CBSE Class 10 Maths Notes Chapter 6 Triangles

SIMILAR FIGURES

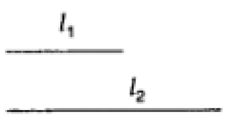
- Two figures having the same shape but not necessary the same size are called similar figures.
- All congruent figures are similar but all similar figures are not congruent.

SIMILAR POLYGONS

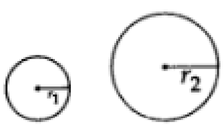
Two polygons are said to be similar to each other, if:
(i) their corresponding angles are equal, and
(ii) the lengths of their corresponding sides are proportional

Example:

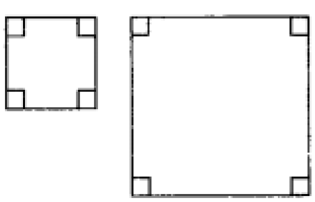
Any two line segments are similar since length are proportional



Any two circles are similar since radii are proportional



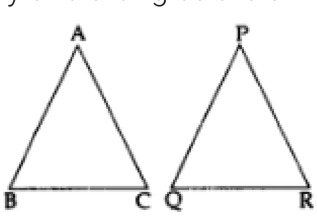
Any two squares are similar since corresponding angles are equal and lengths are proportional.



Note:

Similar figures are congruent if there is one to one correspondence between the figures.
∴ From above we deduce:

Any two triangles are similar, if their



(i) Corresponding angles are equal

∠A = ∠P

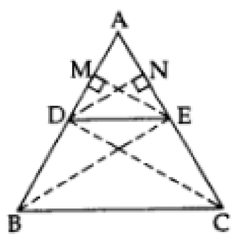
∠B = ∠Q

∠C = ∠R

(ii) Corresponding sides are proportional
$\frac{AB}{PQ} = \frac{AC}{PR} = \frac{BC}{QR}$

THALES THEOREM OR BASIC PROPORTIONALITY THEORY

Theorem 1:
State and prove Thales' Theorem.
Statement:
If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the same ratio.



Given: In ΔABC, DE || BC.
To prove: $\frac{AD}{DB} = \frac{AE}{EC}$
Const.: Draw EM ⊥ AD and DN ⊥ AE. Join B to E and C to D.
Proof: In ΔADE and ΔBDE,
$\frac{ar(\Delta ADE)}{ar(\Delta BDE)} = \frac{\frac{1}{2} \times AD \times EM}{\frac{1}{2} \times DB \times EM} = \frac{AD}{DB}$ ……..(i) [Area of Δ = $\frac{1}{2}$ x base x corresponding altitude
In ΔADE and ΔCDE,
$\frac{ar(\Delta ADE)}{ar(\Delta CDE)} = \frac{\frac{1}{2} \times AE \times DN}{\frac{1}{2} \times EC \times DN} = \frac{AE}{EC}$
∵ DE || BC …[Given
∴ ar(ΔBDE) = ar(ΔCDE)
…[∵ As on the same base and between the same parallel sides are equal in area
From (i), (ii) and (iii),
$\frac{AD}{DB} = \frac{AE}{EC}$

CRITERION FOR SIMILARITY OF TRIANGLES

Two triangles are similar if either of the following three criterion's are satisfied:

- **AAA similarity Criterion.** If two triangles are equiangular, then they are similar.
- **Corollary(AA similarity).** If two angles of one triangle are respectively equal to two angles of another triangle, then the two triangles are similar.
- **SSS Similarity Criterion.** If the corresponding sides of two triangles are proportional, then they are similar.
- **SAS Similarity Criterion.** If in two triangles, one pair of corresponding sides are proportional and the included angles are equal, then the two triangles are similar.

Results in Similar Triangles based on Similarity Criterion:

1. Ratio of corresponding sides = Ratio of corresponding perimeters
2. Ratio of corresponding sides = Ratio of corresponding medians
3. Ratio of corresponding sides = Ratio of corresponding altitudes
4. Ratio of corresponding sides = Ratio of corresponding angle bisector segments.

AREA OF SIMILAR TRIANGLES

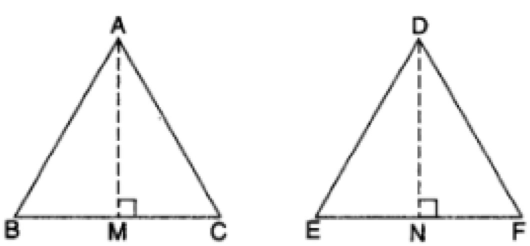
Theorem 2.
The ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding sides.

Given: ΔABC ~ ΔDEF

To prove: $\frac{ar(\Delta ABC)}{ar(\Delta DEF)} = \frac{AB^2}{DE^2} = \frac{BC^2}{EF^2} = \frac{AC^2}{DF^2}$

Const.: Draw AM ⊥ BC and DN ⊥ EF.

Proof: In ΔABC and ΔDEF



$\frac{ar(\Delta ABC)}{ar(\Delta DEF)} = \frac{\frac{1}{2} \times BC \times AM}{\frac{1}{2} \times EF \times DN} = \frac{BC}{EF} . \frac{AM}{DN}$ ...(i) ......[Area of Δ = $\frac{1}{2}$ x base x corresponding altitude

∵ ΔABC ~ ΔDEF

∴ $\frac{AB}{DE} = \frac{BC}{EF}$ .....(ii) ...[Sides are proportional

∠B = ∠E ....[∵ ΔABC ~ ΔDEF

∠M = ∠N .....[each 90°

∴ ΔABM ~ ΔDEN ....[AA similarity

∴ $\frac{AB}{DE} = \frac{AM}{DN}$ .....(iii) ...[Sides are proportional

From (ii) and (iii), we have: $\frac{BC}{EF} = \frac{AM}{DN}$ ...(iv)

From (i) and (iv), we have: $\frac{ar(\Delta ABC)}{ar(\Delta DEF)} = \frac{BC}{EF} . \frac{BC}{EF} = \frac{BC^2}{EF^2}$

Similarly, we can prove that

$\frac{ar(\Delta ABC)}{ar(\Delta DEF)} = \frac{AB^2}{DE^2} = \frac{AC^2}{DF^2}$

$\therefore \frac{ar(\Delta ABC)}{ar(\Delta DEF)} = \frac{AB^2}{DE^2} = \frac{BC^2}{EF^2} = \frac{AC^2}{DF^2}$

Results based on Area Theorem:

1. Ratio of areas of two similar triangles = Ratio of squares of corresponding altitudes
2. Ratio of areas of two similar triangles = Ratio of squares of corresponding medians
3. Ratio of areas of two similar triangles = Ratio of squares of corresponding angle bisector segments.

Note:
If the areas of two similar triangles are equal, the triangles are congruent.

PYTHAGORAS THEOREM

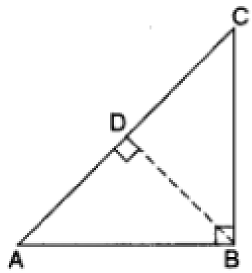
Theorem 3:
State and prove Pythagoras' Theorem.
Statement:
Prove that, in a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

Given: ΔABC is a right triangle right-angled at B.

To prove: $AB^2 + BC^2 = AC^2$

Const.: Draw BD ⊥ AC

Proof: In Δs ABC and ADB,



∠A = ∠A ...[common

∠ABC = ∠ADB ...[each 90°

∴ ΔABC ~ ΔADB ...[AA Similarity

∴ $\frac{AB}{AD} = \frac{AC}{AB}$ ....[sides are proportional]

⇒ $AB^2 = AC.AD$

Now in ΔABC and ΔBDC

∠C = ∠C .....[common]

∠ABC = ∠BDC ....[each 90°]

∴ ΔABC ~ ΔBDC .....[AA similarity]

∴ $\frac{BC}{DC} = \frac{AC}{BC}$ ....[sides are proportional]

$BC^2 = AC.DC$ ...(ii)

On adding (i) and (ii), we get

$AB^2 + BC^2 = ACAD + AC.DC$

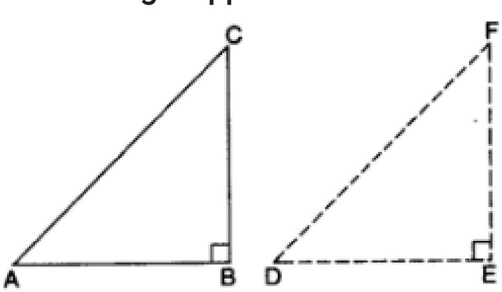
⇒ $AB^2 + BC^2 = AC.(AD + DC)$

$AB^2 + BC^2 = AC.AC$

∴$AB^2 + BC^2 = AC^2$

CONVERSE OF PYTHAGORAS THEOREM

Theorem 4:
State and prove the converse of Pythagoras' Theorem.
Statement:
Prove that, in a triangle, if square of one side is equal to the sum of the squares of the other two sides, then the angle opposite the first side is a right angle.



Given: In ΔABC, $AB^2 + BC^2 = AC^2$

To prove: $\angle ABC = 90°$

Const.: Draw a right angled ΔDEF in which DE = AB and EF = BC

Proof: In ΔABC,

$AB^2 + BC^2 = AC^2$ …(i) [given]

In rt. ΔDEF

$DE^2 + EF^2 = DF^2$ …[by pythagoras theorem]

$AB^2 + BC^2 = DF^2$ …..(ii) …[DE = AB, EF = BC]

From (i) and (ii), we get

$AC^2 = DF^2$

$\Rightarrow AC = DF$

Now, DE = AB …[by cont]

EF = BC …[by cont]

DF = AC …….[proved above]

$\therefore \Delta DEF \cong \Delta ABC$ ……[sss congruence]

$\therefore \angle DEF = \angle ABC$ …..[CPCT]

$\angle DEF = 90°$ …[by cont]

$\therefore \angle ABC = 90°$

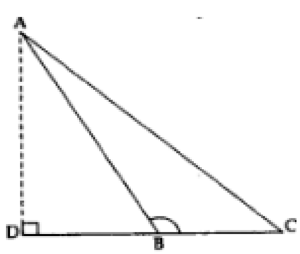
Results based on Pythagoras' Theorem:

(i) Result on obtuse Triangles.

If ΔABC is an obtuse angled triangle, obtuse angled at B,

If $AD \perp CB$, then

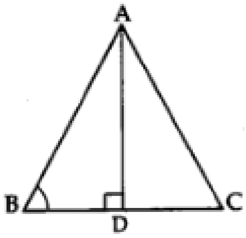
$AC^2 = AB^2 + BC^2 + 2\, BC.BD$



(ii) Result on Acute Triangles.

If ΔABC is an acute angled triangle, acute angled at B, and $AD \perp BC$, then

$AC^2 = AB^2 + BC^2 - 2\, BD.BC$.


A
B
D
C