

1. Fundamental Principle of Counting
2. Permutations
3. Combinations

Fundamental Principle of Counting

- **Addition Law:** If there are two operations such that they can be performed independently in m and n ways respectively, then either of the two operations can be performed in $(m + n)$ ways.
- **Multiplication:** If one operation can be performed in m ways and if corresponding to each of the m ways of performing this operation, there are n ways of performing a second operation, then the number of ways of performing two operations together in $m \times n$.
- **Factorial Notation:** The continued product of first n natural numbers is called the ' n factorial' and is denoted by $n!$.
- $0! = 1$
- **Permutations:** The number of permutations of n different things taken r at a time, where repetition is not allowed, is denoted by ${}^n P_r$ and is given by ${}^n P_r = \frac{n!}{(n-r)!}$

where $0 \leq r \leq n$.

$$n! = 1 \times 2 \times 3 \times \dots \times n$$

$$n! = n \times (n - 1) !$$

- The number of permutations of n different things, taken r at a time, where repetition is allowed, is n^r .
- The number of permutations of n objects taken all at a time, where p_1 objects are of first kind, p_2 objects are of the second kind, ..., p_k objects are of the k^{th} kind and rest,

if any, are all different is $\frac{n!}{p_1!p_2!\dots p_k!}$.

Combinations:

- The number of combinations of n different things taken r at a time, denoted by nC_r is given by ${}^nC_r = \frac{n!}{r!(n-r)!}$, $0 \leq r \leq n$.
- ${}^nC_0 = 1$
- ${}^nC_n = 1$
- ${}^nC_r = {}^nC_{n-r}$
- ${}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$
- ${}^nC_r = \frac{n}{r} \cdot {}^{n-1}C_{r-1}$
- $n \cdot {}^{n-1}C_{r-1} = (n-r+1) {}^nC_{r-1}$
- Division into Groups: The number of ways $m + n$ things can be divided into two groups containing m and n things respectively $= {}^{m+n}C_m = \frac{(m+n)!}{m!n!}$