

CBSE Class 11 Mathematics
Revision Notes
Chapter - 3
TRIGONOMETRIC FUNCTIONS

1. **Angles**
2. **Trigonometric Functions**
3. **Sum and Difference of Two Angles**
4. **Trigonometric Equations**

- **Measurement of an angle:** The measure of an angle is the amount of rotation from the initial side to the terminal side.
- **Right angle:** If the rotating ray starting from its initial position to final position, describes one quarter of a circle, then we say that the measure of the angle formed is a right angle.
- If in a circle of radius r , an arc of length l subtends an angle of θ radians, then $l = r\theta$
- Radian measure = $\frac{\pi}{180} \times \text{Degree measure}$
- Degree measure = $\frac{180}{\pi} \times \text{Radian measure}$

Trigonometric Functions

- **Quadrant:**

t -ratios	I	II	III	IV
$\sin \theta = y$	+	+	-	-
$\cos \theta = x$	+	-	-	+
$\tan \theta = \frac{y}{x}$	+	-	+	-

- $\operatorname{cosec} \theta = \frac{1}{\sin \theta}$, $\sec \theta = \frac{1}{\cos \theta}$, $\cot \theta = \frac{1}{\tan \theta}$
- **Trigonometric values of some angles:**

	0°	30°	45°	60°	90°	180°
$\sin \theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	0

$\cos \theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	-1
$\tan \theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞	0

Trigonometric Identities:

- $\cos^2 x + \sin^2 x = 1$
- $1 + \tan^2 x = \sec^2 x$
- $1 + \cot^2 x = \operatorname{cosec}^2 x$

Trigonometric ratio of $(90^\circ + x)$ in terms of x :

- $\cos\left(\frac{\pi}{2} + x\right) = -\sin x$
- $\sin\left(\frac{\pi}{2} + x\right) = \cos x$
- $\tan\left(\frac{\pi}{2} + x\right) = -\cot x$

Trigonometric ratio of $(90^\circ - x)$ in terms of x :

- $\cos\left(\frac{\pi}{2} - x\right) = \sin x$
- $\sin\left(\frac{\pi}{2} - x\right) = \cos x$
- $\tan\left(\frac{\pi}{2} - x\right) = \cot x$

Trigonometric ratio of $(180^\circ - x)$ in terms of x :

- $\cos(\pi - x) = -\cos x$
- $\sin(\pi - x) = \sin x$
- $\tan(\pi - x) = -\tan x$

Trigonometric ratio of $(270^\circ - x)$ in terms of x :

- $\sin\left(\frac{3\pi}{2} - x\right) = -\cos x$
- $\cos\left(\frac{3\pi}{2} - x\right) = -\sin x$
- $\tan\left(\frac{3\pi}{2} - x\right) = \cot x$

Trigonometric ratio of $(270^\circ + x)$ in terms of x :

- $\sin\left(\frac{3\pi}{2} + x\right) = -\cos x$

- $\cos\left(\frac{3\pi}{2} + x\right) = \sin x$
- $\tan\left(\frac{3\pi}{2} + x\right) = -\cot x$

Trigonometric ratio of $(360^\circ - x)$ in terms of x :

- $\cos(2\pi - x) = \cos x$
- $\sin(2\pi - x) = -\sin x$
- $\tan(2\pi - x) = -\tan x$

Trigonometric ratio of $(360^\circ + x)$ in terms of x :

- $\cos(2\pi + x) = \cos x$
- $\sin(2\pi + x) = \sin x$
- $\tan(2\pi + x) = \tan x$
- $\cos(2n\pi + x) = \cos x$
- $\sin(2n\pi + x) = \sin x$

Trigonometric Ratios of Compound Angles:

Sum Formulae:

- $\sin(x + y) = \sin x \cos y + \cos x \sin y$
- $\cos(x + y) = \cos x \cos y - \sin x \sin y$
- $\tan(x + y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$
- $\cot(x + y) = \frac{\cot x \cot y - 1}{\cot y + \cot x}$

Difference Formulae:

- $\sin(x - y) = \sin x \cos y - \cos x \sin y$
- $\cos(x - y) = \cos x \cos y + \sin x \sin y$
- $\tan(x - y) = \frac{\tan x - \tan y}{1 + \tan x \tan y}$
- $\cot(x - y) = \frac{\cot x \cot y + 1}{\cot y - \cot x}$

Some Useful Results:

- $\sin(x + y) \sin(x - y) = \sin^2 x - \sin^2 y = \cos^2 y - \cos^2 x$
- $\cos(x + y) \cos(x - y) = \cos^2 x - \sin^2 y = \cos^2 y - \sin^2 x$

- $\tan(x + y + z) = \frac{\tan x + \tan y + \tan z - \tan x \tan y \tan z}{1 - \tan x \tan y - \tan y \tan z - \tan z \tan x}$

Transformation Formulae:

Product Formulae (on the basis of L.H.S.) or A-B formulae:

- $2 \sin x \cos y = \sin(x + y) + \sin(x - y)$
- $2 \cos x \sin y = \sin(x + y) - \sin(x - y)$
- $2 \cos x \cos y = \cos(x + y) + \cos(x - y)$
- $2 \sin x \sin y = \cos(x - y) - \cos(x + y)$

Sum and Difference Formulae (on the basis of L.H.S.) or C-D formulae:

- $\sin C + \sin D = 2 \sin \frac{C+D}{2} \cos \frac{C-D}{2}$
- $\sin C - \sin D = 2 \cos \frac{C+D}{2} \sin \frac{C-D}{2}$
- $\cos C + \cos D = 2 \cos \frac{C+D}{2} \cos \frac{C-D}{2}$
- $\cos C - \cos D = 2 \sin \frac{C+D}{2} \sin \frac{D-C}{2}$

Trigonometric Functions of Multiple and Sub-multiples of Angles:

- $\sin 2x = 2 \sin x \cos x = \frac{2 \tan x}{1 + \tan^2 x}$
- $\cos 2x = \cos^2 x - \sin^2 x = 2 \cos^2 x - 1 = 1 - 2 \sin^2 x = \frac{1 + \tan^2 x}{1 + \tan^2 x}$
- $\tan 2x = \frac{2 \tan x}{1 - \tan^2 x}$
- $\sin 3x = 3 \sin x - 4 \sin^3 x$
- $\cos 3x = 4 \cos^3 x - 3 \cos x$
- $\tan 3x = \frac{3 \tan x - \tan^3 x}{1 - 3 \tan^2 x}$
- $\sin \frac{x}{2} = \pm \sqrt{\frac{1 - \cos x}{2}}$
- $\cos \frac{x}{2} = \pm \sqrt{\frac{1 + \cos x}{2}}$
- $\tan \frac{x}{2} = \sqrt{\frac{1 - \cos x}{1 + \cos x}}$
- $\sin 18^\circ = \cos 72^\circ = \frac{\sqrt{5}-1}{4}$
- $\cos 18^\circ = \sin 72^\circ = \frac{1}{4} \sqrt{10 + 2\sqrt{5}}$
- $\cos 36^\circ = \frac{\sqrt{5}+1}{4}$
- $\sin 36^\circ = \frac{1}{4} \sqrt{10 - 2\sqrt{5}}$

Trigonometric Equations:

- **Principle Solutions:** The solutions of a trigonometric equation, for which $0 \leq x < 2\pi$ are called the principle solutions.
- **General Solutions:** The solution, consisting of all possible solutions of a trigonometric equation is called its general solutions>
- **Some General Solutions:**
 - $\sin x = 0$ gives $x = n\pi$, where $n \in \mathbb{Z}$.
 - $\cos x = 0$ gives $x = (2n + 1) \frac{\pi}{2}$, where $n \in \mathbb{Z}$.
 - $\tan x = 0$ gives $x = n\pi$
 - $\cot x = 0$ gives $x = (2n + 1) \frac{\pi}{2}$
 - $\sec x = 0$ gives no solution
 - $\operatorname{cosec} x = 0$ gives no solution
 - $\sin x = \sin y$ gives $x = n\pi + (-1)^n y$
 - $\cos x = \cos y$, implies $x = 2n\pi \pm y$, where $n \in \mathbb{Z}$.
 - $\tan x = \tan y$ implies $x = n\pi + y$, where $n \in \mathbb{Z}$.
 - $\sin^2 x = \sin^2 y$ gives $x = n\pi \pm y$
 - $\cos^2 x = \cos^2 y$ gives $x = n\pi \pm y$
 - $\tan^2 x = \tan^2 y$ gives $x = n\pi \pm y$