-7}{20}$$

are rational numbers.

Multiplication:

$$\frac{-5}{7} \times \frac{2}{9} = \frac{-10}{63}$$

$$\frac{2}{3} \times \frac{5}{11} = \frac{10}{33}$$

are rational numbers.

Rational numbers are closed under addition subtraction and multiplication.

Division : eq. $\frac{-3}{5} \div \frac{2}{3} = \frac{-9}{10}$, which is also a rational number. For any rational number a, $a \div 0$ is not defined. So, rational number are not closed under division.

However, if we exclude zero then the rational numbers are closed under division.

(ii) Commutativity:

Addition: Two rational numbers can be added in any order, i.e., commutativity holds for rational numbers

under addition, i.e., for any two rational number a and b, $a + b = b + a$.

$$\frac{-3}{4} + \frac{5}{11} = \frac{-13}{44}$$

$$\frac{5}{11} + \left(\frac{-3}{4}\right) = -\frac{13}{44}$$

Subtraction:

$$\frac{2}{5} - \frac{5}{6} = \frac{12 - 25}{30} = \frac{-13}{30}$$

$$\frac{5}{6} - \frac{2}{5} = \frac{25 - 12}{30} = \frac{13}{30}$$

Hence, subtraction is not associative for rational numbers.

(iii) Multiplication: Multiplication is commutative for rational numbers. In general, $a \times b = b \times a$, for any two rational numbers a and b .

$$\frac{-3}{4} \times \frac{5}{6} = \frac{5}{6} \times \left(\frac{-3}{4}\right) = \frac{-15}{24}$$

Division:

$$\frac{-3}{7} \div \frac{2}{5} = \frac{-3}{7} \times \frac{5}{2} = \frac{-15}{14}$$

$$\frac{2}{5} \div \left(\frac{-3}{7}\right) = \frac{2}{5} \times \frac{7}{-3} = \frac{14}{-15}$$

$$\frac{-3}{7} \div \frac{2}{5} \neq \frac{2}{5} \div \left(\frac{-3}{7}\right)$$

Hence, division is not Cumulative for rational numbers.

(iii) Associativity:

Addition:

$$\text{eg. } \frac{-2}{5} + \left[\frac{3}{4} + \left(\frac{-7}{8}\right)\right] = \frac{-2}{5} + \left(\frac{-1}{8}\right) = \frac{-21}{40} = \left[\frac{-2}{5} + \frac{3}{4}\right] + \left(\frac{-7}{8}\right) = \frac{7}{20} - \frac{7}{8} = \frac{-21}{40}$$

So, addition is associative for rational numbers, i.e., for any three rational numbers a , b and c , $a + (b + c) = (a + b) + c$.

Subtraction:

$$\text{eg. } \frac{-3}{4} - \left[\frac{-5}{6} - \frac{2}{3}\right] = \frac{-3}{4} - \left(\frac{-9}{6}\right) = \left(\frac{-9}{6}\right) = \frac{9}{12} = \frac{3}{4}$$

and

$$\left[\frac{-3}{4} - \left(\frac{-5}{6}\right)\right] - \frac{2}{3} = \frac{1}{12} - \frac{2}{3} = \frac{-7}{12}$$

$$\text{i.e., } \frac{3}{4} \neq \frac{-7}{12}.$$

Hence, subtraction is not associative for rational numbers.

Multiplication:

$$\text{eg. } \frac{-2}{3} \times \left(\frac{2}{5} \times \frac{6}{7}\right) = \frac{-2}{3} \times \frac{12}{35} = \frac{-24}{105} = \frac{-8}{35}$$

and

$$\left(\frac{-2}{3} \times \frac{2}{5}\right) \times \frac{6}{7} = \frac{-4}{15} \times \frac{6}{7} = \frac{-24}{105} = \frac{-8}{35}$$

So, multiplication is associative for rational number, i.e., for any three rational numbers a, b and c, $a \times (b \times c) = (a \times b) \times c$.

Division:

eg.
$$\frac{2}{3} \div \left[\frac{-1}{2} \div \frac{2}{5} \right] = \left[\frac{2}{3} \div \left(\frac{-1}{2} \right) \right] \div \frac{2}{5}$$

We have, LHS
$$\frac{2}{3} \div \left[\frac{-1}{2} \div \frac{2}{5} \right] = \left[\frac{2}{3} \div \frac{1}{2} \times \frac{5}{2} \right]$$
$$= \frac{2}{3} \div \left(\frac{-5}{4} \right) = \frac{2}{3} \times \frac{4}{-5} = \frac{8}{-15}$$

RHS
$$\left[\frac{2}{3} \div \left(\frac{-1}{2} \right) \right] \div \frac{2}{5} = \left[\frac{2}{3} \times \left(\frac{-2}{1} \right) \right] \div \frac{2}{5} = \left(\frac{-4}{3} \right) \div \frac{2}{5}$$
$$= \frac{-4}{3} \times \frac{5}{2} = \frac{-20}{6} = \frac{-10}{3}$$

But **LHS \neq RHS**

Hence, the division is not associative for rational numbers.

Distributivity of multiplication over addition for rational number : For all rational numbers a, b and c, $a(b + c) = ab + ac$

eg. Let $\frac{-2}{5}, \frac{3}{8}$ and $\frac{-3}{4}$ are any three rational numbers, then

$$\frac{-2}{5} \times \left\{ \frac{3}{8} + \left(\frac{-3}{4} \right) \right\} = \frac{-2}{5} \times \left\{ \frac{3 + (-6)}{8} \right\}$$
$$= \frac{-2}{5} \times \left(\frac{-3}{8} \right) = \frac{6}{40} = \frac{3}{20}$$

Also,
$$\frac{-2}{5} \times \frac{3}{8} = \frac{-6}{40} = \frac{-3}{20}$$

and
$$\frac{-2}{5} \times \frac{-3}{4} = \frac{6}{20}$$

Therefore,
$$\left(\frac{-2}{5} \times \frac{3}{8} \right) + \left(\frac{-2}{5} \times \frac{-3}{4} \right) = \frac{-3}{20} + \frac{6}{20} = \frac{3}{20}$$

Thus,
$$\frac{2}{5} \times \left\{ \frac{3}{8} + \left(\frac{-3}{4} \right) \right\} = \left(\frac{-2}{5} \times \frac{3}{8} \right) + \left\{ \frac{-2}{5} \times \left(\frac{-3}{4} \right) \right\}$$

Distributivity of multiplication over subtraction for rational number: For any three rational numbers a, b and c, $a(b - c) = ab - ac$

eg. Let $\frac{1}{2}$, $\frac{-2}{5}$ and $\frac{-3}{10}$ are any three rational numbers, then

$$\begin{aligned} \frac{1}{2} \times \left\{ \frac{-2}{5} - \left(\frac{-3}{10} \right) \right\} &= \frac{1}{2} \times \left\{ \frac{-4 - (-3)}{10} \right\} = \frac{1}{2} \times \left(\frac{-4 + 3}{10} \right) \\ &= \frac{1}{2} \times \left(\frac{-1}{10} \right) = \frac{-1}{20} \end{aligned}$$

Also, $\frac{1}{2} \times \left(\frac{-2}{5} \right) = \frac{-2}{10} = -\frac{1}{5}$

and $\frac{1}{2} \times \left(\frac{-3}{10} \right) = \frac{-3}{10}$

Therefore,
$$\begin{aligned} \left(\frac{1}{2} \times \frac{-2}{5} \right) - \left(\frac{1}{2} \times \frac{-3}{10} \right) &= \frac{-1}{5} - \left(\frac{-3}{10} \right) \\ &= \frac{-1}{5} + \frac{3}{10} \\ &= \frac{-4 + 3}{10} = \frac{-1}{10} \end{aligned}$$

Thus,
$$\frac{1}{2} \times \left\{ \frac{-2}{5} - \left(\frac{-3}{10} \right) \right\} = \left(\frac{1}{2} \times \frac{-2}{5} \right) - \left(\frac{1}{2} \times \frac{-3}{10} \right)$$

The numbers 1, 2, 3, 4, are called natural numbers.

If we add 0 to the collection of natural numbers, what we get is called the collection of whole numbers.

Thus, 0, 1, 2, 3, 4, are whole numbers.

Natural numbers are also known as positive integers. If we put a negative sign before each positive integer, we get negative integers. Thus, -1, -2,

-3, -4, are negative integers. A number of the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$ is called a rational number. All the above types of numbers are needed to solve various types of simple algebraic equations.

Properties of Rational Numbers

The list of properties of rational numbers can be given as follows:

- Closure
- Commutativity
- Associativity
- The role of zero (0)
- The role of 1
- Negative of a number
- Reciprocal
- Distributivity of multiplication over addition for rational numbers.

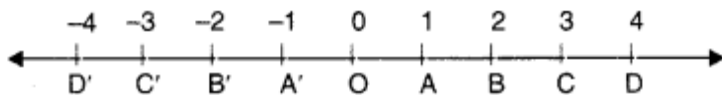
Distributivity of Multiplication Over Addition for Rational Numbers

For all rational numbers a , b and c ,

$$a(b + c) = ab + ac$$

$$a(b - c) = ab - ac.$$

Representation of Rational Numbers on the Number Line



- We draw a line.
- We mark a point O on it and name it 0 . Mark a point to the right of 0 . Name it 1 . The distance between these two points is called unit distance.
- Mark a point to the right of 1 at unit distance and name it 2 .
- Proceeding in this manner, we can mark points $3, 4, 5,$
- Similarly we can mark $-1, -2, -3, -4, -5, \dots$ to the left of 0 . This line is called the number line.
- This line extends indefinitely on both sides.

The positive rational numbers are represented by points on the number line to the right of O whereas the negative rational numbers are represented by points on the number line to the left of O .

Any rational number can be represented on this line. The denominator of the rational number indicates the number of equal parts into which the first unit has been divided whereas the numerator indicates as to how many of these parts are to be taken into consideration.