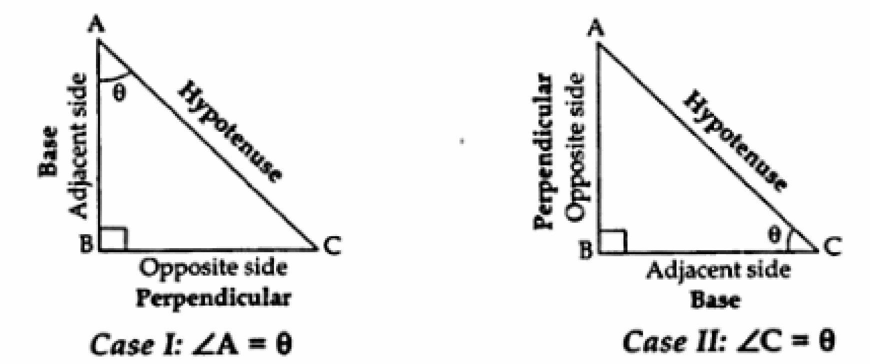
CBSE Class 10 Maths Notes Chapter 8 Introduction to Trigonometry

- Position of a point P in the Cartesian plane with respect to co-ordinate axes is represented by the ordered pair (x, y).
- Trigonometry is the science of relationships between the sides and angles of a right-angled triangle.
- Trigonometric Ratios: Ratios of sides of right triangle are called trigonometric ratios.
  Consider triangle ABC right-angled at B. These ratios are always defined with respect to acute angle 'A' or angle 'C.
- If one of the trigonometric ratios of an acute angle is known, the remaining trigonometric ratios of an angle can be easily determined.
- How to identify sides: Identify the angle with respect to which the t-ratios have to be calculated. Sides are always labelled with respect to the 'θ' being considered.

Let us look at both cases:



In a right triangle ABC, right-angled at B. Once we have identified the sides, we can define six t-Ratios with respect to the sides.

case I	case II
(i) sine A = $\frac{perpendicular}{hypotenuse} = \frac{BC}{AC}$	(i) sine C = $\frac{perpendicular}{hypotenuse} = \frac{AB}{AC}$
(ii) cosine A = $\frac{base}{hypotenuse} = \frac{AB}{AC}$	(ii) cosine C = $\frac{base}{hypotenuse} = \frac{BC}{AC}$
(iii) tangent A = $\frac{perpendicular}{base} = \frac{BC}{AB}$	(iii) tangent C = $\frac{perpendicular}{base} = \frac{AB}{BC}$
(iv) cosecant A = $\frac{hypotenuse}{perpendicular} = \frac{AC}{BC}$	(iv) cosecant C = $\frac{hypotenuse}{perpendicular} = \frac{AC}{AB}$
(v) secant A = $\frac{hypotenuse}{base} = \frac{AC}{AB}$	(v) secant C = $\frac{hypotenuse}{base} = \frac{AC}{BC}$
(v) cotangent A = $\frac{base}{perpendicular} = \frac{AB}{BC}$	(v) cotangent C = $\frac{base}{perpendicular} = \frac{BC}{AB}$

Note from above six relationships:

cosecant A = $\frac{1}{sinA}$, secant A = $\frac{1}{cosineA}$, cotangent A = $\frac{1}{tanA}$,

However, it is very tedious to write full forms of t-ratios, therefore the abbreviated notations are:
sine A is sin A
cosine A is cos A
tangent A is tan A
cosecant A is cosec A
secant A is sec A
cotangent A is cot A

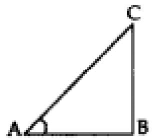
TRIGONOMETRIC IDENTITIES

An equation involving trigonometric ratio of angle(s) is called a trigonometric identity, if it is true for all values of the angles involved. These are:
tan θ = $\frac{sin\theta}{cos\theta}$
cot θ = $\frac{cos\theta}{sin\theta}$

- $\sin^2\theta + \cos^2\theta = 1 \Rightarrow \sin^2\theta = 1 - \cos^2\theta \Rightarrow \cos^2\theta = 1 - \sin^2\theta$
- $\text{cosec}^2\theta - \cot^2\theta = 1 \Rightarrow \text{cosec}^2\theta = 1 + \cot^2\theta \Rightarrow \cot^2\theta = \text{cosec}^2\theta - 1$
- $\sec^2\theta - \tan^2\theta = 1 \Rightarrow \sec^2\theta = 1 + \tan^2\theta \Rightarrow \tan^2\theta = \sec^2\theta - 1$
- $\sin\theta\,\text{cosec}\,\theta = 1 \Rightarrow \cos\theta\sec\theta = 1 \Rightarrow \tan\theta\cot\theta = 1$

ALERT:

A t-ratio only depends upon the angle ‘θ’ and stays the same for same angle of different sized right triangles.

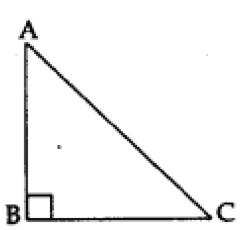


Value of t-ratios of specified angles:

∠A	0°	30°	45°	60°	90°
sin A	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos A	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan A	0	$\frac{1}{\sqrt{3}}$	1	√3	not defined
cosec A	not defined	2	√2	$\frac{2}{\sqrt{3}}$	1
sec A	1	$\frac{2}{\sqrt{3}}$	√2	2	not defined
cot A	not defined	√3	1	$\frac{1}{\sqrt{3}}$	0

The value of sin θ and cos θ can never exceed 1 (one) as opposite side is 1. Adjacent side can never be greater than hypotenuse since hypotenuse is the longest side in a right-angled Δ.

‘t-RATIOS’ OF COMPLEMENTARY ANGLES



If ΔABC is a right-angled triangle, right-angled at B, then

∠A + ∠C = 90° [∵ ∠A + ∠B + ∠C = 180° angle-sum-property]

or ∠C = (90° − ∠A)

Thus, ∠A and ∠C are known as complementary angles and are related by the following relationships:

sin (90° -A) = cos A; cosec (90° – A) = sec A

cos (90° – A) = sin A; sec (90° – A) = cosec A

tan (90° – A) = cot A; cot (90° – A) = tan A