

1. Binomial Theorem for Positive Integral Indices

2. General and Middle Terms

- **Binomial Theorem:** The expansion of a binomial for any positive integral n is given by Binomial Theorem, which is

$$(a + b)^n = {}^nC_0 a^n + {}^nC_1 a^{n-1} b + {}^nC_2 a^{n-2} b^2 + \dots + {}^nC_{n-1} a b^{n-1} + {}^nC_n b^n.$$

- The coefficients of the expansions are arranged in an array. This array is called *Pascal's triangle*.
- The general term of an expansion $(a + b)^n$ is $T_{r+1} = {}^nC_r a^{n-r} b^r$
- The general term of an expansion $(a - b)^n = (-1)^r \cdot {}^nC_r \cdot a^{n-r} \cdot b^r$
- The general term of $(1 + x)^n = {}^nC_r \cdot x^r$
- The general term of $(1 - x)^n = (-1)^r \cdot {}^nC_r \cdot x^r$
- In the expansion $(a + b)^n$, if n is even, then the middle term is the $\left(\frac{n}{2} + 1\right)^{th}$ term.
If n is odd, then the middle terms are $\left(\frac{n}{2} + 1\right)^{th}$ and $\left(\frac{n+1}{2} + 1\right)^{th}$ terms.
- r^{th} term from the end in $(a + b)^n = (n + 2 - r)^{th}$ term from the beginning.
- Method to prove Binomial Theorem:

(a) Principle of Mathematical Induction.

(b) Combinatorial Method.

- Factorial notation:
 - (i) $n! = 1 \times 2 \times 3 \times 4 \dots \times n$; $0! = 1$
 - (ii) ${}^nC_r = \frac{n!}{r!(n-r)!}$
 - (iii) ${}^nC_r = {}^nC_{n-r}$
 - (iv) ${}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$