CBSE Class 7 Maths Notes Chapter 8 Rational Numbers

The numbers used for counting objects are called counting numbers or natural numbers. These are: 1, 2, 3, 4, ………

If we include 0 to natural numbers, we get whole numbers. Thus, 0, 1, 2, 3, 4, ….. are whole numbers.

If we include the negatives of natural numbers to the whole numbers, we get integers. Thus, ….., -3, -2, -1, 0, 1, 2, 3, …… are integers.
We see that we have extended the number system from natural numbers to whole numbers and then from whole numbers to integers.

The numbers of the form $\frac{numerator}{denominator}$ where the numerator is either 0 or a positive integer and the denominator is a positive integer, are called fractions.

We compare two fractions by finding their equivalent forms. We have studied all the four basic operations of addition, subtraction, multiplication, and division on them. In this chapter, we shall further extend the number system by introducing rational numbers.

Need for Rational Numbers
There are many situations which involve fractional numbers. To include such numbers, we need to extend our number system by introducing rational numbers.

What are Rational Numbers?
A number of the form $\frac{p}{q}$ where p and q (≠0) are integers, is called a rational number.

Numerator and Denominator
In $\frac{p}{q}$, the integer p is the numerator, and the integer q (≠0) is the denominator.
Thus in $\frac{-3}{7}$, the numerator is -3 and the denominator is 7.

Equivalent Rational Numbers
If we multiply the numerator and denominator of a rational number by the same non-zero integer, we obtain another rational number equivalent to the given rational number.

Positive and Negative Rational Numbers

A rational number whose numerator and denominator both are positive integers is called a positive rational number.

A rational number, whose numerator is a negative integer and denominator is a positive integer, is called a negative rational number. Similarly, if the numerator is positive integer and denominator is a negative integer; is also a negative rational number.

Rational Numbers on a Number Line

Positive rational numbers are marked on the right of 0 on the number line whereas negative rational numbers are marked on the left of 0 on the number line.

The method of representation is the same as the method of representation of fractions on the number line.

Rational Numbers in Standard Form

A rational number is said to be in the standard form if its denominator is a positive integer and the numerator and the denominator have no common factor other than 1. Note that the negative sign occurs only in the numerator.

A rational number in standard form is said to be in its lowest form.

Reduction of a Rational Number to its Lowest Form

To reduce a rational number to its standard form (or lowest form), we divide its numerator and denominator by their HCF ignoring the negative sign, if any.

However, if there is a negative sign in the denominator, we divide by -HCF'.

Comparison of Rational Numbers

Two positive rational numbers can be compared exactly as we compare two fractions.

Two negative rational numbers can be compared by ignoring their negative signs and then reversing the order.

Comparison of a negative and a positive rational number is obvious as a negative rational number is always less than a positive rational number.

Rational Numbers Between Two Rational Numbers

There exist an unlimited number of rational numbers between any two rational numbers.

Operations on Rational Numbers

Addition

Addition of two rational numbers with same denominators: Two rational numbers with the same denominators can be added by adding their numerators, keeping the denominator same.

Addition of two rational numbers with different denominators: As in the case of fractions, we first find the LCM of the two denominators. Then we find the rational numbers equivalent to the given rational numbers with this LCM as the denominator. Now, we add the two rational numbers as in (A).

Additive Inverse

The additive inverse of the rational number $\frac{p}{q}$ is $-\frac{p}{q}$

Subtraction

While subtracting two rational numbers, we add the additive inverse of the rational number to be subtracted to the other rational number.

Multiplication

Multiplication of a rational number by a positive integer:

While multiplying a rational number by a positive integer, we multiply the numerator by that integer, keeping the denominator unchanged.

Multiplication of rational number by a negative integer:

While multiplying a rational number by a negative integer, we multiply the numerator by that integer, keeping the denominator unchanged.

Multiplication of two rational numbers (none of which is an integer):

Based on the above observations,

So, as done in fractions we multiply two rational numbers as follows:

- Step 1. Multiply the numerators of the two rational numbers.
- Step 2. Multiply the denominators of the two rational numbers.
- Step 3. Write the product as $\frac{Result \quad of \quad Step1}{Result \quad of \quad Step2}$

Division

The reciprocal of the rational number $\frac{p}{q}$ is $\frac{q}{p}$

To divide one rational number by other rational number, we multiply one rational number by the reciprocal of the other.

Product of Reciprocals

The product of a rational number with its reciprocal is always 1.

A rational number is defined as a number that can be expressed in the form $\frac{p}{q}$, where p and q are integers and q ≠ 0.

$e.g. \frac{4}{5}, \frac{2}{3}, \frac{6}{13},$ etc.

Rational Numbers include integers and fractions.

In $\frac{p}{q}$, p is the numerator and q is the denominator.

Zero is a rational number. We can write $\frac{0}{1}$.

By multiplying the numerator and denominator of a rational number by the same non – zero integer, we obtained another rational number equivalent to the given rational number.

$$e.g. \frac{-10}{14} = \frac{-10 \times 2}{14 \times 2} = \frac{-20}{28}$$

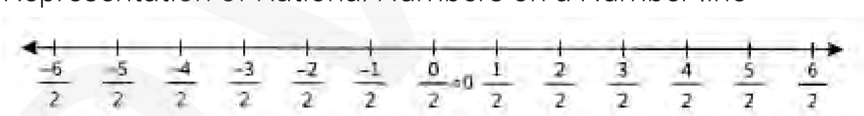
If both the numerator and denominator are either positive or negative integers, then they are said to be a positive rational number.

$$e.g. \frac{5}{7}, \frac{-13}{-8}, \frac{17}{9}, \frac{-40}{-72}, \text{ etc.}$$

A rational number is said to be negative if its numerator and denominator are such that one of them is a positive integer and the other is a negative integer.

$$e.g. \frac{-3}{5}, \frac{3}{-5}, \frac{-18}{7}, \frac{11}{-13} \text{ etc.}$$

Zero is neither positive nor negative rational numbers.
Representation of Rational Numbers on a Number line



To reduce the rational number to its standard form, we divide its numerator and denominator by their HCF ignoring the negative sign.

$e.g.\ \frac{36}{-24}$

HCF of 36 and 24 = 12

$$\frac{36}{-24} = \frac{36 \div (-12)}{-24 \div (-12)} = \frac{-3}{2}$$

To compare two negative rational numbers, we compare them ignoring their negative signs and then reverse the order.

e.g. To compare $\frac{-7}{5}$ and $\frac{-5}{3}$, first compare $\frac{7}{5}$ and $\frac{5}{3}$.

We get $\frac{7}{5} < \frac{5}{3}$ and conclude that $\frac{-7}{5} > \frac{-5}{3}$.

We can find an unlimited number of rational numbers between any two rational numbers.

While adding rational numbers with same denominators, we add the numerators keeping the denominators same.

$$e.g.\quad \frac{-2}{5} + \frac{3}{5} = \frac{-2+3}{5} = \frac{1}{5}.$$

While subtracting two rational numbers, we add the additive inverse of the rational, number that is being Subtracted, to the other rational number.

$$e.g.\quad \frac{5}{7} - \frac{3}{8} = \frac{5}{7} + \left(\frac{-3}{8}\right) = \frac{19}{56}.$$

While multiplying a rational number by a positive integer, we multiply the numerator by that integer, keeping the denominator unchanged.

$$e.g.\quad \frac{-2}{9} \times (-5) = \frac{-2 \times (-5)}{9} = \frac{10}{9}.$$

Product of reciprocals is always equal to 1.

$$e.g.\quad \frac{-4}{9} \times \frac{-9}{4} = 1$$

To divide one rational number by the other non – zero rational number, we multiply the rational number by the reciprocal of the other.

$$e.g.\quad \frac{-6}{5} \div \frac{-2}{3} = \frac{-6}{5} \times \frac{3}{-2} = \frac{-6 \times 3}{5 \times -2} = \frac{18}{10}.$$