

1. Slope of a Line
2. Various Forms of the Equation of a Line
3. General Equation of a Line and Distance of a Point From a Line

First Degree Equation

Every first degree equation like $ax + by + c = 0$ would be the equation of a straight line.

Slope of a line

- Slope (m) of a non-vertical line passing through the points (x_1, y_1) and (x_2, y_2) is given by $m = \frac{y_2 - y_1}{x_2 - x_1}$, $x_1 \neq x_2$.
- If a line makes an angle α with the positive direction of x-axis, then the slope of the line is given by $m = \tan \alpha$, $\alpha \neq 90^\circ$
- Slope of horizontal line is zero and slope of vertical line is undefined.
- An acute angle (say θ) between lines L_1 and L_2 with slopes m_1 and m_2 is given by $\tan \theta = \left| \frac{m_2 - m_1}{1 + m_1 m_2} \right|$, $1 + m_1 m_2 \neq 0$
- Two lines are parallel if and only if their slopes are equal i.e., $m_1 = m_2$
- Two lines are *perpendicular* if and only if product of their slopes is -1 , i.e., $m_1 \cdot m_2 = -1$
- Three points A, B and C are collinear, if and only if slope of AB = slope of BC.
- Equation of the horizontal line having distance a from the x-axis is either $y = a$ or $y = -a$.
- Equation of the vertical line having distance b from the y-axis is either $x = b$ or $x = -b$.
- The point (x, y) lies on the line with slope m and through the fixed point (x_0, y_0) , if and only if its coordinates satisfy the equation.

Various forms of equations of a line:

- **Two points form:** Equation of the line passing through the points (x_1, y_1) and (x_2, y_2) is given by $y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$
- **Slope-Intercept form:** The point (x, y) on the line with slope m and y -intercept c lies on the line if and only if $y = mx + c$.
- If a line with slope m makes x -intercept d . Then equation of the line is $y = m(x - d)$.
- **Intercept form:** Equation of a line making intercepts a and b on the x - and y -axis, respectively, is $\frac{x}{a} + \frac{y}{b} = 1$.
- **Normal form:** The equation of the line having normal distance from origin p and angle between normal and the positive x -axis ω is given by $x \cos \omega + y \sin \omega = p$
- **General Equation of a Line:** Any equation of the form $Ax + By + C = 0$, with A and B are not zero, simultaneously, is called the general linear equation or general equation of a line.
- **Working Rule for reducing general form into the normal form:**

(i) Shift constant ' C ' to the R.H.S. and get $Ax + By = -C$

(ii) If the R.H.S. is not positive, then make it positive by multiplying the whole equation by -1 .

(iii) Divide both sides of equation by $\sqrt{A^2 + B^2}$.

The equation so obtained is in the normal form.

- **Parametric Equation (Symmetric Form):** $\frac{x - x_1}{\cos \theta} = \frac{y - y_1}{\sin \theta} = r$
- **Equation of a line through origin:** $y = mx$ or $y = x \tan \theta$.
- The perpendicular distance (d) of a line $Ax + By + C = 0$ from a point (x_1, y_1) is given by $d = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}$
- Distance between the parallel lines $Ax + By + C_1 = 0$ and $Ax + By + C_2 = 0$, is given by $d = \frac{|C_1 - C_2|}{\sqrt{A^2 + B^2}}$

Concurrent Lines

Three or more straight lines are said to be concurrent if they pass through a common point i.e., they meet at a point. Thus, if three lines are concurrent the point of intersection of two lines lies on the third line.

Condition of concurrency of three lines:

$$a_1 (b_2 c_3 - b_3 c_2) + b_1 (c_2 a_3 - c_3 a_2) + c_1 (a_2 b_3 - a_3 b_2) = 0$$

EQUATIONS OF FAMILY OF LINES THROUGH THE INTERSECTION OF TWO LINES

$$A_1 x + B_1 y + C_1 + k (A_2 x + B_2 y + C_2) = 0$$

where k is a constant and also called parameter.

This equation is of first degree of x and y , therefore, it represents a family of lines.

DISTANCE BETWEEN TWO PARALLEL LINES

Working Rule to find the distance between two parallel lines:

- (i) Find the co-ordinates of any point on one of the given line, preferably by putting $x = 0$ and $y = 0$.
- (ii) The perpendicular distance of this point from the other line is the required distance between the lines.

CBSE Class 11 Mathematics

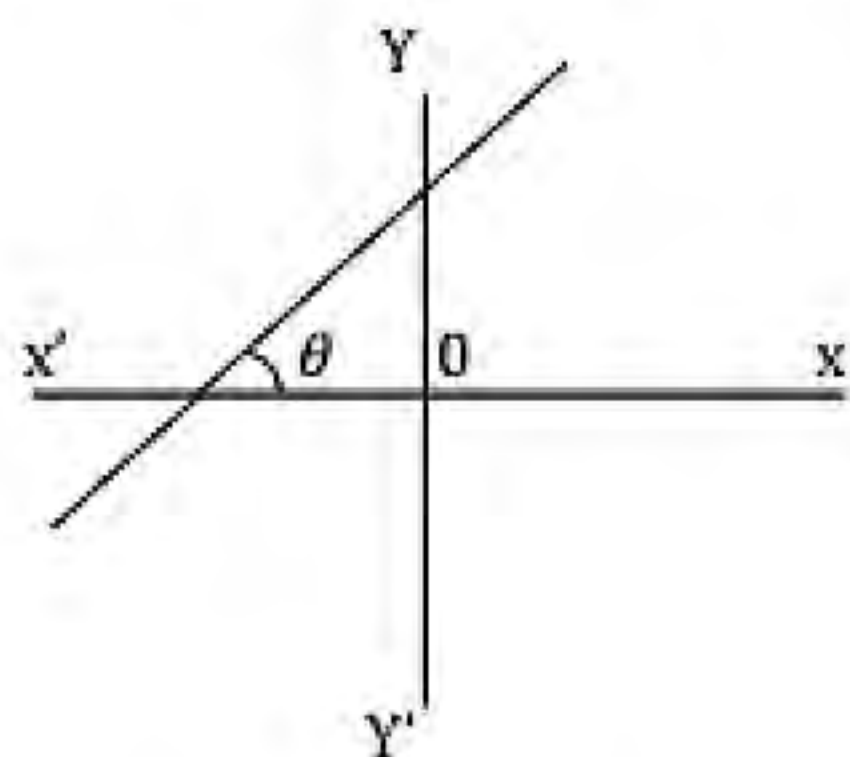
Revision Notes

Chapter-10

STRAIGHT LINE

Definition: A straight line is a curve such that every point on the line segment joining any two points on it lies on it. (No turning point b/w two points called a straight line)

Slope of Line (Gradient): A line makes with the +ve direction of the x – axis in anticlockwise sense is called the slope or gradient of the line.



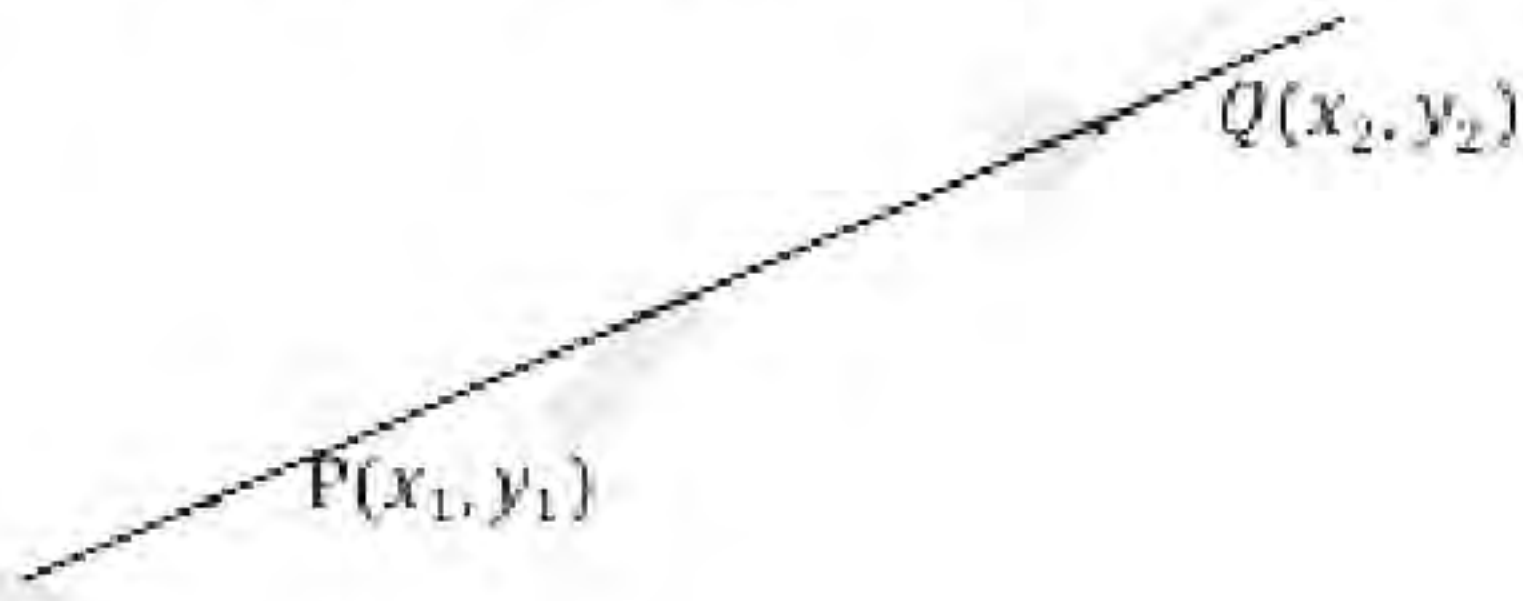
The slope of a line is generally denoted by m . Thus, $m = \tan \theta$.

1. Since a line parallel to x – axis makes an angle of 0° with x – axis, therefore its slope is $\tan 0^\circ = 0$.

2. A line parallel to y – axis makes an angle of 90° with x – axis, so its slope is $\tan \frac{\pi}{2} = \infty$.

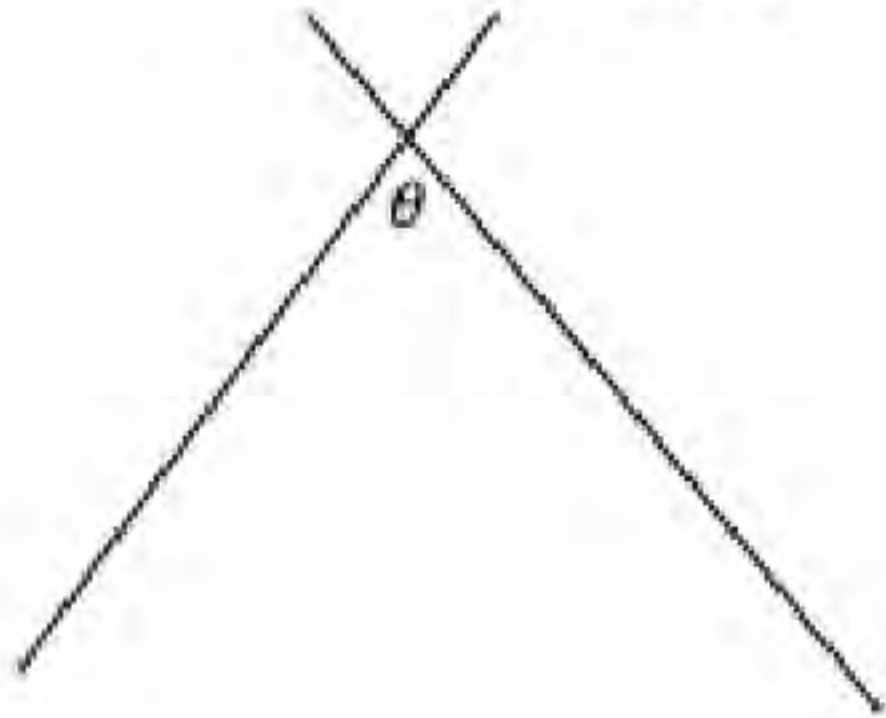
Slope of Line when Passing from two given points:

If $P(x_1, y_1)$ & $Q(x_2, y_2)$ So, $m = \frac{y_2 - y_1}{x_2 - x_1}$



Angle between two Lines:

$\tan \theta = \left| \frac{m_2 - m_1}{1 + m_1 m_2} \right|$ here m_1, m_2 are slope of lines and θ is angle between two lines.



NOTE: 1. If two lines are parallel to each other $\Rightarrow m_1 = m_2$ because $\theta = 0$

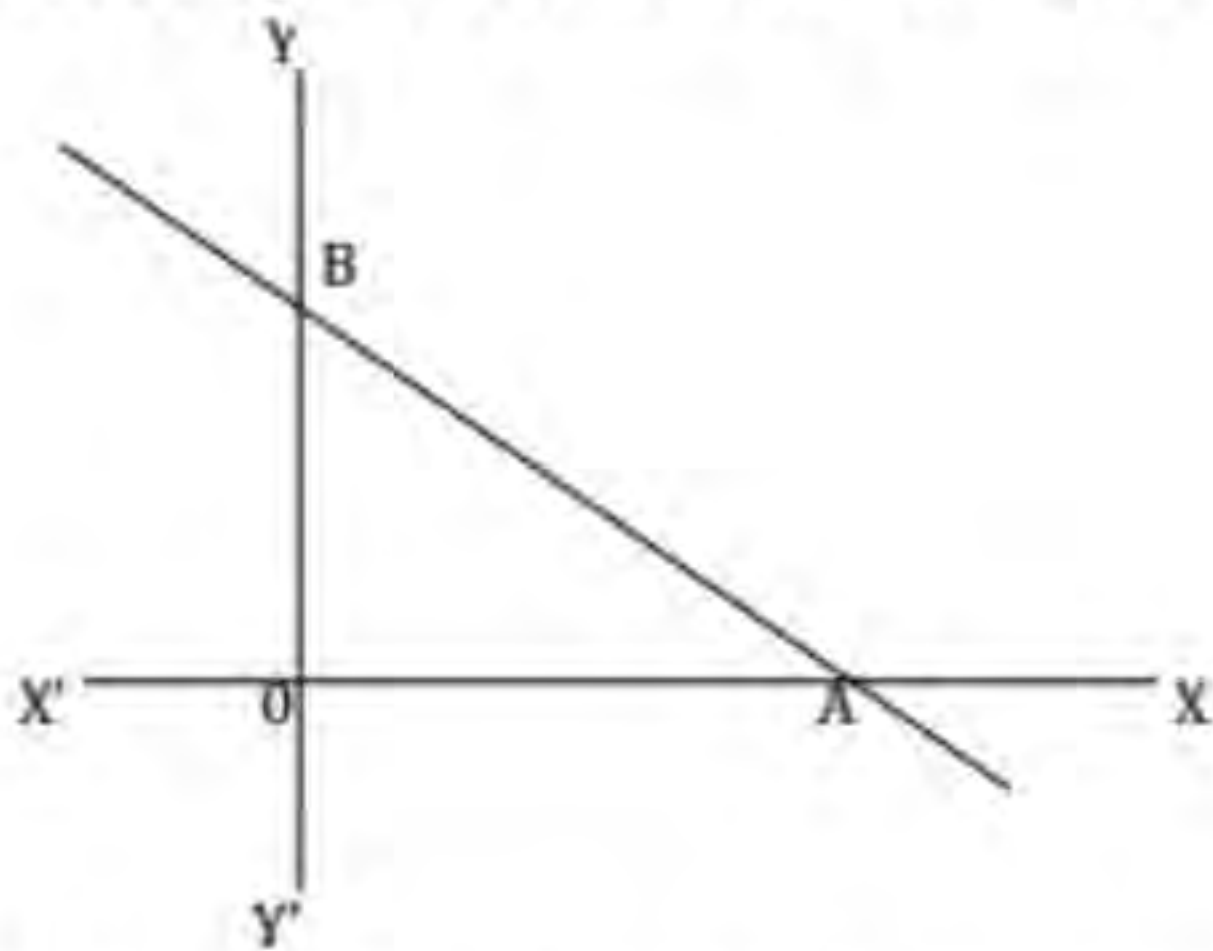
1. if two line are perpendicular to each other $\Rightarrow m_1 m_2 = -1$ because $\theta = 90$

2. if line parallel to x - axis $\Rightarrow$ equation of line $y = k$

3. if line parallel to y - axis $\Rightarrow$ equation of line $x = k$

4. every linear equation of two variable represent a line e.g. $ax + by + c = 0$

Intercepts of line on the Axes:



Here OA = X axis intercepts

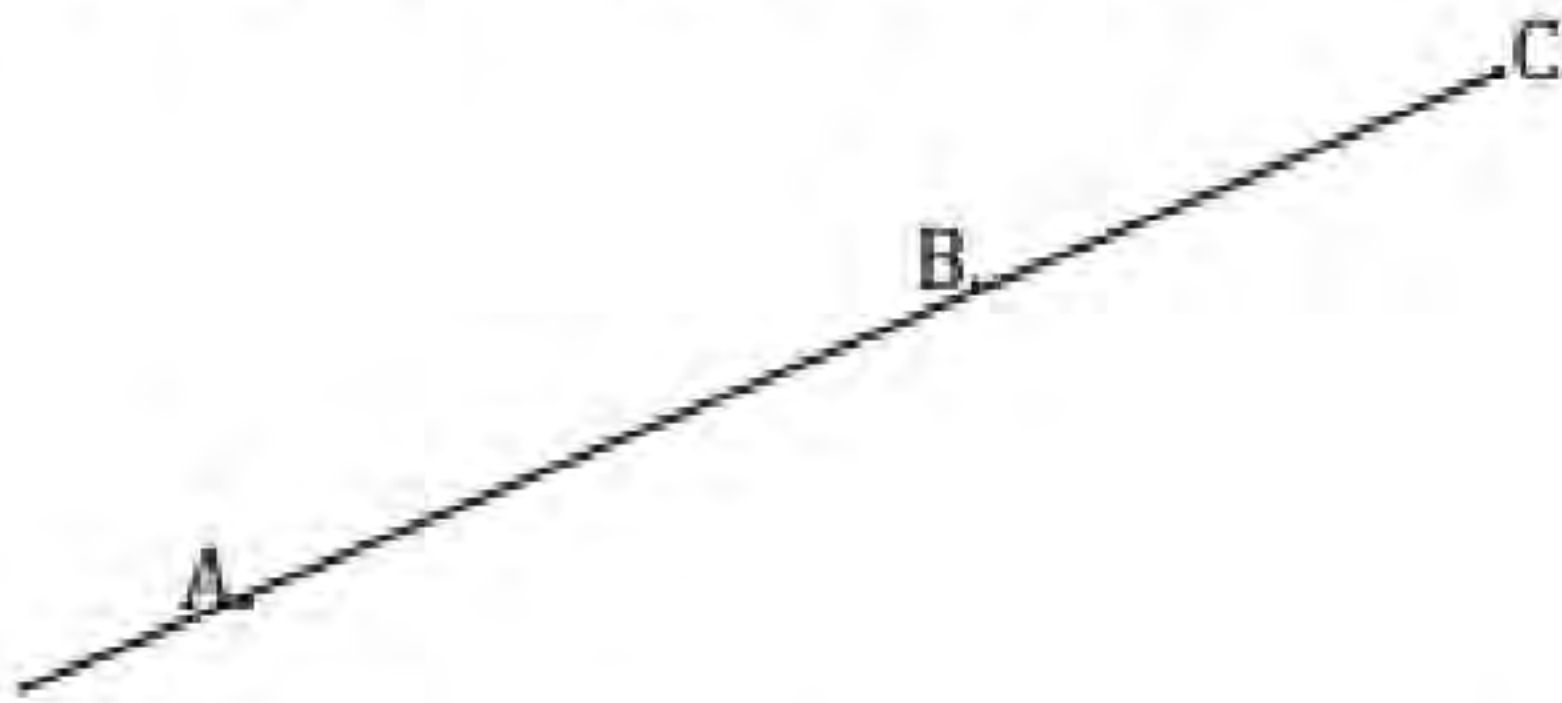
And OB = Y axis intercepts

Let OA = a and OB = b

So, $A(a, 0)$ and $B(0, b)$

NOTE: If three points are collinear then slope is equal b/w any two points of line

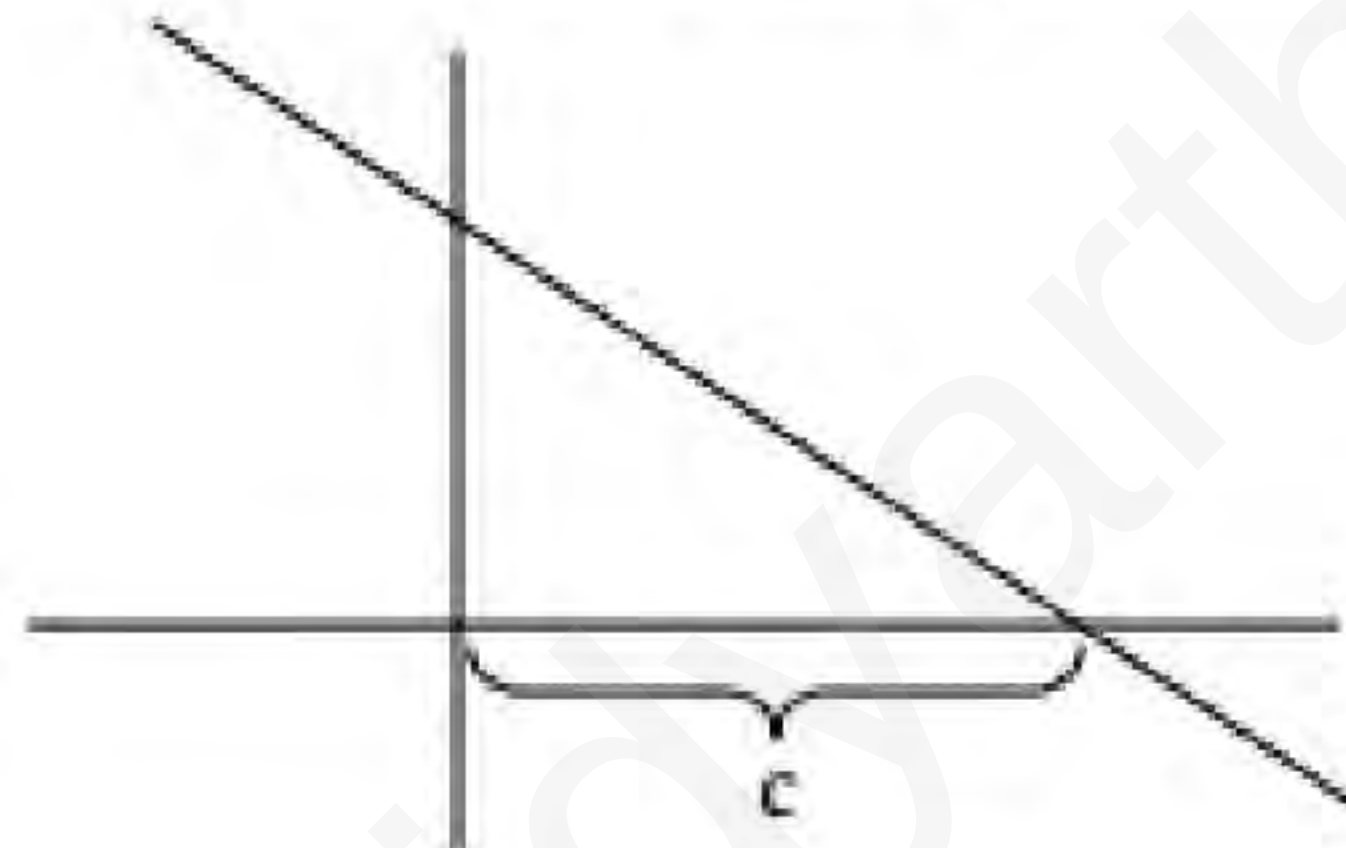
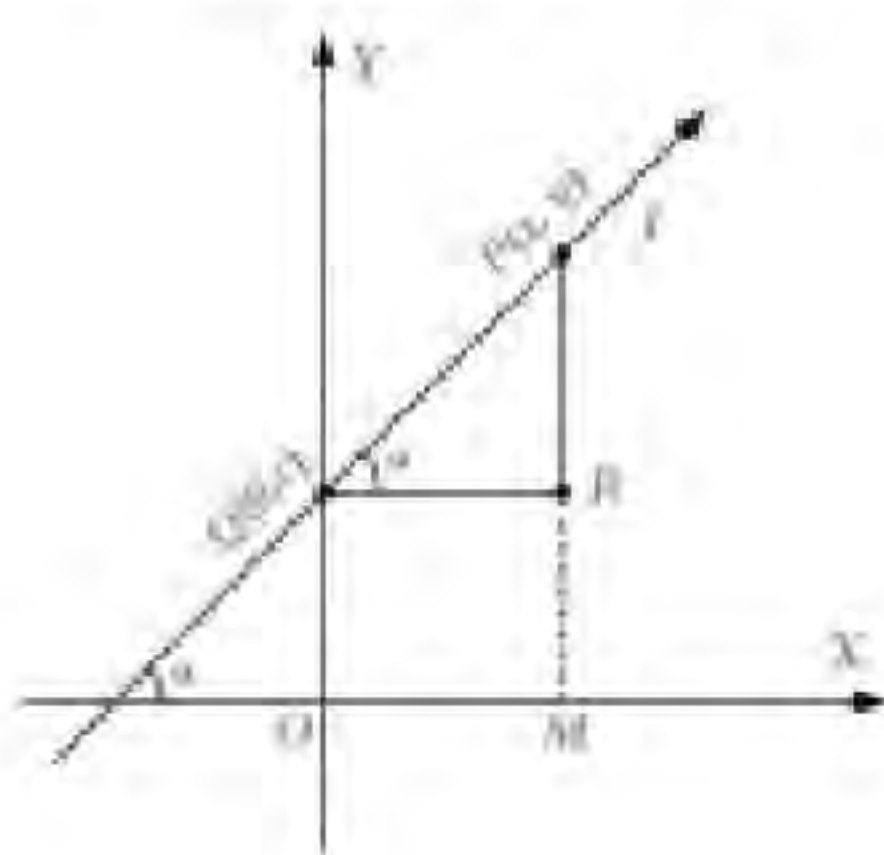
let $A(x_1, y_1)$, $B(x_2, y_2)$ & $(x_3, y_3) \Rightarrow$ slope of BC = slope of AC



Different forms of the equation of a straight line:

1. Slope intercept form of a line:

The equation of a line with slope m and making an intercept c on y -axis is $y = mx + c$



The equation of a line with slope m and making an intercept c on x -axis is $y = m(x - c)$

2. Point - slope form of a line:

The equation of a line which passes through the point (given) $P(x_1, y_1)$ and has the slope ' m ' is

$$y - y_1 = m(x - x_1).$$

3. Two point form of a line:

The equation of a line passing through two points $P(x_1, y_1)$ and $Q(x_2, y_2)$ is

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1} (x - x_1)$$

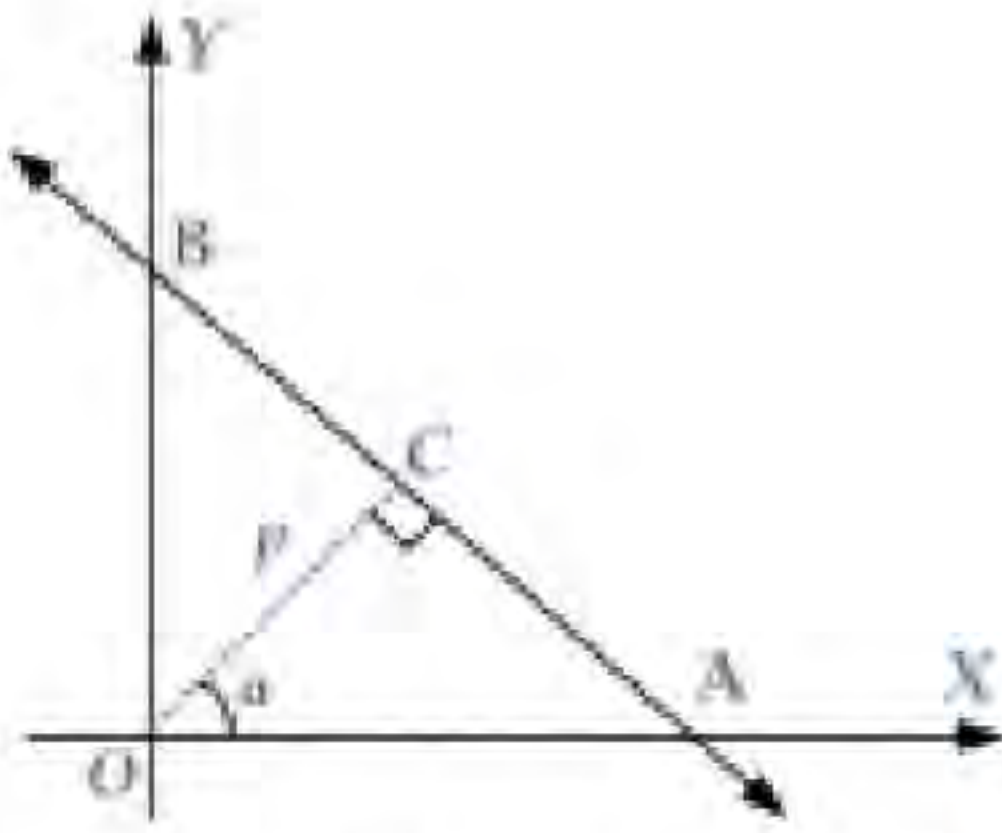
4. Intercept form of a line:

The equation of a line which cuts off intercepts ' a ' and ' b ' respectively from the x -axis and y -axis is $\frac{x}{a} + \frac{y}{b} = 1$.

5. Normal form or Perpendicular form of a line:

The equation of the straight line upon which the length of the perpendicular from the origin is p and this Perpendicular makes an angle α with x -axis is

$$x \cos \alpha + y \sin \alpha = p$$

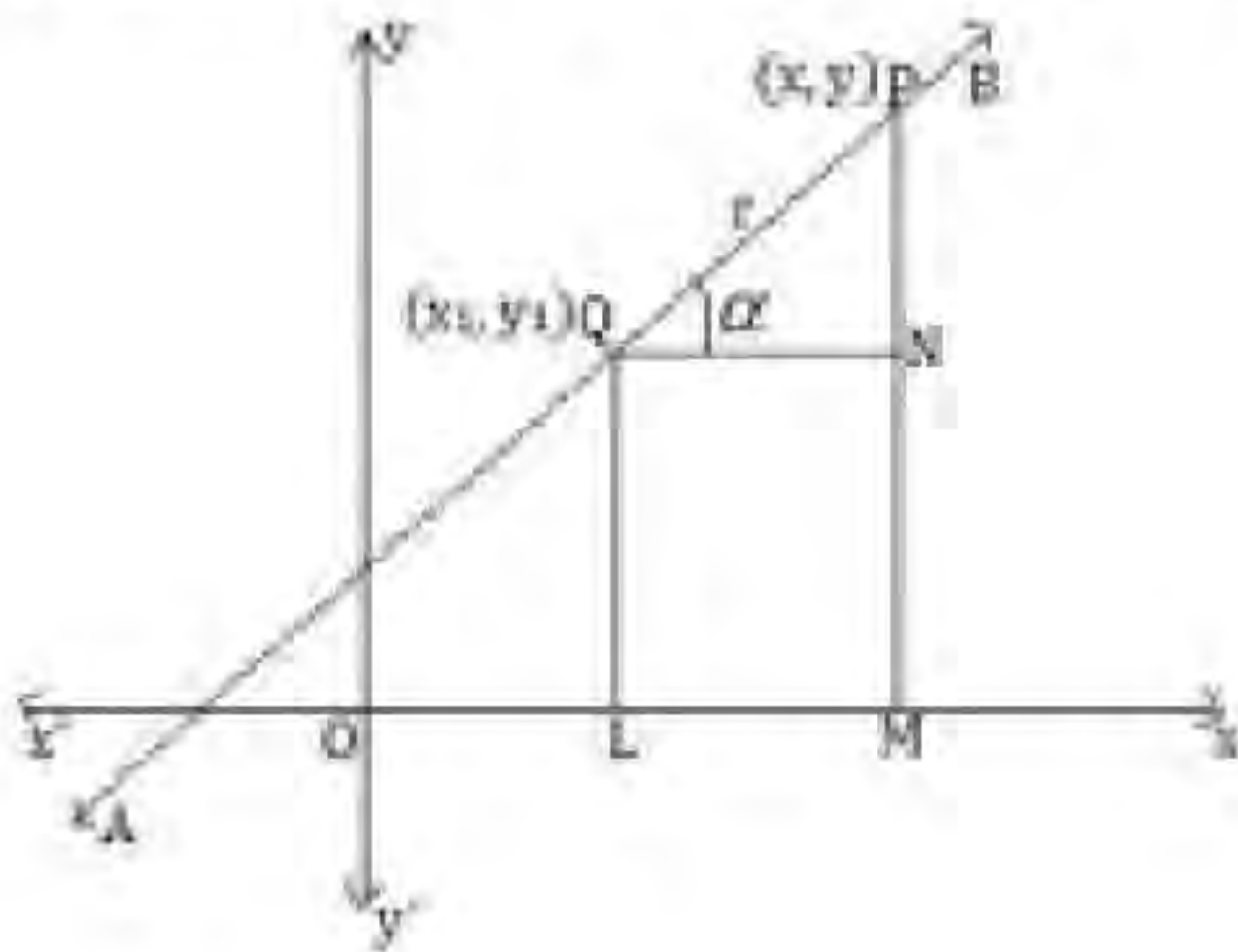


6. Distance form of a line:

The equation of the straight line passing through (x_1, y_1) and making an angle θ with the

+ve direction of x - axis is $\frac{x-x_1}{\cos \theta} = \frac{y-y_1}{\sin \theta} = r$

Where r is the distance of the point (x, y) on the line from the point (x_1, y_1)



Transformation of general equation in different standard forms:

1. Transformation of $Ax + By + C = 0$ in the slope intercept form $y = m x + c$

$$y = \left(-\frac{A}{B}\right)x + \left(-\frac{C}{B}\right)$$

This is of the form $y = m x + c$, where

$$m = -\frac{A}{B} = -\frac{\text{coefficient of } x}{\text{coefficient of } y}, \text{ and intercept on } y\text{-axis} = -\frac{C}{B} = -\frac{\text{constant}}{\text{coefficient of } y}$$

2. Transformation of $Ax + By + C = 0$ in intercept form $\frac{x}{a} + \frac{y}{b} = 1$

$$\frac{x}{\left(-\frac{C}{A}\right)} + \frac{y}{\left(-\frac{C}{B}\right)} = 1$$

Intercept on x - axis $= -\frac{C}{A} = -\frac{\text{constant term}}{\text{coefficient of } x}$, Intercept on y - axis

$$= -\frac{C}{B} = -\frac{\text{constant term}}{\text{coefficient of } y}$$

3. Transformation of $Ax + By + C = 0$ in intercept form $x \cos \alpha + y \sin \alpha = p$

$$-\frac{A}{\sqrt{A^2+B^2}}x - \frac{B}{\sqrt{A^2+B^2}}y = \frac{C}{\sqrt{A^2+B^2}}$$

$$\text{Here } \cos \alpha = -\frac{A}{\sqrt{A^2+B^2}} \text{ and } \sin \alpha = -\frac{B}{\sqrt{A^2+B^2}}; p = \pm \frac{C}{\sqrt{A^2+B^2}}$$

NOTE: Three lines are said to be concurrent if they pass through a common point OR they meet at a point.

Lines parallel and Perpendicular to a given line:

1. Line parallel to a guven line

To write a line parallel to a given line we keep the expression containing x and y same and simply replace The given constant by an unknown constant k. the value of k can be determined by some given condition.

2. Line perpendicular to a guven line

The equation of a line perpendicular to a given line $ax + by + c = 0$ is $bx - ay + k = 0$.

Distance of a point from a line:

The length of the perpendicular from a point (α, β) to a line $ax + by + c = 0$ is

Distance B/W Parallel lines:

The distance between two parallel lines $ax + by + c_1 = 0$ and $ax + by + c_2 = 0$ is

$$\left| \frac{c_1 - c_2}{\sqrt{a^2 + b^2}} \right|$$