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## CHAPTER NINE

# MECHANICAL PROPERTIES OF FLUIDS

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### 9.1 INTRODUCTION

In this chapter, we shall study some common physical properties of liquids and gases. Liquids and gases can flow and are therefore, called fluids. It is this property that distinguishes liquids and gases from solids in a basic way.

Fluids are everywhere around us. Earth has an envelop of air and two-thirds of its surface is covered with water. Water is not only necessary for our existence; every mammalian body constitute mostly of water. All the processes occurring in living beings including plants are mediated by fluids. Thus understanding the behaviour and properties of fluids is important.

How are fluids different from solids? What is common in liquids and gases? Unlike a solid, a fluid has no definite shape of its own. Solids and liquids have a fixed volume, whereas a gas fills the entire volume of its container. We have learnt in the previous chapter that the volume of solids can be changed by stress. The volume of solid, liquid or gas depends on the stress or pressure acting on it. When we talk about fixed volume of solid or liquid, we mean its volume under atmospheric pressure. The difference between gases and solids or liquids is that for solids or liquids the change in volume due to change of external pressure is rather small. In other words solids and liquids have much lower compressibility as compared to gases.

Shear stress can change the shape of a solid keeping its volume fixed. The key property of fluids is that they offer very little resistance to shear stress; their shape changes by application of very small shear stress. The shearing stress of fluids is about million times smaller than that of solids.

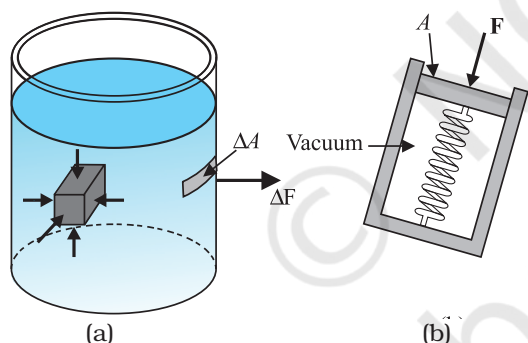
### 9.2 PRESSURE

A sharp needle when pressed against our skin pierces it. Our skin, however, remains intact when a blunt object with a wider contact area (say the back of a spoon) is pressed against it with the same force. If an elephant were to step on a man's chest, his ribs would crack. A circus performer across whose

chest a large, light but strong wooden plank is placed first, is saved from this accident. Such everyday experiences convince us that both the force and its coverage area are important. Smaller the area on which the force acts, greater is the impact. This impact is known as pressure.

When an object is submerged in a fluid at rest, the fluid exerts a force on its surface. This force is always normal to the object's surface. This is so because if there were a component of force parallel to the surface, the object will also exert a force on the fluid parallel to it; as a consequence of Newton's third law. This force will cause the fluid to flow parallel to the surface. Since the fluid is at rest, this cannot happen. Hence, the force exerted by the fluid at rest has to be perpendicular to the surface in contact with it. This is shown in Fig. 9.1(a).

The normal force exerted by the fluid at a point may be measured. An idealised form of one such pressure-measuring device is shown in Fig. 9.1(b). It consists of an evacuated chamber with a spring that is calibrated to measure the force acting on the piston. This device is placed at a point inside the fluid. The inward force exerted by the fluid on the piston is balanced by the outward spring force and is thereby measured.



**Fig. 9.1** (a) The force exerted by the liquid in the beaker on the submerged object or on the walls is normal (perpendicular) to the surface at all points.  
(b) An idealised device for measuring pressure.

If  $F$  is the magnitude of this normal force on the piston of area  $A$  then the **average pressure**  $P_{av}$  is defined as the normal force acting per unit area.

$$P_{av} = \frac{F}{A} \quad (9.1)$$

In principle, the piston area can be made arbitrarily small. The pressure is then defined in a limiting sense as

$$P = \lim_{\Delta A \rightarrow 0} \frac{\Delta F}{\Delta A} \quad (9.2)$$

Pressure is a scalar quantity. We remind the reader that it is the component of the force normal to the area under consideration and not the (vector) force that appears in the numerator in Eqs. (9.1) and (9.2). Its dimensions are  $[ML^{-1}T^{-2}]$ . The SI unit of pressure is  $N\,m^{-2}$ . It has been named as pascal (Pa) in honour of the French scientist Blaise Pascal (1623-1662) who carried out pioneering studies on fluid pressure. A common unit of pressure is the atmosphere (atm), i.e. the pressure exerted by the atmosphere at sea level ( $1\,atm = 1.013 \times 10^5\,Pa$ ).

Another quantity, that is indispensable in describing fluids, is the density  $\rho$ . For a fluid of mass  $m$  occupying volume  $V$ ,

$$\rho = \frac{m}{V} \quad (9.3)$$

The dimensions of density are  $[ML^{-3}]$ . Its SI unit is  $kg\,m^{-3}$ . It is a positive scalar quantity. A liquid is largely incompressible and its density is therefore, nearly constant at all pressures. Gases, on the other hand exhibit a large variation in densities with pressure.

The density of water at  $4^\circ C$  ( $277\,K$ ) is  $1.0 \times 10^3\,kg\,m^{-3}$ . The relative density of a substance is the ratio of its density to the density of water at  $4^\circ C$ . It is a dimensionless positive scalar quantity. For example the relative density of aluminium is 2.7. Its density is  $2.7 \times 10^3\,kg\,m^{-3}$ . The densities of some common fluids are displayed in Table 9.1.

**Table 9.1** Densities of some common fluids at STP\*

Fluid	$\rho\,(kg\,m^{-3})$
Water	$1.00 \times 10^3$
Sea water	$1.03 \times 10^3$
Mercury	$13.6 \times 10^3$
Ethyl alcohol	$0.806 \times 10^3$
Whole blood	$1.06 \times 10^3$
Air	1.29
Oxygen	1.43
Hydrogen	$9.0 \times 10^{-2}$
Interstellar space	$\approx 10^{-20}$

\* STP means standard temperature ( $0^\circ C$ ) and 1 atm pressure.

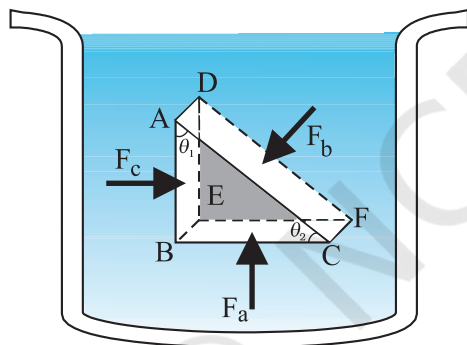
**Example 9.1** The two thigh bones (femurs), each of cross-sectional area  $10 \text{ cm}^2$  support the upper part of a human body of mass  $40 \text{ kg}$ . Estimate the average pressure sustained by the femurs.

**Answer** Total cross-sectional area of the femurs is  $A = 2 \times 10 \text{ cm}^2 = 20 \times 10^{-4} \text{ m}^2$ . The force acting on them is  $F = 40 \text{ kg wt} = 400 \text{ N}$  (taking  $g = 10 \text{ m s}^{-2}$ ). This force is acting vertically down and hence, normally on the femurs. Thus, the average pressure is

$$P_{av} = \frac{F}{A} = 2 \times 10^5 \text{ N m}^{-2}$$

### 9.2.1 Pascal's Law

The French scientist Blaise Pascal observed that the pressure in a fluid at rest is the same at all points if they are at the same height. This fact may be demonstrated in a simple way.



**Fig. 9.2** Proof of Pascal's law. ABC-DEF is an element of the interior of a fluid at rest. This element is in the form of a right-angled prism. The element is small so that the effect of gravity can be ignored, but it has been enlarged for the sake of clarity.

Fig. 9.2 shows an element in the interior of a fluid at rest. This element ABC-DEF is in the form of a right-angled prism. In principle, this prismatic element is very small so that every part of it can be considered at the same depth from the liquid surface and therefore, the effect of the gravity is the same at all these points. But for clarity we have enlarged this element. The forces on this element are those exerted by the rest of the fluid and they must be normal to the surfaces of the element as discussed above. Thus, the fluid exerts pressures  $P_a$ ,  $P_b$  and  $P_c$  on

this element of area corresponding to the normal forces  $F_a$ ,  $F_b$  and  $F_c$  as shown in Fig. 9.2 on the faces BEFC, ADFC and ADEB denoted by  $A_a$ ,  $A_b$  and  $A_c$  respectively. Then

$$F_b \sin \theta = F_c, \quad F_b \cos \theta = F_a \quad (\text{by equilibrium})$$

$$A_b \sin \theta = A_c, \quad A_b \cos \theta = A_a \quad (\text{by geometry})$$

Thus,

$$\frac{F_b}{A_b} = \frac{F_c}{A_c} = \frac{F_a}{A_a}; \quad P_b = P_c = P_a \quad (9.4)$$

Hence, pressure exerted is same in all directions in a fluid at rest. It again reminds us that like other types of stress, pressure is not a vector quantity. No direction can be assigned to it. The force against any area within (or bounding) a fluid at rest and under pressure is normal to the area, regardless of the orientation of the area.

Now consider a fluid element in the form of a horizontal bar of uniform cross-section. The bar is in equilibrium. The horizontal forces exerted at its two ends must be balanced or the pressure at the two ends should be equal. This proves that for a liquid in equilibrium the pressure is same at all points in a horizontal plane. Suppose the pressure were not equal in different parts of the fluid, then there would be a flow as the fluid will have some net force acting on it. Hence in the absence of flow the pressure in the fluid must be same everywhere in a horizontal plane.

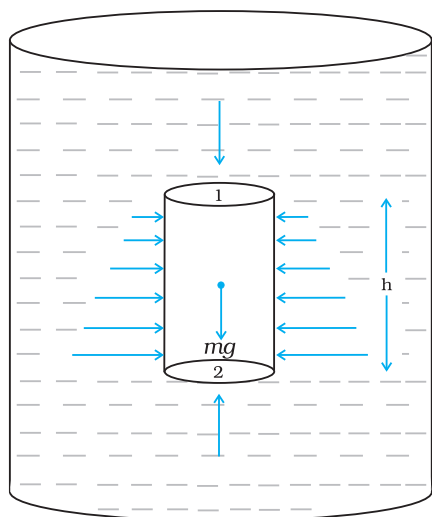
### 9.2.2 Variation of Pressure with Depth

Consider a fluid at rest in a container. In Fig. 9.3 point 1 is at height  $h$  above a point 2. The pressures at points 1 and 2 are  $P_1$  and  $P_2$  respectively. Consider a cylindrical element of fluid having area of base  $A$  and height  $h$ . As the fluid is at rest the resultant horizontal forces should be zero and the resultant vertical forces should balance the weight of the element. The forces acting in the vertical direction are due to the fluid pressure at the top ( $P_1 A$ ) acting downward, at the bottom ( $P_2 A$ ) acting upward. If  $mg$  is weight of the fluid in the cylinder we have

$$(P_2 - P_1) A = mg \quad (9.5)$$

Now, if  $\rho$  is the mass density of the fluid, we have the mass of fluid to be  $m = \rho V = \rho h A$  so that

$$P_2 - P_1 = \rho gh \quad (9.6)$$



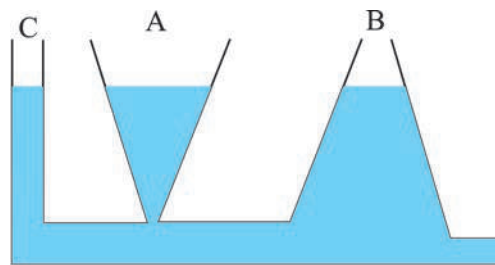
**Fig. 9.3** Fluid under gravity. The effect of gravity is illustrated through pressure on a vertical cylindrical column.

Pressure difference depends on the vertical distance  $h$  between the points (1 and 2), mass density of the fluid  $\rho$  and acceleration due to gravity  $g$ . If the point 1 under discussion is shifted to the top of the fluid (say, water), which is open to the atmosphere,  $P_1$  may be replaced by atmospheric pressure ( $P_a$ ) and we replace  $P_2$  by  $P$ . Then Eq. (9.6) gives

$$P = P_a + \rho gh \quad (9.7)$$

Thus, the pressure  $P$ , at depth below the surface of a liquid open to the atmosphere is greater than atmospheric pressure by an amount  $\rho gh$ . The excess of pressure,  $P - P_a$ , at depth  $h$  is called a **gauge pressure** at that point.

The area of the cylinder is not appearing in the expression of absolute pressure in Eq. (9.7). Thus, the height of the fluid column is important and not cross-sectional or base area or the shape of the container. The liquid pressure is the same at all points at the same horizontal level (same depth). The result is appreciated through the example of **hydrostatic paradox**. Consider three vessels A, B and C [Fig. 9.4] of different shapes. They are connected at the bottom by a horizontal pipe. On filling with water, the level in the three vessels is the same, though they hold different amounts of water. This is so because water at the bottom has the same pressure below each section of the vessel.



**Fig 9.4** Illustration of hydrostatic paradox. The three vessels A, B and C contain different amounts of liquids, all upto the same height.

**Example 9.2** What is the pressure on a swimmer 10 m below the surface of a lake?

**Answer** Here

$h = 10 \text{ m}$  and  $\rho = 1000 \text{ kg m}^{-3}$ . Take  $g = 10 \text{ m s}^{-2}$

From Eq. (9.7)

$$\begin{aligned} P &= P_a + \rho gh \\ &= 1.01 \times 10^5 \text{ Pa} + 1000 \text{ kg m}^{-3} \times 10 \text{ m s}^{-2} \times 10 \text{ m} \\ &= 2.01 \times 10^5 \text{ Pa} \\ &\approx 2 \text{ atm} \end{aligned}$$

This is a 100% increase in pressure from surface level. At a depth of 1 km, the increase in pressure is 100 atm! Submarines are designed to withstand such enormous pressures. ◀

### 9.2.3 Atmospheric Pressure and Gauge Pressure

The pressure of the atmosphere at any point is equal to the weight of a column of air of unit cross-sectional area extending from that point to the top of the atmosphere. At sea level, it is  $1.013 \times 10^5 \text{ Pa}$  (1 atm). Italian scientist Evangelista Torricelli (1608–1647) devised for the first time a method for measuring atmospheric pressure. A long glass tube closed at one end and filled with mercury is inverted into a trough of mercury as shown in Fig. 9.5 (a). This device is known as ‘mercury barometer’. The space above the mercury column in the tube contains only mercury vapour whose pressure  $P$  is so small that it may be neglected. Thus, the pressure at Point A = 0. The pressure inside the column at Point B must be the same as the pressure at Point C, which is atmospheric pressure,  $P_a$ .

$$P_a = \rho gh \quad (9.8)$$

where  $\rho$  is the density of mercury and  $h$  is the height of the mercury column in the tube.



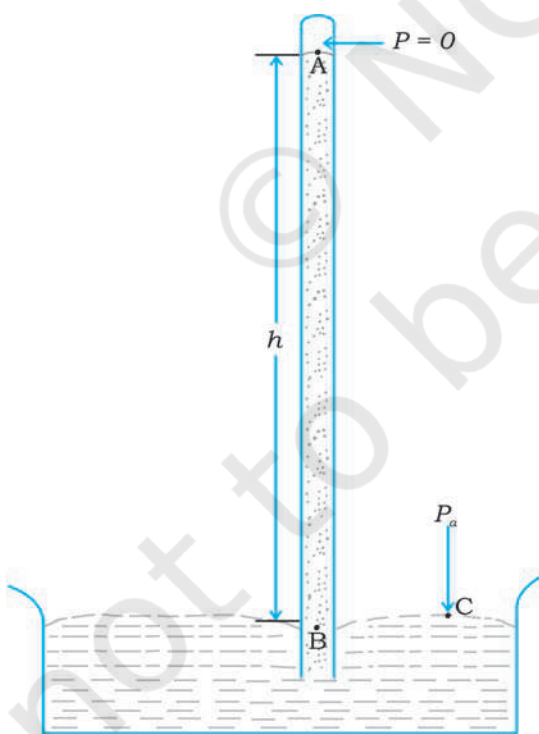
In the experiment it is found that the mercury column in the barometer has a height of about 76 cm at sea level equivalent to one atmosphere (1 atm). This can also be obtained using the value of  $\rho$  in Eq. (9.8). A common way of stating pressure is in terms of cm or mm of mercury (Hg). A pressure equivalent of 1 mm is called a torr (after Torricelli).

$$1 \text{ torr} = 133 \text{ Pa.}$$

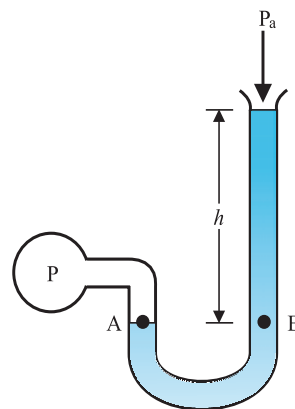
The mm of Hg and torr are used in medicine and physiology. In meteorology, a common unit is the bar and millibar.

$$1 \text{ bar} = 10^5 \text{ Pa}$$

An open tube manometer is a useful instrument for measuring pressure differences. It consists of a U-tube containing a suitable liquid i.e., a low density liquid (such as oil) for measuring small pressure differences and a high density liquid (such as mercury) for large pressure differences. One end of the tube is open to the atmosphere and the other end is connected to the system whose pressure we want to measure [see Fig. 9.5 (b)]. The pressure  $P$  at A is equal to pressure at point B. What we normally measure is the gauge pressure, which is  $P - P_a$ , given by Eq. (9.8) and is proportional to manometer height  $h$ .



**Fig 9.5** (a) The mercury barometer.



(b) The open tube manometer

**Fig 9.5** Two pressure measuring devices.

Pressure is same at the same level on both sides of the U-tube containing a fluid. For liquids, the density varies very little over wide ranges in pressure and temperature and we can treat it safely as a constant for our present purposes. Gases on the other hand, exhibits large variations of densities with changes in pressure and temperature. Unlike gases, liquids are, therefore, largely treated as incompressible.

**Example 9.3** The density of the atmosphere at sea level is  $1.29 \text{ kg/m}^3$ . Assume that it does not change with altitude. Then how high would the atmosphere extend?

**Answer** We use Eq. (9.7)

$$\rho gh = 1.29 \text{ kg m}^{-3} \times 9.8 \text{ m s}^{-2} \times h \text{ m} = 1.01 \times 10^5 \text{ Pa}$$

$$\therefore h = 7989 \text{ m} \approx 8 \text{ km}$$

In reality the density of air decreases with height. So does the value of  $g$ . The atmospheric cover extends with decreasing pressure over 100 km. We should also note that the sea level atmospheric pressure is not always 760 mm of Hg. A drop in the Hg level by 10 mm or more is a sign of an approaching storm.

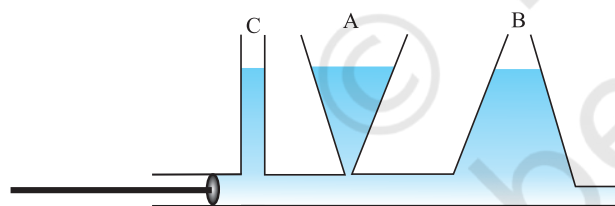
**Example 9.4** At a depth of 1000 m in an ocean (a) what is the absolute pressure? (b) What is the gauge pressure? (c) Find the force acting on the window of area  $20 \text{ cm} \times 20 \text{ cm}$  of a submarine at this depth, the interior of which is maintained at sea-level atmospheric pressure. (The density of sea water is  $1.03 \times 10^3 \text{ kg m}^{-3}$ ,  $g = 10 \text{ m s}^{-2}$ .)

**Answer** Here  $h = 1000 \text{ m}$  and  $\rho = 1.03 \times 10^3 \text{ kg m}^{-3}$ .

- (a) From Eq. (9.6), absolute pressure  
 $P = P_a + \rho gh$   
 $= 1.01 \times 10^5 \text{ Pa}$   
 $+ 1.03 \times 10^3 \text{ kg m}^{-3} \times 10 \text{ m s}^{-2} \times 1000 \text{ m}$   
 $= 104.01 \times 10^5 \text{ Pa}$   
 $\approx 104 \text{ atm}$
- (b) Gauge pressure is  $P - P_a = \rho gh = P_g$   
 $P_g = 1.03 \times 10^3 \text{ kg m}^{-3} \times 10 \text{ m s}^{-2} \times 1000 \text{ m}$   
 $= 103 \times 10^5 \text{ Pa}$   
 $\approx 103 \text{ atm}$
- (c) The pressure outside the submarine is  $P = P_a + \rho gh$  and the pressure inside it is  $P_a$ . Hence, the net pressure acting on the window is gauge pressure,  $P_g = \rho gh$ . Since the area of the window is  $A = 0.04 \text{ m}^2$ , the force acting on it is  
 $F = P_g A = 103 \times 10^5 \text{ Pa} \times 0.04 \text{ m}^2 = 4.12 \times 10^5 \text{ N}$

### 9.2.4 Hydraulic Machines

Let us now consider what happens when we change the pressure on a fluid contained in a vessel. Consider a horizontal cylinder with a piston and three vertical tubes at different points [Fig. 9.6 (a)]. The pressure in the horizontal cylinder is indicated by the height of liquid column in the vertical tubes. It is necessarily the same in all. If we push the piston, the fluid level rises in all the tubes, again reaching the same level in each one of them.



**Fig 9.6 (a)** Whenever external pressure is applied on any part of a fluid in a vessel, it is equally transmitted in all directions.

This indicates that when the pressure on the cylinder was increased, it was distributed uniformly throughout. We can say **whenever external pressure is applied on any part of a fluid contained in a vessel, it is transmitted undiminished and equally in all directions. This is another form of the Pascal's law and it has many applications in daily life.**

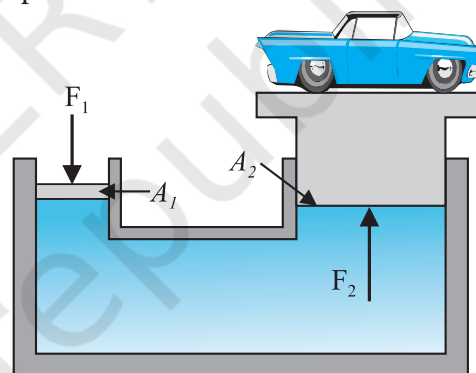
A number of devices, such as **hydraulic lift** and **hydraulic brakes**, are based on the Pascal's

law. In these devices, fluids are used for transmitting pressure. In a hydraulic lift, as shown in Fig. 9.6 (b), two pistons are separated by the space filled with a liquid. A piston of small cross-section  $A_1$  is used to exert a force  $F_1$  directly

on the liquid. The pressure  $P = \frac{F_1}{A_1}$  is transmitted throughout the liquid to the larger cylinder attached with a larger piston of area  $A_2$ , which results in an upward force of  $P \times A_2$ . Therefore, the piston is capable of supporting a large force (large weight of, say a car, or a truck,

placed on the platform)  $F_2 = PA_2 = \frac{F_1 A_2}{A_1}$ . By changing the force at  $A_1$ , the platform can be moved up or down. Thus, the applied force has

been increased by a factor of  $\frac{A_2}{A_1}$  and this factor is the mechanical advantage of the device. The example below clarifies it.



**Fig 9.6 (b)** Schematic diagram illustrating the principle behind the hydraulic lift, a device used to lift heavy loads.

**Example 9.5** Two syringes of different cross-sections (without needles) filled with water are connected with a tightly fitted rubber tube filled with water. Diameters of the smaller piston and larger piston are 1.0 cm and 3.0 cm respectively. (a) Find the force exerted on the larger piston when a force of 10 N is applied to the smaller piston. (b) If the smaller piston is pushed in through 6.0 cm, how much does the larger piston move out?

**Answer** (a) Since pressure is transmitted undiminished throughout the fluid,

$$F_2 = \frac{A_2}{A_1} F_1 = \frac{\pi(3/2 \times 10^{-2} \text{ m})^2}{\pi(1/2 \times 10^{-2} \text{ m})^2} \times 10 \text{ N} \\ = 90 \text{ N}$$

(b) Water is considered to be perfectly incompressible. Volume covered by the movement of smaller piston inwards is equal to volume moved outwards due to the larger piston.

$$L_1 A_1 = L_2 A_2$$

$$L_2 = \frac{A_1}{A_2} L_1 = \frac{\pi(1/2 \times 10^{-2} \text{ m})^2}{\pi(3/2 \times 10^{-2} \text{ m})^2} \times 6 \times 10^{-2} \text{ m} \\ \approx 0.67 \times 10^{-2} \text{ m} = 0.67 \text{ cm}$$

Note, atmospheric pressure is common to both pistons and has been ignored.

**Example 9.6** In a car lift compressed air exerts a force  $F_1$  on a small piston having a radius of 5.0 cm. This pressure is transmitted to a second piston of radius 15 cm (Fig 9.7). If the mass of the car to be lifted is 1350 kg, calculate  $F_1$ . What is the pressure necessary to accomplish this task? ( $g = 9.8 \text{ ms}^{-2}$ ).

**Answer** Since pressure is transmitted undiminished throughout the fluid,

$$F_1 = \frac{A_1}{A_2} F_2 = \frac{\pi(5 \times 10^{-2} \text{ m})^2}{\pi(15 \times 10^{-2} \text{ m})^2} (1350 \text{ kg} \times 9.8 \text{ m s}^{-2}) \\ = 1470 \text{ N} \\ \approx 1.5 \times 10^3 \text{ N}$$

The air pressure that will produce this force is

$$P = \frac{F_1}{A_1} = \frac{1.5 \times 10^3 \text{ N}}{\pi(5 \times 10^{-2})^2 \text{ m}} = 1.9 \times 10^5 \text{ Pa}$$

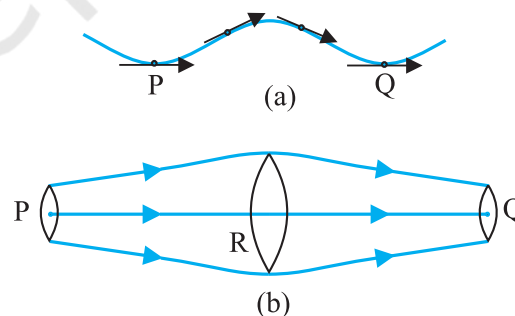
This is almost double the atmospheric pressure.

Hydraulic brakes in automobiles also work on the same principle. When we apply a little force on the pedal with our foot the master piston moves inside the master cylinder, and the pressure caused is transmitted through the brake oil to act on a piston of larger area. A large force acts on the piston and is pushed down expanding the brake shoes against brake lining. In this way, a small force on the pedal produces a large retarding force on the wheel. An

important advantage of the system is that the pressure set up by pressing pedal is transmitted equally to all cylinders attached to the four wheels so that the braking effort is equal on all wheels.

### 9.3 STREAMLINE FLOW

**So far we have studied fluids at rest.** The study of the fluids in motion is known as fluid dynamics. When a water tap is turned on slowly, the water flow is smooth initially, but loses its smoothness when the speed of the outflow is increased. In studying the motion of fluids, we focus our attention on what is happening to various fluid particles at a particular point in space at a particular time. The flow of the fluid is said to be **steady** if at any given point, the velocity of each passing fluid particle remains constant in time. This does not mean that the velocity at different points in space is same. The velocity of a particular particle may change as it moves from one point to another. That is, at some other point the particle may have a different velocity, but every other particle which passes the second point behaves exactly as the previous particle that has just passed that point. Each particle follows a smooth path, and the paths of the particles do not cross each other.



**Fig. 9.7** The meaning of streamlines. (a) A typical trajectory of a fluid particle. (b) A region of streamline flow.

The path taken by a fluid particle under a steady flow is a **streamline**. It is defined as a curve whose tangent at any point is in the direction of the fluid velocity at that point. Consider the path of a particle as shown in Fig.9.7 (a), the curve describes how a fluid particle moves with time. The curve PQ is like a

permanent map of fluid flow, indicating how the fluid streams. No two streamlines can cross, for if they do, an oncoming fluid particle can go either one way or the other and the flow would not be steady. Hence, in steady flow, the map of flow is stationary in time. How do we draw closely spaced streamlines? If we intend to show streamline of every flowing particle, we would end up with a continuum of lines. Consider planes perpendicular to the direction of fluid flow e.g., at three points P, R and Q in Fig. 9.7 (b). The plane pieces are so chosen that their boundaries be determined by the same set of streamlines. This means that number of fluid particles crossing the surfaces as indicated at P, R and Q is the same. If area of cross-sections at these points are  $A_P$ ,  $A_R$  and  $A_Q$  and speeds of fluid particles are  $v_P$ ,  $v_R$  and  $v_Q$ , then mass of fluid  $\Delta m_P$  crossing at  $A_P$  in a small interval of time  $\Delta t$  is  $\rho_P A_P v_P \Delta t$ . Similarly mass of fluid  $\Delta m_R$  flowing or crossing at  $A_R$  in a small interval of time  $\Delta t$  is  $\rho_R A_R v_R \Delta t$  and mass of fluid  $\Delta m_Q$  is  $\rho_Q A_Q v_Q \Delta t$  crossing at  $A_Q$ . The mass of liquid flowing out equals the mass flowing in, holds in all cases. Therefore,

$$\rho_P A_P v_P \Delta t = \rho_R A_R v_R \Delta t = \rho_Q A_Q v_Q \Delta t \quad (9.9)$$

For flow of incompressible fluids

$$\rho_P = \rho_R = \rho_Q$$

Equation (9.9) reduces to

$$A_P v_P = A_R v_R = A_Q v_Q \quad (9.10)$$

which is called the **equation of continuity** and it is a statement of conservation of mass in flow of incompressible fluids. In general

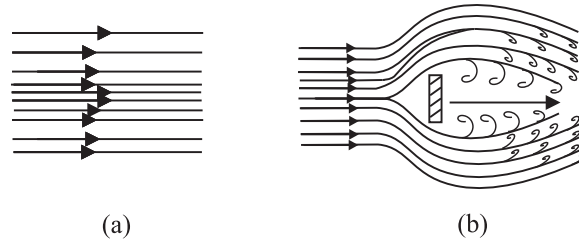
$$Av = \text{constant} \quad (9.11)$$

$Av$  gives the volume flux or flow rate and remains constant throughout the pipe of flow. Thus, at narrower portions where the streamlines are closely spaced, velocity increases and its vice versa. From (Fig 9.7b) it is clear that  $A_R > A_Q$  or  $v_R < v_Q$ , the fluid is accelerated while passing from R to Q. This is associated with a change in pressure in fluid flow in horizontal pipes.

Steady flow is achieved at low flow speeds. Beyond a limiting value, called critical speed, this flow loses steadiness and becomes **turbulent**. One sees this when a fast flowing stream encounters rocks, small foamy whirlpool-like regions called 'white water rapids' are formed.

Figure 9.8 displays streamlines for some typical flows. For example, Fig. 9.8(a) describes a laminar flow where the velocities at different points in the fluid may have different magnitudes

but their directions are parallel. Figure 9.8 (b) gives a sketch of turbulent flow.



**Fig. 9.8** (a) Some streamlines for fluid flow. (b) A jet of air striking a flat plate placed perpendicular to it. This is an example of turbulent flow.

## 9.4 BERNOULLI'S PRINCIPLE

Fluid flow is a complex phenomenon. But we can obtain some useful properties for steady or streamline flows using the conservation of energy.

Consider a fluid moving in a pipe of varying cross-sectional area. Let the pipe be at varying heights as shown in Fig. 9.9. We now suppose that an incompressible fluid is flowing through the pipe in a steady flow. Its velocity must change as a consequence of equation of continuity. A force is required to produce this acceleration, which is caused by the fluid surrounding it, the pressure must be different in different regions. Bernoulli's equation is a general expression that relates the pressure difference between two points in a pipe to both velocity changes (kinetic energy change) and elevation (height) changes (potential energy change). The Swiss Physicist Daniel Bernoulli developed this relationship in 1738.

Consider the flow at two regions 1 (i.e., BC) and 2 (i.e., DE). Consider the fluid initially lying between B and D. In an infinitesimal time interval  $\Delta t$ , this fluid would have moved. Suppose  $v_1$  is the speed at B and  $v_2$  at D, then fluid initially at B has moved a distance  $v_1 \Delta t$  to C ( $v_1 \Delta t$  is small enough to assume constant cross-section along BC). In the same interval  $\Delta t$  the fluid initially at D moves to E, a distance equal to  $v_2 \Delta t$ . Pressures  $P_1$  and  $P_2$  act as shown on the plane faces of areas  $A_1$  and  $A_2$  binding the two regions. The work done on the fluid at left end (BC) is  $W_1 = P_1 A_1 (v_1 \Delta t) = P_1 \Delta V$ . Since the same volume  $\Delta V$  passes through both the regions (from the equation of continuity) the work done by the fluid at the other end (DE) is  $W_2 = P_2 A_2 (v_2 \Delta t) = P_2 \Delta V$  or,



the work done on the fluid is  $-P_2\Delta V$ . So the total work done on the fluid is

$$W_1 - W_2 = (P_1 - P_2) \Delta V$$

Part of this work goes into changing the kinetic energy of the fluid, and part goes into changing the gravitational potential energy. If the density of the fluid is  $\rho$  and  $\Delta m = \rho A_1 v_1 \Delta t = \rho \Delta V$  is the mass passing through the pipe in time  $\Delta t$ , then change in gravitational potential energy is

$$\Delta U = \rho g \Delta V (h_2 - h_1)$$

The change in its kinetic energy is

$$\Delta K = \frac{1}{2} \rho \Delta V (v_2^2 - v_1^2)$$

We can employ the work - energy theorem (Chapter 6) to this volume of the fluid and this yields

$$(P_1 - P_2) \Delta V = \frac{1}{2} \rho \Delta V (v_2^2 - v_1^2) + \rho g \Delta V (h_2 - h_1)$$

We now divide each term by  $\Delta V$  to obtain

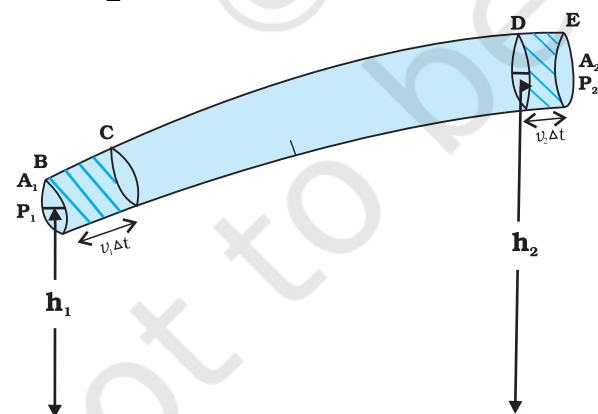
$$(P_1 - P_2) = \frac{1}{2} \rho (v_2^2 - v_1^2) + \rho g (h_2 - h_1)$$

We can rearrange the above terms to obtain

$$P_1 + \frac{1}{2} \rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2} \rho v_2^2 + \rho g h_2 \quad (9.12)$$

This is **Bernoulli's equation**. Since 1 and 2 refer to any two locations along the pipeline, we may write the expression in general as

$$P + \frac{1}{2} \rho v^2 + \rho g h = \text{constant} \quad (9.13)$$



**Fig. 9.9** The flow of an ideal fluid in a pipe of varying cross section. The fluid in a section of length  $v_1\Delta t$  moves to the section of length  $v_2\Delta t$  in time  $\Delta t$ .

In words, the Bernoulli's relation may be stated as follows: As we move along a streamline the sum of the pressure ( $P$ ), the kinetic energy

per unit volume  $\left(\frac{\rho v^2}{2}\right)$  and the potential energy

per unit volume ( $\rho g h$ ) remains a constant.

Note that in applying the energy conservation principle, there is an assumption that no energy is lost due to friction. But in fact, when fluids flow, some energy does get lost due to internal friction. This arises due to the fact that in a fluid flow, the different layers of the fluid flow with different velocities. These layers exert frictional forces on each other resulting in a loss of energy. This property of the fluid is called viscosity and is discussed in more detail in a later section. The lost kinetic energy of the fluid gets converted into heat energy. Thus, Bernoulli's equation ideally applies to fluids with zero viscosity or non-viscous fluids. Another restriction on application of Bernoulli theorem is that the fluids must be incompressible, as the elastic energy of the fluid is also not taken into consideration. In practice, it has a large number of useful applications and can help explain a wide variety of phenomena for low viscosity incompressible fluids. Bernoulli's equation also does not hold for non-steady or turbulent flows, because in that situation velocity and pressure are constantly fluctuating in time.

When a fluid is at rest i.e., its velocity is zero everywhere, Bernoulli's equation becomes

$$P_1 + \rho g h_1 = P_2 + \rho g h_2$$

$$(P_1 - P_2) = \rho g (h_2 - h_1)$$

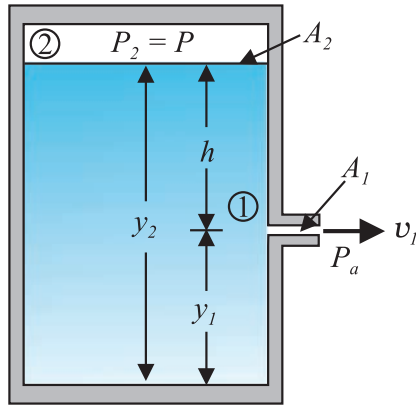
which is same as Eq. (9.6).

#### 9.4.1 Speed of Efflux: Torricelli's Law

The word efflux means fluid outflow. Torricelli discovered that the speed of efflux from an open tank is given by a formula identical to that of a freely falling body. Consider a tank containing a liquid of density  $\rho$  with a small hole in its side at a height  $y_1$  from the bottom (see Fig. 9.10). The air above the liquid, whose surface is at height  $y_2$ , is at pressure  $P$ . From the equation of continuity [Eq. (9.10)] we have

$$v_1 A_1 = v_2 A_2$$

$$v_2 = \frac{A_1}{A_2} v_1$$



**Fig. 9.10** Torricelli's law. The speed of efflux,  $v_1$ , from the side of the container is given by the application of Bernoulli's equation. If the container is open at the top to the atmosphere then  $v_1 = \sqrt{2gh}$ .

If the cross-sectional area of the tank  $A_2$  is much larger than that of the hole ( $A_2 \gg A_1$ ), then we may take the fluid to be approximately at rest at the top, i.e.,  $v_2 = 0$ . Now, applying the Bernoulli equation at points 1 and 2 and noting that at the hole  $P_1 = P_a$ , the atmospheric pressure, we have from Eq. (9.12)

$$P_a + \frac{1}{2}\rho v_1^2 + \rho g y_1 = P + \rho g y_2$$

Taking  $y_2 - y_1 = h$  we have

$$v_1 = \sqrt{2gh + \frac{2(P - P_a)}{\rho}} \quad (9.14)$$

When  $P \gg P_a$  and  $2gh$  may be ignored, the speed of efflux is determined by the container pressure. Such a situation occurs in rocket propulsion. On the other hand, if the tank is open to the atmosphere, then  $P = P_a$  and

$$v_1 = \sqrt{2gh} \quad (9.15)$$

This is also the speed of a freely falling body. Equation (9.15) represents **Torricelli's law**.

#### 9.4.2 Dynamic Lift

Dynamic lift is the force that acts on a body, such as airplane wing, a hydrofoil or a spinning ball, by virtue of its motion through a fluid. In many games such as cricket, tennis, baseball, or golf, we notice that a spinning ball deviates

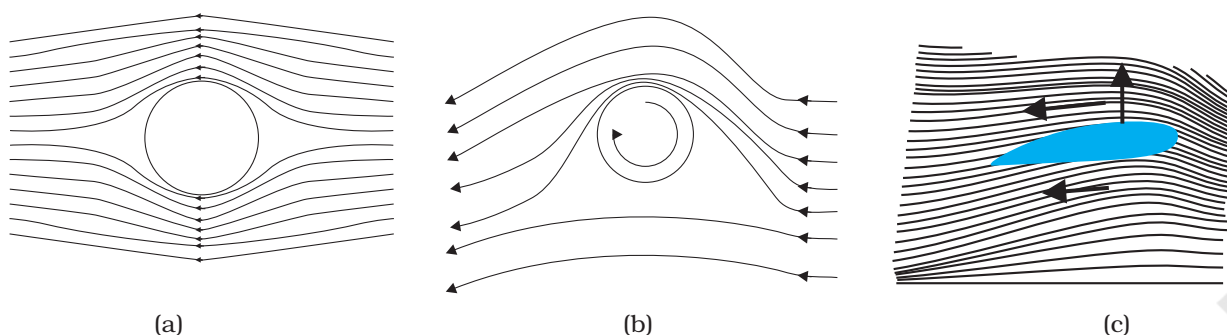
from its parabolic trajectory as it moves through air. This deviation can be partly explained on the basis of Bernoulli's principle.

(i) **Ball moving without spin:** Fig. 9.11(a) shows the streamlines around a non-spinning ball moving relative to a fluid. From the symmetry of streamlines it is clear that the velocity of fluid (air) above and below the ball at corresponding points is the same resulting in zero pressure difference. The air therefore, exerts no upward or downward force on the ball.

(ii) **Ball moving with spin:** A ball which is spinning drags air along with it. If the surface is rough more air will be dragged. Fig 9.11(b) shows the streamlines of air for a ball which is moving and spinning at the same time. The ball is moving forward and relative to it the air is moving backwards. Therefore, the velocity of air above the ball relative to the ball is larger and below it is smaller (see Section 9.3). The stream lines, thus, get crowded above and rarified below.

This difference in the velocities of air results in the pressure difference between the lower and upper faces and there is a net upward force on the ball. This dynamic lift due to spinning is called **Magnus effect**.

**Aerofoil or lift on aircraft wing:** Figure 9.11 (c) shows an aerofoil, which is a solid piece shaped to provide an upward dynamic lift when it moves horizontally through air. The cross-section of the wings of an aeroplane looks somewhat like the aerofoil shown in Fig. 9.11 (c) with streamlines around it. When the aerofoil moves against the wind, the orientation of the wing relative to flow direction causes the streamlines to crowd together above the wing more than those below it. The flow speed on top is higher than that below it. There is an upward force resulting in a dynamic lift of the wings and this balances the weight of the plane. The following example illustrates this.



**Fig 9.11** (a) Fluid streaming past a static sphere. (b) Streamlines for a fluid around a sphere spinning clockwise. (c) Air flowing past an aerofoil.

**Example 9.7** A fully loaded Boeing aircraft has a mass of  $3.3 \times 10^5$  kg. Its total wing area is  $500 \text{ m}^2$ . It is in level flight with a speed of  $960 \text{ km/h}$ . (a) Estimate the pressure difference between the lower and upper surfaces of the wings (b) Estimate the fractional increase in the speed of the air on the upper surface of the wing relative to the lower surface. [The density of air is  $\rho = 1.2 \text{ kg m}^{-3}$ ]

**Answer** (a) The weight of the Boeing aircraft is balanced by the upward force due to the pressure difference

$$\Delta P \cdot A = 3.3 \times 10^5 \text{ kg} \times 9.8$$

$$\Delta P = (3.3 \times 10^5 \text{ kg} \times 9.8 \text{ m s}^{-2}) / 500 \text{ m}^2 \\ = 6.5 \times 10^3 \text{ Nm}^{-2}$$

(b) We ignore the small height difference between the top and bottom sides in Eq. (9.12). The pressure difference between them is then

$$\Delta P = \frac{\rho}{2} (v_2^2 - v_1^2)$$

where  $v_2$  is the speed of air over the upper surface and  $v_1$  is the speed under the bottom surface.

$$(v_2 - v_1) = \frac{2\Delta P}{\rho(v_2 + v_1)}$$

Taking the average speed

$$v_{av} = (v_2 + v_1)/2 = 960 \text{ km/h} = 267 \text{ m s}^{-1},$$

we have

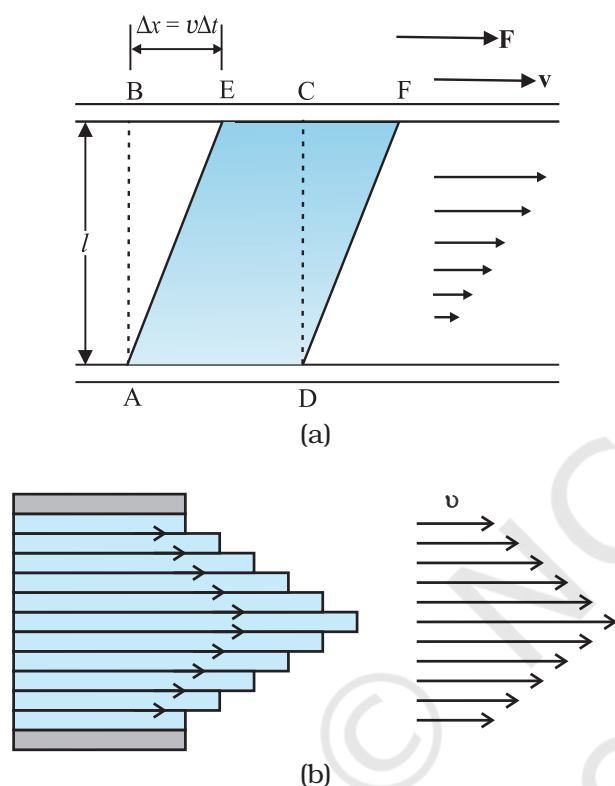
$$(v_2 - v_1)/v_{av} = \frac{\Delta P}{\rho v_{av}^2} \approx 0.08$$

The speed above the wing needs to be only 8 % higher than that below.

## 9.5 VISCOSITY

Most of the fluids are not ideal ones and offer some resistance to motion. This resistance to fluid motion is like an internal friction analogous to friction when a solid moves on a surface. It is called viscosity. This force exists when there is relative motion between layers of the liquid. Suppose we consider a fluid like oil enclosed between two glass plates as shown in Fig. 9.12 (a). The bottom plate is fixed while the top plate is moved with a constant velocity  $\mathbf{v}$  relative to the fixed plate. If oil is replaced by honey, a greater force is required to move the plate with the same velocity. Hence we say that honey is more viscous than oil. The fluid in contact with a surface has the same velocity as that of the surfaces. Hence, the layer of the liquid in contact with top surface moves with a velocity  $\mathbf{v}$  and the layer of the liquid in contact with the fixed surface is stationary. The velocities of layers increase uniformly from bottom (zero velocity) to the top layer (velocity  $\mathbf{v}$ ). For any layer of liquid, its upper layer pulls it forward while lower layer pulls it backward. This results in force between the layers. This

type of flow is known as laminar. The layers of liquid slide over one another as the pages of a book do when it is placed flat on a table and a horizontal force is applied to the top cover. When a fluid is flowing in a pipe or a tube, then velocity of the liquid layer along the axis of the tube is maximum and decreases gradually as we move towards the walls where it becomes zero, Fig. 9.12 (b). The velocity on a cylindrical surface in a tube is constant.



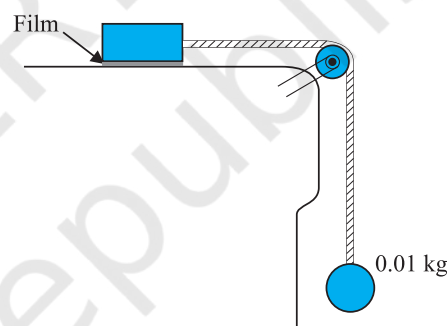
**Fig 9.12** (a) A layer of liquid sandwiched between two parallel glass plates, in which the lower plate is fixed and the upper one is moving to the right with velocity  $v$ . (b) velocity distribution for viscous flow in a pipe.

On account of this motion, a portion of liquid, which at some instant has the shape ABCD, take the shape of AEDF after short interval of time ( $\Delta t$ ). During this time interval the liquid has undergone a shear strain of  $\Delta x/l$ . Since, the strain in a flowing fluid increases with time continuously. Unlike a solid, here the stress is found experimentally to depend on 'rate of

change of strain' or 'strain rate' i.e.  $\Delta x/(l \Delta t)$  or  $v/l$  instead of strain itself. The coefficient of viscosity (pronounced 'eta') for a fluid is defined as the ratio of shearing stress to the strain rate.

$$\eta = \frac{F/A}{v/l} = \frac{Fl}{vA} \quad (9.16)$$

The SI unit of viscosity is poiseuille (Pl). Its other units are  $\text{N s m}^{-2}$  or  $\text{Pa s}$ . The dimensions of viscosity are  $[\text{ML}^{-1}\text{T}^{-1}]$ . Generally, thin liquids, like water, alcohol, etc., are less viscous than thick liquids, like coal tar, blood, glycerine, etc. The coefficients of viscosity for some common fluids are listed in Table 9.2. We point out two facts about blood and water that you may find interesting. As Table 9.2 indicates, blood is 'thicker' (more viscous) than water. Further, the relative viscosity ( $\eta/\eta_{\text{water}}$ ) of blood remains constant between  $0^\circ\text{C}$  and  $37^\circ\text{C}$ .



**Fig. 9.13** Measurement of the coefficient of viscosity of a liquid.

The viscosity of liquids decreases with temperature, while it increases in the case of gases.

**Example 9.8** A metal block of area  $0.10 \text{ m}^2$  is connected to a  $0.010 \text{ kg}$  mass via a string that passes over an ideal pulley (considered massless and frictionless), as in Fig. 9.13. A liquid with a film thickness of  $0.30 \text{ mm}$  is placed between the block and the table. When released the block moves to the right with a constant speed of  $0.085 \text{ m s}^{-1}$ . Find the coefficient of viscosity of the liquid.



**Answer** The metal block moves to the right because of the tension in the string. The tension  $T$  is equal in magnitude to the weight of the suspended mass  $m$ . Thus, the shear force  $F$  is  $F = T = mg = 0.010 \text{ kg} \times 9.8 \text{ m s}^{-2} = 9.8 \times 10^{-2} \text{ N}$

$$\text{Shear stress on the fluid} = F/A = \frac{9.8 \times 10^{-2}}{0.10} \text{ N/m}^2$$

$$\text{Strain rate} = \frac{v}{l} = \frac{0.085}{0.30 \times 10^{-3}}$$

$$h = \frac{\text{stress}}{\text{strain rate}} \text{ s}^{-1}$$

$$= \frac{(9.8 \times 10^{-2} \text{ N}) (0.30 \times 10^{-3} \text{ m})}{(0.085 \text{ m s}^{-1}) (0.10 \text{ m}^2)}$$

$$= 3.46 \times 10^{-3} \text{ Pa s}$$

**Table 9.2 The viscosities of some fluids**

Fluid	T(°C)	Viscosity (mPl)
Water	20	1.0
	100	0.3
Blood	37	2.7
	16	113
Machine Oil	38	34
	20	830
Glycerine	–	200
Honey	0	0.017
Air	40	0.019

### 9.5.1 Stokes' Law

When a body falls through a fluid it drags the layer of the fluid in contact with it. A relative motion between the different layers of the fluid is set and, as a result, the body experiences a retarding force. Falling of a raindrop and swinging of a pendulum bob are some common examples of such motion. It is seen that the viscous force is proportional to the velocity of the object and is opposite to the direction of motion. The other quantities on which the force  $F$  depends are viscosity  $\eta$  of the fluid and radius  $a$  of the sphere. Sir George G. Stokes (1819–1903), an English scientist enunciated clearly the viscous drag force  $F$  as

$$F = 6\pi\eta av \quad (9.17)$$

This is known as Stokes' law. We shall not derive Stokes' law.

This law is an interesting example of retarding force, which is proportional to velocity. We can study its consequences on an object falling through a viscous medium. We consider a raindrop in air. It accelerates initially due to gravity. As the velocity increases, the retarding force also increases. Finally, when viscous force plus buoyant force becomes equal to the force due to gravity, the net force becomes zero and so does the acceleration. The sphere (raindrop) then descends with a constant velocity. Thus, in equilibrium, this terminal velocity  $v_t$  is given by

$$6\pi\eta av_t = (4\pi/3) a^3 (\rho - \sigma)g$$

where  $\rho$  and  $\sigma$  are mass densities of sphere and the fluid, respectively. We obtain

$$v_t = 2a^2 (\rho - \sigma)g / (9\eta) \quad (9.18)$$

So the terminal velocity  $v_t$  depends on the square of the radius of the sphere and inversely on the viscosity of the medium.

You may like to refer back to Example 6.2 in this context.

**Example 9.9** The terminal velocity of a copper ball of radius 2.0 mm falling through a tank of oil at 20°C is 6.5 cm s<sup>-1</sup>. Compute the viscosity of the oil at 20°C. Density of oil is 1.5 × 10<sup>3</sup> kg m<sup>-3</sup>, density of copper is 8.9 × 10<sup>3</sup> kg m<sup>-3</sup>.

**Answer** We have  $v_t = 6.5 \times 10^{-2} \text{ m s}^{-1}$ ,  $a = 2 \times 10^{-3} \text{ m}$ ,  $g = 9.8 \text{ m s}^{-2}$ ,  $\rho = 8.9 \times 10^3 \text{ kg m}^{-3}$ ,

$\sigma = 1.5 \times 10^3 \text{ kg m}^{-3}$ . From Eq. (9.18)

$$\eta = \frac{2}{9} \times \frac{(2 \times 10^{-3})^2 \text{ m}^2 \times 9.8 \text{ m s}^{-2}}{6.5 \times 10^{-2} \text{ m s}^{-1}} \times 7.4 \times 10^3 \text{ kg m}^{-3}$$

$$= 9.9 \times 10^{-1} \text{ kg m}^{-1} \text{ s}^{-1}$$

### 9.6 SURFACE TENSION

You must have noticed that, oil and water do not mix; water wets you and me but not ducks; mercury does not wet glass but water sticks to it, oil rises up a cotton wick, inspite of gravity,

Sap and water rise up to the top of the leaves of the tree, hair of a paint brush do not cling together when dry and even when dipped in water but form a fine tip when taken out of it. All these and many more such experiences are related with the free surfaces of liquids. As liquids have no definite shape but have a definite volume, they acquire a free surface when poured in a container. These surfaces possess some additional energy. This phenomenon is known as surface tension and it is concerned with only liquid as gases do not have free surfaces. Let us now understand this phenomena.

### 9.6.1 Surface Energy

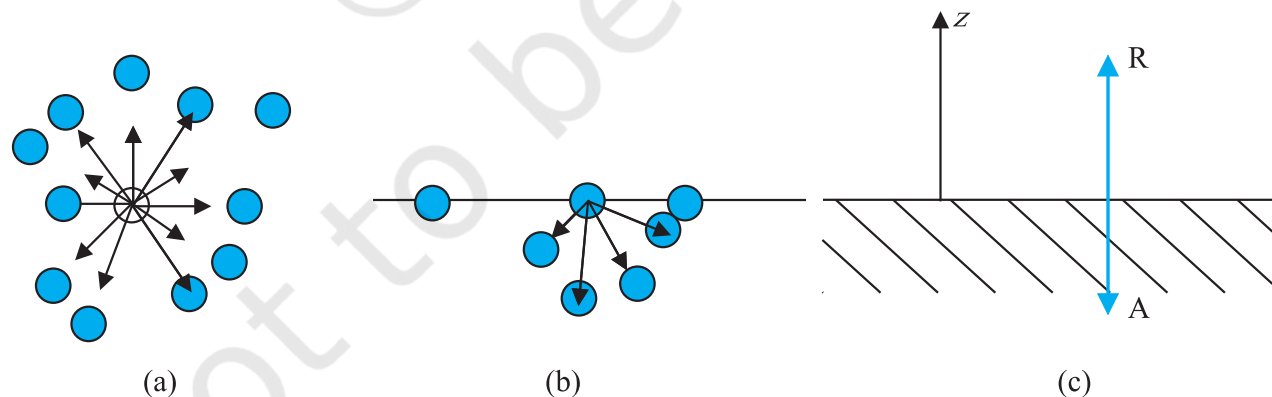
A liquid stays together because of attraction between molecules. Consider a molecule well inside a liquid. The intermolecular distances are such that it is attracted to all the surrounding molecules [Fig. 9.14(a)]. This attraction results in a negative potential energy for the molecule, which depends on the number and distribution of molecules around the chosen one. But the average potential energy of all the molecules is the same. This is supported by the fact that to take a collection of such molecules (the liquid) and to disperse them far away from each other in order to evaporate or vaporise, the heat of evaporation required is quite large. For water it is of the order of 40 kJ/mol.

Let us consider a molecule near the surface Fig. 9.14(b). Only lower half side of it is surrounded by liquid molecules. There is some negative potential energy due to these, but obviously it is less than that of a molecule in bulk, i.e., the one fully inside. Approximately it is half of the latter. Thus, molecules on a liquid surface have some extra energy in comparison to molecules in the interior. A liquid, thus, tends to have the least surface area which external conditions permit. Increasing surface area requires energy. Most surface phenomenon can be understood in terms of this fact. What is the energy required for having a molecule at the surface? As mentioned above, roughly it is half the energy required to remove it entirely from the liquid i.e., half the heat of evaporation.

Finally, what is a surface? Since a liquid consists of molecules moving about, there cannot be a perfectly sharp surface. The density of the liquid molecules drops rapidly to zero around  $z = 0$  as we move along the direction indicated Fig 9.14 (c) in a distance of the order of a few molecular sizes.

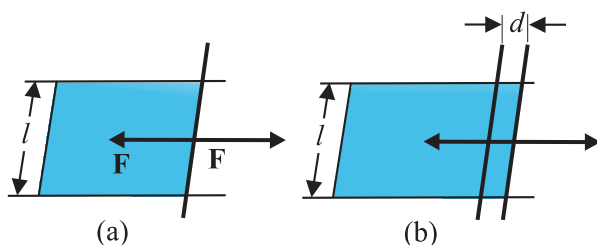
### 9.6.2 Surface Energy and Surface Tension

As we have discussed that an extra energy is associated with surface of liquids, the creation of more surface (spreading of surface) keeping other things like volume fixed requires a



**Fig. 9.14** Schematic picture of molecules in a liquid, at the surface and balance of forces. (a) Molecule inside a liquid. Forces on a molecule due to others are shown. Direction of arrows indicates attraction of repulsion. (b) Same, for a molecule at a surface. (c) Balance of attractive (A) and repulsive (R) forces.

horizontal liquid film ending in bar free to slide over parallel guides Fig (9.15).



**Fig. 9.15** Stretching a film. (a) A film in equilibrium; (b) The film stretched an extra distance.

Suppose that we move the bar by a small distance  $d$  as shown. Since the area of the surface increases, the system now has more energy, this means that some work has been done against an internal force. Let this internal force be  $\mathbf{F}$ , the work done by the applied force is  $\mathbf{F} \cdot \mathbf{d} = Fd$ . From conservation of energy, this is stored as additional energy in the film. If the surface energy of the film is  $S$  per unit area, the extra area is  $2dl$ . A film has two sides and the liquid in between, so there are two surfaces and the extra energy is

$$S(2dl) = Fd \quad (9.19)$$

$$\text{Or, } S = Fd/2dl = F/2l \quad (9.20)$$

This quantity  $S$  is the magnitude of surface tension. It is equal to the surface energy per unit area of the liquid interface and is also equal to the force per unit length exerted by the fluid on the movable bar.

So far we have talked about the surface of one liquid. More generally, we need to consider fluid surface in contact with other fluids or solid surfaces. The surface energy in that case depends on the materials on both sides of the surface. For example, if the molecules of the materials attract each other, surface energy is reduced while if they repel each other the surface energy is increased. Thus, more appropriately, the surface energy is the energy of the interface between two materials and depends on both of them.

We make the following observations from above:

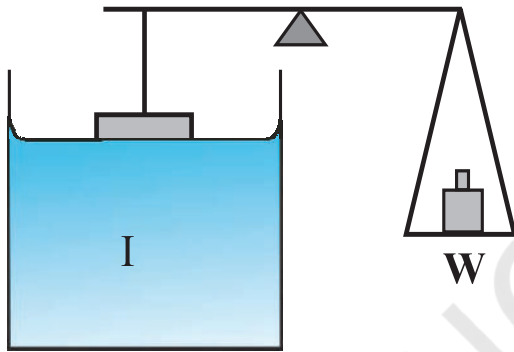
- (i) Surface tension is a force per unit length (or surface energy per unit area) acting in the plane of the interface between the plane of the liquid and any other substance; it also is the extra energy that the molecules at the interface have as compared to molecules in the interior.
- (ii) At any point on the interface besides the boundary, we can draw a line and imagine equal and opposite surface tension forces  $S$  per unit length of the line acting perpendicular to the line, in the plane of the interface. The line is in equilibrium. To be more specific, imagine a line of atoms or molecules at the surface. The atoms to the left pull the line towards them; those to the right pull it towards them! This line of atoms is in equilibrium under tension. If the line really marks the end of the interface, as in Figure 9.14 (a) and (b) there is only the force  $S$  per unit length acting inwards.

Table 9.3 gives the surface tension of various liquids. The value of surface tension depends on temperature. Like viscosity, the surface tension of a liquid usually falls with temperature.

**Table 9.3** Surface tension of some liquids at the temperatures indicated with the heats of the vaporisation

Liquid	Temp (°C)	Surface Tension (N/m)	Heat of vaporisation (kJ/mol)
Helium	-270	0.000239	0.115
Oxygen	-183	0.0132	7.1
Ethanol	20	0.0227	40.6
Water	20	0.0727	44.16
Mercury	20	0.4355	63.2

A fluid will stick to a solid surface if the surface energy between fluid and the solid is smaller than the sum of surface energies between solid-air, and fluid-air. Now there is attraction between the solid surface and the liquid. It can be directly measured experimentally as schematically shown in Fig. 9.16. A flat vertical glass plate, below which a vessel of some liquid is kept, forms one arm of the balance. The plate is balanced by weights on the other side, with its horizontal edge just over water. The vessel is raised slightly till the liquid just touches the glass plate and pulls it down a little because of surface tension. Weights are added till the plate just clears water.



**Fig. 9.16** Measuring Surface Tension.

Suppose the additional weight required is  $W$ . Then from Eq. 9.20 and the discussion given there, the surface tension of the liquid-air interface is

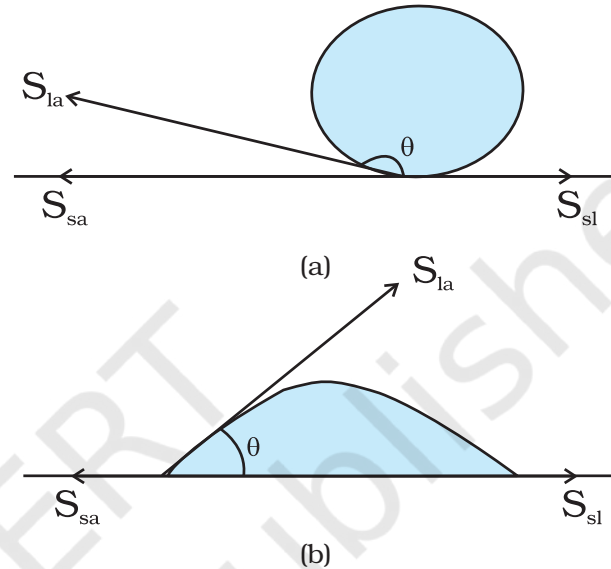
$$S_{la} = (W/2l) = (mg/2l) \quad (9.21)$$

where  $m$  is the extra mass and  $l$  is the length of the plate edge. The subscript (la) emphasises the fact that the liquid-air interface tension is involved.

### 9.6.3 Angle of Contact

The surface of liquid near the plane of contact, with another medium is in general curved. The angle between tangent to the liquid surface at the point of contact and solid surface inside the liquid is termed as angle of contact. It is denoted

by  $\theta$ . It is different at interfaces of different pairs of liquids and solids. The value of  $\theta$  determines whether a liquid will spread on the surface of a solid or it will form droplets on it. For example, water forms droplets on lotus leaf as shown in Fig. 9.17 (a) while spreads over a clean plastic plate as shown in Fig. 9.17(b).



**Fig. 9.17** Different shapes of water drops with interfacial tensions (a) on a lotus leaf (b) on a clean plastic plate.

We consider the three interfacial tensions at all the three interfaces, liquid-air, solid-air and solid-liquid denoted by  $S_{la}$ ,  $S_{sa}$  and  $S_{sl}$ , respectively as given in Fig. 9.17 (a) and (b). At the line of contact, the surface forces between the three media must be in equilibrium. From the Fig. 9.17(b) the following relation is easily derived.

$$S_{la} \cos \theta + S_{sl} = S_{sa} \quad (9.22)$$

The angle of contact is an obtuse angle if  $S_{sl} > S_{la}$  as in the case of water-leaf interface while it is an acute angle if  $S_{sl} < S_{la}$  as in the case of water-plastic interface. When  $\theta$  is an obtuse angle then molecules of liquids are attracted strongly to themselves and weakly to those of solid, it costs a lot of energy to create a liquid-solid surface, and liquid then does not wet the solid. This is what happens with water on a waxy or oily surface, and with mercury on



any surface. On the other hand, if the molecules of the liquid are strongly attracted to those of the solid, this will reduce  $S_{sl}$  and therefore,  $\cos \theta$  may increase or  $\theta$  may decrease. In this case  $\theta$  is an acute angle. This is what happens for water on glass or on plastic and for kerosene oil on virtually anything (it just spreads). Soaps, detergents and dyeing substances are wetting agents. When they are added the angle of contact becomes small so that these may penetrate well and become effective. Water proofing agents on the other hand are added to create a large angle of contact between the water and fibres.

#### 9.6.4 Drops and Bubbles

One consequence of surface tension is that free liquid drops and bubbles are spherical if effects of gravity can be neglected. You must have seen this especially clearly in small drops just formed in a high-speed spray or jet, and in soap bubbles blown by most of us in childhood. Why are drops and bubbles spherical? What keeps soap bubbles stable?

As we have been saying repeatedly, a liquid-air interface has energy, so for a given volume the surface with minimum energy is the one with the least area. The sphere has this property. Though it is out of the scope of this book, but you can check that a sphere is better than at least a cube in this respect! So, if gravity and other forces (e.g. air resistance) were ineffective, liquid drops would be spherical.

Another interesting consequence of surface tension is that the pressure inside a spherical drop Fig. 9.18(a) is more than the pressure outside. Suppose a spherical drop of radius  $r$  is in equilibrium. If its radius increase by  $\Delta r$ . The extra surface energy is

$$[4\pi(r + \Delta r)^2 - 4\pi r^2] S_{la} = 8\pi r \Delta r S_{la} \quad (9.23)$$

If the drop is in equilibrium this energy cost is balanced by the energy gain due to expansion under the pressure difference ( $P_i - P_o$ ) between the inside of the bubble and the outside. The work done is

$$W = (P_i - P_o) 4\pi r^2 \Delta r \quad (9.24)$$

so that

$$(P_i - P_o) = (2 S_{la} / r) \quad (9.25)$$

In general, for a liquid-gas interface, the convex side has a higher pressure than the concave side. For example, an air bubble in a liquid, would have higher pressure inside it. See Fig 9.18 (b).

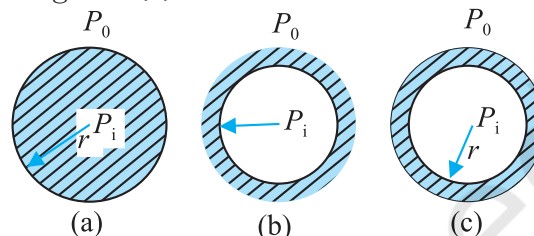


Fig. 9.18 Drop, cavity and bubble of radius  $r$ .

A bubble Fig 9.18 (c) differs from a drop and a cavity; in this it has two interfaces. Applying the above argument we have for a bubble

$$(P_i - P_o) = (4 S_{la} / r) \quad (9.26)$$

This is probably why you have to blow hard, but not too hard, to form a soap bubble. A little extra air pressure is needed inside!

#### 9.6.5 Capillary Rise

One consequence of the pressure difference across a curved liquid-air interface is the well-known effect that water rises up in a narrow tube in spite of gravity. The word capilla means hair in Latin; if the tube were hair thin, the rise would be very large. To see this, consider a vertical capillary tube of circular cross section (radius  $a$ ) inserted into an open vessel of water (Fig. 9.19). The contact angle between water and

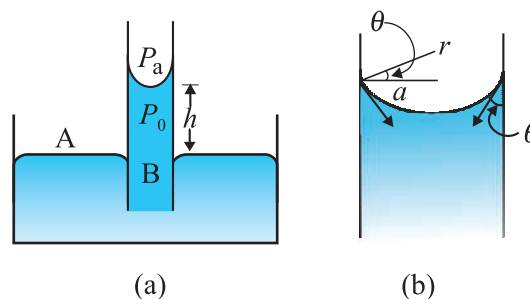


Fig. 9.19 Capillary rise, (a) Schematic picture of a narrow tube immersed water. (b) Enlarged picture near interface.

glass is acute. Thus the surface of water in **the capillary is concave. This means that there is a pressure difference between the two sides of the top surface. This is given by**

$$(P_i - P_o) = (2S/r) = 2S/(a \sec \theta) \\ = (2S/a) \cos \theta \quad (9.27)$$

**Thus the pressure of the water inside the tube, just at the meniscus (air-water interface) is less than the atmospheric pressure. Consider the two points A and B in Fig. 9.19(a). They must be at the same pressure, namely**

$$P_o + h \rho g = P_i = P_A \quad (9.28)$$

**where  $\rho$  is the density of water and  $h$  is called the capillary rise** [Fig. 9.19(a)]. Using Eq. (9.27) and (9.28) we have

$$h \rho g = (P_i - P_o) = (2S \cos \theta)/a \quad (9.29)$$

The discussion here, and the Eqs. (9.24) and (9.25) make it clear that the capillary rise is due to surface tension. It is larger, for a smaller  $a$ . Typically it is of the order of a few cm for fine capillaries. For example, if  $a = 0.05$  cm, using the value of surface tension for water (Table 9.3), we find that

$$h = 2S/(\rho g a) \\ = \frac{2 \times (0.073 \text{ N m}^{-1})}{(10^3 \text{ kg m}^{-3})(9.8 \text{ m s}^{-2})(5 \times 10^{-4} \text{ m})} \\ = 2.98 \times 10^{-2} \text{ m} = 2.98 \text{ cm}$$

Notice that if the liquid meniscus is convex, as for mercury, i.e., if  $\cos \theta$  is negative then from Eq. (9.28) for example, it is clear that the liquid will be lower in the capillary!

**Example 9.10** The lower end of a capillary tube of diameter 2.00 mm is dipped 8.00 cm below the surface of water in a beaker. What is the pressure required in the tube in order to blow a hemispherical bubble at its end in water? The surface tension of water at temperature of the experiments is  $7.30 \times 10^{-2} \text{ Nm}^{-1}$ . 1 atmospheric pressure =  $1.01 \times 10^5 \text{ Pa}$ , density of water =  $1000 \text{ kg/m}^3$ ,  $g = 9.80 \text{ m s}^{-2}$ . Also calculate the excess pressure.

**Answer** The excess pressure in a bubble of gas in a liquid is given by  $2S/r$ , where  $S$  is the surface tension of the liquid-gas interface. You should note there is only one liquid surface in this case. (For a bubble of liquid in a gas, there are two liquid surfaces, so the formula for excess pressure in that case is  $4S/r$ .) The radius of the bubble is  $r$ . Now the pressure outside the bubble  $P_o$  equals atmospheric pressure plus the pressure due to 8.00 cm of water column. That is

$$P_o = (1.01 \times 10^5 \text{ Pa} + 0.08 \text{ m} \times 1000 \text{ kg m}^{-3} \times 9.80 \text{ m s}^{-2}) \\ = 1.01784 \times 10^5 \text{ Pa}$$

$$\text{Therefore, the pressure inside the bubble is} \\ P_i = P_o + 2S/r \\ = 1.01784 \times 10^5 \text{ Pa} + (2 \times 7.3 \times 10^{-2} \text{ Pa m} / 10^{-3} \text{ m}) \\ = (1.01784 + 0.00146) \times 10^5 \text{ Pa} \\ = 1.02 \times 10^5 \text{ Pa}$$

where the radius of the bubble is taken to be equal to the radius of the capillary tube, since the bubble is hemispherical! (The answer has been rounded off to three significant figures.) The excess pressure in the bubble is 146 Pa. ◀

### SUMMARY

1. The basic property of a fluid is that it can flow. The fluid does not have any resistance to change of its shape. Thus, the shape of a fluid is governed by the shape of its container.
2. A liquid is incompressible and has a free surface of its own. A gas is compressible and it expands to occupy all the space available to it.
3. If  $F$  is the normal force exerted by a fluid on an area  $A$  then the average pressure  $P_{av}$  is defined as the ratio of the force to area

$$P_{av} = \frac{F}{A}$$

4. The unit of the pressure is the pascal (Pa). It is the same as  $\text{N m}^{-2}$ . Other common units of pressure are  
 $1 \text{ atm} = 1.01 \times 10^5 \text{ Pa}$   
 $1 \text{ bar} = 10^5 \text{ Pa}$   
 $1 \text{ torr} = 133 \text{ Pa} = 0.133 \text{ kPa}$   
 $1 \text{ mm of Hg} = 1 \text{ torr} = 133 \text{ Pa}$
5. *Pascal's law* states that: Pressure in a fluid at rest is same at all points which are at the same height. A change in pressure applied to an enclosed fluid is transmitted undiminished to every point of the fluid and the walls of the containing vessel.
6. The pressure in a fluid varies with depth  $h$  according to the expression  
 $P = P_a + \rho gh$   
 where  $\rho$  is the density of the fluid, assumed uniform.
7. The volume of an incompressible fluid passing any point every second in a pipe of non uniform cross-section is the same in the steady flow.  
 $vA = \text{constant}$  ( $v$  is the velocity and  $A$  is the area of cross-section)  
 The equation is due to mass conservation in incompressible fluid flow.
8. *Bernoulli's principle* states that as we move along a streamline, the sum of the pressure ( $P$ ), the kinetic energy per unit volume ( $\rho v^2/2$ ) and the potential energy per unit volume ( $\rho gy$ ) remains a constant.  
 $P + \rho v^2/2 + \rho gy = \text{constant}$   
 The equation is basically the conservation of energy applied to non viscous fluid motion in steady state. There is no fluid which has zero viscosity, so the above statement is true only approximately. The viscosity is like friction and converts the kinetic energy to heat energy.
9. Though shear strain in a fluid does not require shear stress, when a shear stress is applied to a fluid, the motion is generated which causes a shear strain growing with time. The ratio of the shear stress to the time rate of shearing strain is known as coefficient of viscosity,  $\eta$ .  
 where symbols have their usual meaning and are defined in the text.
10. *Stokes' law* states that the viscous drag force  $\mathbf{F}$  on a sphere of radius  $a$  moving with velocity  $\mathbf{v}$  through a fluid of viscosity is,  $\mathbf{F} = 6\pi\eta a\mathbf{v}$ .
11. Surface tension is a force per unit length (or surface energy per unit area) acting in the plane of interface between the liquid and the bounding surface. It is the extra energy that the molecules at the interface have as compared to the interior.

#### POINTS TO PONDER

1. Pressure is a *scalar quantity*. The definition of the pressure as “force per unit area” may give one false impression that pressure is a vector. The “force” in the numerator of the definition is the component of the force normal to the area upon which it is impressed. While describing fluids as a concept, shift from particle and rigid body mechanics is required. We are concerned with properties that vary from point to point in the fluid.
2. One should not think of pressure of a fluid as being exerted only on a solid like the walls of a container or a piece of solid matter immersed in the fluid. Pressure exists at all points in a fluid. An element of a fluid (such as the one shown in Fig. 9.4) is in equilibrium because the pressures exerted on the various faces are equal.

3. The expression for pressure  
 $P = P_a + \rho gh$   
holds true if fluid is incompressible. Practically speaking it holds for liquids, which are largely incompressible and hence  $\rho$  is a constant with height.
4. The gauge pressure is the difference of the actual pressure and the atmospheric pressure.  
 $P - P_a = P_g$   
Many pressure-measuring devices measure the gauge pressure. These include the tyre pressure gauge and the blood pressure gauge (sphygmomanometer).
5. A streamline is a map of fluid flow. In a steady flow two streamlines do not intersect as it means that the fluid particle will have two possible velocities at the point.
6. Bernoulli's principle does not hold in presence of viscous drag on the fluid. The work done by this dissipative viscous force must be taken into account in this case, and  $P_2$  [Fig. 9.9] will be lower than the value given by Eq. (9.12).
7. As the temperature rises the atoms of the liquid become more mobile and the coefficient of viscosity,  $\eta$  falls. In a gas the temperature rise increases the random motion of atoms and  $\eta$  increases.
8. Surface tension arises due to excess potential energy of the molecules on the surface in comparison to their potential energy in the interior. Such a surface energy is present at the interface separating two substances at least one of which is a fluid. It is not the property of a single fluid alone.

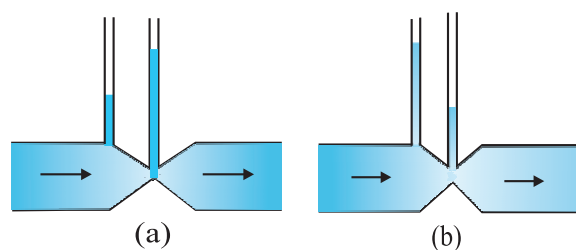
Physical Quantity	Symbol	Dimensions	Unit	Remarks
Pressure	$P$	$[M L^{-1} T^{-2}]$	pascal (Pa)	1 atm = $1.013 \times 10^5$ Pa, Scalar
Density	$\rho$	$[M L^{-3}]$	$kg m^{-3}$	Scalar
Specific Gravity		No	No	$\frac{\rho_{\text{substance}}}{\rho_{\text{water}}}$ , Scalar
Co-efficient of viscosity	$\eta$	$[M L^{-1} T^{-1}]$	Pa s or poiseulles (Pl)	Scalar
Surface Tension	$S$	$[M T^{-2}]$	$N m^{-1}$	Scalar

## EXERCISES

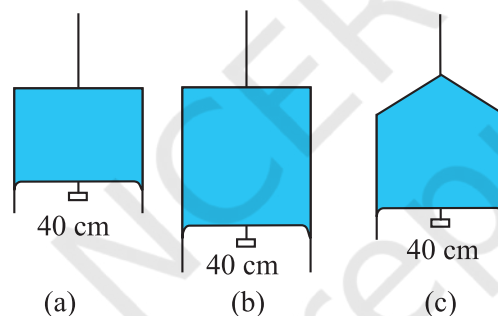
- 9.1 Explain why
  - (a) The blood pressure in humans is greater at the feet than at the brain
  - (b) Atmospheric pressure at a height of about 6 km decreases to nearly half of its value at the sea level, though the height of the atmosphere is more than 100 km
  - (c) Hydrostatic pressure is a scalar quantity even though pressure is force divided by area.
- 9.2 Explain why
  - (a) The angle of contact of mercury with glass is obtuse, while that of water with glass is acute.
  - (b) Water on a clean glass surface tends to spread out while mercury on the same surface tends to form drops. (Put differently, water wets glass while mercury does not.)



- (c) Surface tension of a liquid is independent of the area of the surface
- (d) Water with detergent dissolved in it should have small angles of contact.
- (e) A drop of liquid under no external forces is always spherical in shape
- 9.3** Fill in the blanks using the word(s) from the list appended with each statement:
  - (a) Surface tension of liquids generally ... with temperatures (increases / decreases)
  - (b) Viscosity of gases ... with temperature, whereas viscosity of liquids ... with temperature (increases / decreases)
  - (c) For solids with elastic modulus of rigidity, the shearing force is proportional to ... , while for fluids it is proportional to ... (shear strain / rate of shear strain)
  - (d) For a fluid in a steady flow, the increase in flow speed at a constriction follows (conservation of mass / Bernoulli's principle)
  - (e) For the model of a plane in a wind tunnel, turbulence occurs at a ... speed for turbulence for an actual plane (greater / smaller)
- 9.4** Explain why
  - (a) To keep a piece of paper horizontal, you should blow over, not under, it
  - (b) When we try to close a water tap with our fingers, fast jets of water gush through the openings between our fingers
  - (c) The size of the needle of a syringe controls flow rate better than the thumb pressure exerted by a doctor while administering an injection
  - (d) A fluid flowing out of a small hole in a vessel results in a backward thrust on the vessel
  - (e) A spinning cricket ball in air does not follow a parabolic trajectory
- 9.5** A 50 kg girl wearing high heel shoes balances on a single heel. The heel is circular with a diameter 1.0 cm. What is the pressure exerted by the heel on the horizontal floor ?
- 9.6** Toricelli's barometer used mercury. Pascal duplicated it using French wine of density  $984 \text{ kg m}^{-3}$ . Determine the height of the wine column for normal atmospheric pressure.
- 9.7** A vertical off-shore structure is built to withstand a maximum stress of  $10^9 \text{ Pa}$ . Is the structure suitable for putting up on top of an oil well in the ocean ? Take the depth of the ocean to be roughly 3 km, and ignore ocean currents.
- 9.8** A hydraulic automobile lift is designed to lift cars with a maximum mass of 3000 kg. The area of cross-section of the piston carrying the load is  $425 \text{ cm}^2$ . What maximum pressure would the smaller piston have to bear ?
- 9.9** A U-tube contains water and methylated spirit separated by mercury. The mercury columns in the two arms are in level with 10.0 cm of water in one arm and 12.5 cm of spirit in the other. What is the specific gravity of spirit ?
- 9.10** In the previous problem, if 15.0 cm of water and spirit each are further poured into the respective arms of the tube, what is the difference in the levels of mercury in the two arms ? (Specific gravity of mercury = 13.6)
- 9.11** Can Bernoulli's equation be used to describe the flow of water through a rapid in a river ? Explain.
- 9.12** Does it matter if one uses gauge instead of absolute pressures in applying Bernoulli's equation ? Explain.
- 9.13** Glycerine flows steadily through a horizontal tube of length 1.5 m and radius 1.0 cm. If the amount of glycerine collected per second at one end is  $4.0 \times 10^{-3} \text{ kg s}^{-1}$ , what is the pressure difference between the two ends of the tube ? (Density of glycerine =  $1.3 \times 10^3 \text{ kg m}^{-3}$  and viscosity of glycerine =  $0.83 \text{ Pa s}$ ). [You may also like to check if the assumption of laminar flow in the tube is correct].
- 9.14** In a test experiment on a model aeroplane in a wind tunnel, the flow speeds on the upper and lower surfaces of the wing are  $70 \text{ m s}^{-1}$  and  $63 \text{ m s}^{-1}$  respectively. What is the lift on the wing if its area is  $2.5 \text{ m}^2$  ? Take the density of air to be  $1.3 \text{ kg m}^{-3}$ .
- 9.15** Figures 9.20(a) and (b) refer to the steady flow of a (non-viscous) liquid. Which of the two figures is incorrect ? Why ?

**Fig. 9.20**

- 9.16** The cylindrical tube of a spray pump has a cross-section of  $8.0 \text{ cm}^2$  one end of which has 40 fine holes each of diameter  $1.0 \text{ mm}$ . If the liquid flow inside the tube is  $1.5 \text{ m min}^{-1}$ , what is the speed of ejection of the liquid through the holes?
- 9.17** A U-shaped wire is dipped in a soap solution, and removed. The thin soap film formed between the wire and the light slider supports a weight of  $1.5 \times 10^{-2} \text{ N}$  (which includes the small weight of the slider). The length of the slider is  $30 \text{ cm}$ . What is the surface tension of the film?
- 9.18** Figure 9.21 (a) shows a thin liquid film supporting a small weight  $= 4.5 \times 10^{-2} \text{ N}$ . What is the weight supported by a film of the same liquid at the same temperature in Fig. (b) and (c)? Explain your answer physically.

**Fig. 9.21**

- 9.19** What is the pressure inside the drop of mercury of radius  $3.00 \text{ mm}$  at room temperature? Surface tension of mercury at that temperature ( $20^\circ\text{C}$ ) is  $4.65 \times 10^{-1} \text{ N m}^{-1}$ . The atmospheric pressure is  $1.01 \times 10^5 \text{ Pa}$ . Also give the excess pressure inside the drop.
- 9.20** What is the excess pressure inside a bubble of soap solution of radius  $5.00 \text{ mm}$ , given that the surface tension of soap solution at the temperature ( $20^\circ\text{C}$ ) is  $2.50 \times 10^{-2} \text{ N m}^{-1}$ ? If an air bubble of the same dimension were formed at depth of  $40.0 \text{ cm}$  inside a container containing the soap solution (of relative density  $1.20$ ), what would be the pressure inside the bubble? (1 atmospheric pressure is  $1.01 \times 10^5 \text{ Pa}$ ).