

1. Limits
2. Derivatives
3. Miscellaneous Questions

Meaning of $x \rightarrow a$ or " x tends to a " or " x approaches a ", x is a variable. The expected value of the function as dictated by the points to the left of a point defines the left hand limit of the function at that point. Similarly the righthand limit. It can be changed so that its value comes nearer and nearer to a . $0 < |x - a| < \delta$

(i) $x \neq a$, (ii) $|x - a|$ becomes smaller and smaller as we please.

Neighbourhood: The set of all real numbers lying between $a - \delta$ and $a + \delta$ is called the neighbourhood of a . Neighbourhood of $a = (a - \delta, a + \delta)$, $x \in (a - \delta, a + \delta)$

- **Limit** of a function at a point is the common value of the left and right hand limits, if they coincide

Left hand limit of f at $x = a$. When x approaches a from left hand side of a , the function $f(x)$ tends to l "a definite number". This definite number l is said to be the left hand limit of f at $x = a$.

Right hand limit of f at $x = a$. When x approaches a from right hand side of a , the function $f(x)$ tends to l "a definite number". This definite number l is said to be the right hand limit of f at $x = a$.

Therefore, if Left hand limit of f at $x = a =$ Right hand limit of f at $x = a$, then the limit of $f(x)$ at $x = a$ exists.

- For function f and a real number a , $\lim_{x \rightarrow \infty} f(x)$ and $f(a)$ may not be same (Infact, one may be defined and not the other one).
- For functions f and g the following holds:.

$$\lim_{x \rightarrow a} [f(x) \pm g(x)] = \lim_{x \rightarrow a} f(x) \pm \lim_{x \rightarrow a} g(x)$$

$$\lim_{x \rightarrow a} [f(x) \cdot g(x)] = \lim_{x \rightarrow a} f(x) \cdot \lim_{x \rightarrow a} g(x)$$

$$\lim_{x \rightarrow a} \left[\frac{f(x)}{g(x)} \right] = \frac{\lim_{x \rightarrow a} f(x)}{\lim_{x \rightarrow a} g(x)}$$

Following are some of the standard limits

$$\lim_{x \rightarrow a} \frac{x^n - a^n}{x - a} = na^{n-1}$$

$$\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1, \lim_{x \rightarrow a} \frac{\sin(x-a)}{x-a} = 1$$

$$\lim_{x \rightarrow 0} \frac{1 - \cos x}{x} = 0$$

$$\lim_{x \rightarrow 0} \frac{\tan x}{x} = 1, \lim_{x \rightarrow a} \frac{\tan(x-a)}{x-a} = 1$$

$$\lim_{x \rightarrow 0} \frac{\sin^{-1} x}{x} = 1, \lim_{x \rightarrow 0} \frac{\tan^{-1} x}{x} = 1$$

$$\lim_{x \rightarrow 0} \frac{a^x - 1}{x} = \log_e a, a > 0, a \neq 1$$

Derivatives

The derivative of a function f at a is defined by

$$f'(a) = \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}$$

Derivative of a function f at any point x is defined by

$$f'(x) = \frac{df(x)}{dx} = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

For functions u and v the following holds:

$$(u \pm v)' = u' \pm v'$$

$$(uv)' = u'v + uv' \quad \Rightarrow \quad \frac{d}{dx} (uv) = u \cdot \frac{dv}{dx} + v \cdot \frac{du}{dx}$$

$$\left(\frac{u}{v} \right)' = \frac{u'v - uv'}{v^2} \quad \Rightarrow \quad \frac{d}{dx} \left(\frac{u}{v} \right) = \frac{v \cdot \frac{du}{dx} - u \cdot \frac{dv}{dx}}{v^2}$$

provided all are defined.

Following are some of the standard derivatives

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

$$\frac{d}{dx}(\sin x) = \cos x$$

$$\frac{d}{dx}(\cos x) = -\sin x$$

$$\frac{d}{dx}(\tan x) = \sec^2 x$$

$$\frac{d}{dx}(\cot x) = -\operatorname{cosec}^2 x$$

$$\frac{d}{dx}(\sec x) = \sec x \cdot \tan x$$

$$\frac{d}{dx}(\operatorname{cosec} x) = -\operatorname{cosec} x \cdot \cot x$$