# CBSE Class 11 Maths Notes Chapter 4 Complex Numbers and Quadratic Equations

### Imaginary Numbers

The square root of a negative real number is called an imaginary number, e.g. $\sqrt{-2}$, $\sqrt{-5}$ etc.
The quantity $\sqrt{-1}$ is an imaginary unit and it is denoted by 'i' called Iota.

### Integral Power of IOTA (i)

$i = \sqrt{-1}$, $i^2 = -1$, $i^3 = -i$, $i^4 = 1$
So, $i^{4n+1} = i$, $i^{4n+2} = -1$, $i^{4n+3} = -i$, $i^{4n} = 1$

Note:

- For any two real numbers a and b, the result $\sqrt{a} \times \sqrt{b} : \sqrt{ab}$ is true only, when atleast one of the given numbers i.e. either zero or positive.
  $\sqrt{-a} \times \sqrt{-b} \neq \sqrt{ab}$
  So, $i^2 = \sqrt{-1} \times \sqrt{-1} \neq 1$
- 'i' is neither positive, zero nor negative.
- $i^n + i^{n+1} + i^{n+2} + i^{n+3} = 0$

### Complex Number

A number of the form x + iy, where x and y are real numbers, is called a complex number, x is called real part and y is called imaginary part of the complex number i.e. Re(Z) = x and Im(Z) = y.

### Purely Real and Purely Imaginary Complex Number

A complex number Z = x + iy is a purely real if its imaginary part is 0, i.e. Im(z) = 0 and purely imaginary if its real part is 0 i.e. Re (z) = 0.

### Equality of Complex Number

Two complex numbers $z_1 = x_1 + iy_1$ and $z_2 = x_2 + iy_2$ are equal, iff $x_1 = x_2$ and $y_1 = y_2$ i.e. $Re(z_1) = Re(z_2)$ and $Im(z_1) = Im(z_2)$
Note: Order relation "greater than" and "less than" are not defined for complex number.

### Algebra of Complex Numbers

### Addition of complex numbers

Let $z_1 = x_1 + iy_1$ and $z_2 = x_2 + iy_2$ be any two complex numbers, then their sum defined as
$z_1 + z_2 = (x_1 + iy_1) + (x_2 + iy_2) = (x_1 + x_2) + i (y_1 + y_2)$

## Properties of Addition

- Commutative: $z_1 + z_2 = z_2 + z_1$
- Associative: $z_1 + (z_2 + z_3) = (z_1 + z_2) + z_3$
- Additive identity $z + 0 = z = 0 + z$
  Here, 0 is additive identity.

## Subtraction of complex numbers

Let $z_1 = (x_1 + iy_1)$ and $z_2 = (x_2 + iy_2)$ be any two complex numbers, then their difference is defined as
$z_1 - z_2 = (x_1 + iy_1) - (x_2 + iy_2) = (x_1 - x_2) + i(y_1 - y_2)$

## Multiplication of complex numbers

Let $z_1 = (x_1 + iy_1)$ and $z_2 = (x_2 + iy_2)$ be any two complex numbers, then their multiplication is defined as
$z_1z_2 = (x_1 + iy_1)(x_2 + iy_2) = (x_1x_2 - y_1y_2) + i(x_1y_2 + x_2y_1)$

## Properties of Multiplication

- Commutative: $z_1z_2 = z_2z_1$
- Associative: $z_1(z_2z_3) = (z_1z_2)z_3$
- Multiplicative identity: $z . 1 = z = 1 . z$
  Here, 1 is multiplicative identity of an element z.
- Multiplicative inverse: For every non-zero complex number z, there exists a complex number $z_1$ such that $z . z_1 = 1 = z_1 . z$
- Distributive law: $z_1(z_2 + z_3) = z_1z_2 + z_1z_3$

## Division of Complex Numbers

Let $z_1 = x_1 + iy_1$ and $z_2 = x_2 + iy_2$ be any two complex numbers, then their division is defined as

$$\frac{z_1}{z_2} = \frac{x_1 + iy_1}{x_2 + iy_2} = \frac{(x_1x_2 + y_1y_2) + i(x_2y_1 - x_1y_2)}{x_2^2 + y_2^2}$$

where, $z_2 \neq 0$.

## Conjugate of Complex Number

Let $z = x + iy$, if 'i' is replaced by (-i), then said to be conjugate of the complex number z and it is denoted by $\bar{z}$, i.e. $\bar{z} = x - iy$

### Properties of Conjugate

(i) $\overline{(\bar{z})} = z$

(ii) $z + \bar{z} = 2\,\text{Re}(z)$, $z - \bar{z} = 2i\,\text{Im}(z)$

(iii) $z = \bar{z}$, if $z$ is purely real

(iv) $z + \bar{z} = 0 \Leftrightarrow z$ is purely imaginary

(v) $\overline{z_1 + z_2} = \bar{z}_1 + \bar{z}_2$

(vi) $\overline{z_1 - z_2} = \bar{z}_1 - \bar{z}_2$

(vii) $\overline{z_1 z_2} = \bar{z}_1 \cdot \bar{z}_2$

(viii) $\overline{\left(\frac{z_1}{z_2}\right)} = \frac{\bar{z}_1}{\bar{z}_2}$, $\bar{z}_2 \neq 0$

(ix) $z \cdot \bar{z} = \{\text{Re}(z)\}^2 + \{\text{Im}(z)\}^2$

(x) $z_1\bar{z}_2 \pm \bar{z}_1 z_2 = 2\,\text{Re}(\bar{z}_1 z_2) = 2\text{Re}(z_1\bar{z}_2)$

(xi) If $z = f(z_1)$, then $\bar{z} = f(\bar{z}_1)$

(xii) $(\bar{z})^n = (\overline{z^n})$

### Modulus of a Complex Number

Let z = x + iy be a complex number. Then, the positive square root of the sum of square of real part and square of imaginary part is called modulus (absolute values) of z and it is denoted by |z| i.e. |z| = $\sqrt{x^2 + y^2}$
It represents a distance of z from origin in the set of complex number c, the order relation is not defined i.e. $z_1 > z_2$ or $z_1 < z_2$ has no meaning but $|z_1| > |z_2|$ or $|z_1|<|z_2|$ has got its meaning, since $|z_1|$ and $|z_2|$ are real numbers.

### Properties of Modulus of a Complex number

(i) $|z| \geq 0$

(ii) If $|z| = 0$, then $z = 0$ i.e. $\text{Re}(z) = 0 = \text{Im}(z)$

(iii) $-|z| \leq \text{Re}(z) \leq |z|$ and $-|z| \leq \text{Im}(z) \leq |z|$

(iv) $|z| = |\bar{z}| = |-z| = |-\bar{z}|$

(v) $z \cdot \bar{z} = |z|^2$

(vi) $|z_1 z_2| = |z_1||z_2|$

(vii) $\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}, z_2 \neq 0$

(viii) $|z_1 + z_2|^2 = |z_1|^2 + |z_2|^2 + 2\,\text{Re}(z_1\bar{z}_2)$

(ix) $|z_1 - z_2|^2 = |z_1|^2 + |z_2|^2 - 2\,\text{Re}(z_1\bar{z}_2)$

(x) $|z_1 + z_2| \leq |z_1| + |z_2|$

(xi) $|z_1 - z_2| \geq |z_1| - |z_2|$

(xii) $|az_1 - bz_2|^2 + |bz_1 + az_2|^2 = (a^2 + b^2)(|z_1|^2 + |z_2|^2)$

In particular,

$|z_1 - z_2|^2 + |z_1 + z_2|^2 = 2(|z_1|^2 + |z_2|^2)$

(xiii) $|z^n| = |z|^n$

(xiv) $|z_1 + z_2|^2 = |z_1|^2 + |z_2|^2 \Leftrightarrow \frac{z_1}{z_2}$ is purely imaginary.

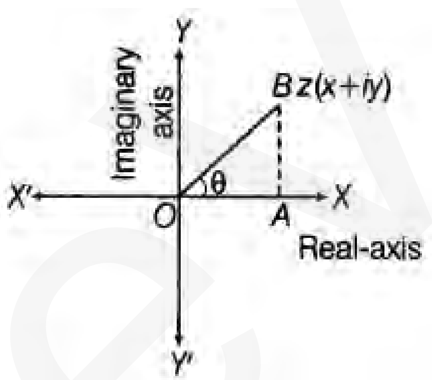
**Argand Plane**

Any complex number z = x + iy can be represented geometrically by a point (x, y) in a plane, called argand plane or gaussian plane. A purely number x, i.e. (x + 0i) is represented by the point (x, 0) on X-axis. Therefore, X-axis is called real axis. A purely imaginary number iy i.e. (0 + iy) is represented by the point (0, y) on the y-axis. Therefore, the y-axis is called the imaginary axis.

**Argument of a complex Number**

The angle made by line joining point z to the origin, with the positive direction of X-axis in an anti-clockwise sense is called argument or amplitude of complex number. It is denoted by the symbol arg(z) or amp(z).

arg(z) = θ = tan-1($\frac{y}{x}$)




Argument of z is not unique, general value of the argument of z is 2nπ + θ, but arg(0) is not defined. The unique value of θ such that -π < θ ≤ π is called the principal value of the amplitude or principal argument.

### Principal Value of Argument

- if x > 0 and y > 0, then arg(z) = θ
- if x < 0 and y > 0, then arg(z) = π − θ
- if x < 0 and y < 0, then arg(z) = -(π − θ)
- if x > 0 and y < 0, then arg(z) = -θ

### Polar Form of a Complex Number

If z = x + iy is a complex number, then z can be written as z = |z| (cosθ + isinθ), where θ = arg(z). This is called polar form. If the general value of the argument is θ, then the polar form of z is z = |z| [cos (2nπ + θ) + isin(2nπ + θ)], where n is an integer.

### Square Root of a Complex Number

If $z = x + iy$, then

$$\sqrt{z} = \sqrt{x + iy}$$

$$= \pm\left[\sqrt{\frac{|z| + x}{2}} + i\sqrt{\frac{|z| - x}{2}}\right], \text{ for } y > 0$$

$$= \pm\left[\sqrt{\frac{|z| + x}{2}} - i\sqrt{\frac{|z| - x}{2}}\right], \text{ for } y < 0$$

### Solution of a Quadratic Equation

The equation $ax^2 + bx + c = 0$, where a, b and c are numbers (real or complex, a ≠ 0) is called the general quadratic equation in variable x. The values of the variable satisfying the given equation are called roots of the equation.

The quadratic equation $ax^2 + bx + c = 0$ with real coefficients has two roots given by $\frac{-b+\sqrt{D}}{2a}$ and $\frac{-b-\sqrt{D}}{2a}$, where $D = b^2 - 4ac$, called the discriminant of the equation.

Note:
(i) When D = 0, roots ore real and equal. When D > 0 roots are real and unequal. Further If a,b, c ∈ Q and D is perfect square, then the roots of quadratic equation are real and unequal and if a, b, c ∈ Q and D is not perfect square, then the roots are irrational and occur in pair. When D < 0, roots of the equation are non real (or complex).
(ii) Let α, β be the roots of quadratic equation $ax^2 + bx + c = 0$, then sum of roots $\alpha + \beta = \frac{-b}{a}$ and the product of roots $\alpha\beta = \frac{c}{a}$.