Algebra

0650CH11

Chapter 11

11.1 Introduction

Our study so far has been with numbers and shapes. We have learnt numbers, operations on numbers and properties of numbers. We applied our knowledge of numbers to various problems in our life. The branch of mathematics in which we studied numbers is **arithmetic**. We have also learnt about figures in two and three dimensions and their properties. The branch of mathematics in which we studied shapes is **geometry**. Now we begin the study of another branch of mathematics. It is called **algebra**.

The main feature of the new branch which we are going to study is the use of letters. Use of letters will allow us to write rules and formulas in a general way. By using letters, we can talk about any number and not just a particular number. Secondly, letters may stand for unknown quantities. By learning methods of determining unknowns, we develop powerful tools for solving puzzles and many problems from daily life. Thirdly, since letters stand for numbers, operations can be performed on them as on numbers. This leads to the study of algebraic expressions and their properties.

You will find algebra interesting and useful. It is very useful in solving problems. Let us begin our study with simple examples.

11.2 Matchstick Patterns

Ameena and Sarita are making patterns with matchsticks. They decide to make simple patterns of the letters of the English alphabet. Ameena takes two matchsticks and forms the letter L as shown in Fig 11.1 (a).

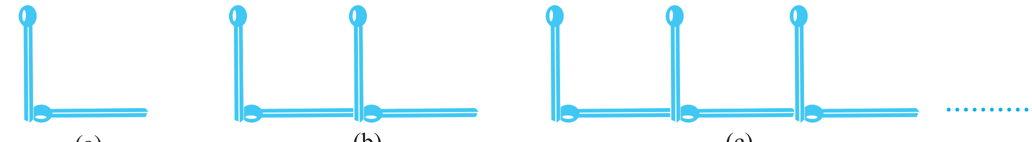


Fig 11.1

Then Sarita also picks two sticks, forms another letter L and puts it next to the one made by Ameena [Fig 11.1 (b)].

Then Ameena adds one more L and this goes on as shown by the dots in Fig 11.1 (c).

Their friend Appu comes in. He looks at the pattern. Appu always asks questions. He asks the girls, "How many matchsticks will be required to make seven Ls"? Ameena and Sarita are systematic. They go on forming the patterns with 1L, 2Ls, 3Ls, and so on and prepare a table.

Table 1

Number of Ls formed	1	2	3	4	5	6	7	8	....	....
Number of matchsticks required	2	4	6	8	10	12	14	16	....	....

Appu gets the answer to his question from the Table 1; 7Ls require 14 matchsticks.

While writing the table, Ameena realises that the number of matchsticks required is twice the number of Ls formed.

Number of matchsticks required = $2 \times$ number of Ls.

For convenience, let us write the letter n for the number of Ls. If one L is made, $n = 1$; if two Ls are made, $n = 2$ and so on; thus, n can be any natural number 1, 2, 3, 4, 5, ... We then write, Number of matchsticks required = $2 \times n$.

Instead of writing $2 \times n$, we write $2n$. Note that $2n$ is same as $2 \times n$.

Ameena tells her friends that her rule gives the number of matchsticks required for forming any number of Ls.

Thus, For $n = 1$, the number of matchsticks required = $2 \times 1 = 2$

For $n = 2$, the number of matchsticks required = $2 \times 2 = 4$

For $n = 3$, the number of matchsticks required = $2 \times 3 = 6$ etc.

These numbers agree with those from Table 1.

Sarita says, "The rule is very powerful! Using the rule, I can say how many matchsticks are required to form even 100 Ls. I do not need to draw the pattern or make a table, once the rule is known".
Do you agree with Sarita?

11.3 The Idea of a Variable

In the above example, we found a rule to give the number of matchsticks required to make a pattern of Ls. The rule was :

Number of matchsticks required = 2*n*

Here, n is the number of Ls in the pattern, and n takes values 1, 2, 3, 4,.... Let us look at Table 1 once again. In the table, the value of n goes on changing (increasing). As a result, the number of matchsticks required also goes on changing (increasing).

***n* is an example of a variable. Its value is not fixed; it can take any value 1, 2, 3, 4, ... We wrote the rule for the number of matchsticks required using the variable *n*.**

The word 'variable' means something that can vary, i.e. change. The value of a variable is not fixed. It can take different values.

We shall look at another example of matchstick patterns to learn more about variables.

11.4 More Matchstick Patterns

Ameena and Sarita have become quite interested in matchstick patterns. They now want to try a pattern of the letter C. To make one C, they use three matchsticks as shown in Fig. 11.2(a).

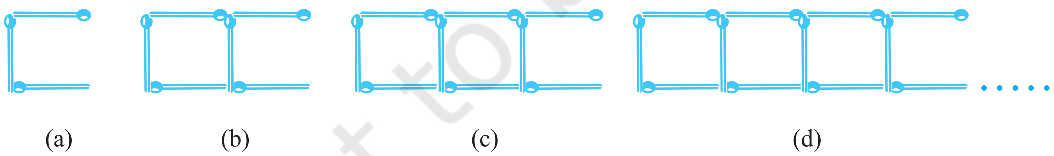


Fig 11.2

Table 2 gives the number of matchsticks required to make a pattern of Cs.

Table 2

Number of Cs formed	1	2	3	4	5	6	7	8	....	....	....
Number of matchsticks required	3	6	9	12	15	18	21	24	....	....	....

Can you complete the entries left blank in the table?

Sarita comes up with the rule :
Number of matchsticks required = 3*n*

She has used the letter n for the number of Cs; n is a variable taking on values 1, 2, 3, 4, ...

Do you agree with Sarita ?

Remember $3n$ is the same as $3 \times n$.

Next, Ameena and Sarita wish to make a pattern of Fs. They make one F using 4 matchsticks as shown in Fig 11.3(a).

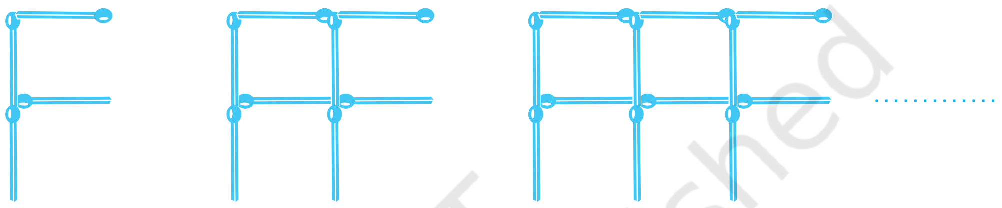


Fig 11.3

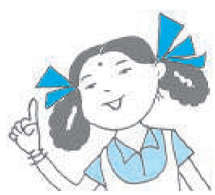
Can you now write the rule for making patterns of F?

Think of other letters of the alphabet and other shapes that can be made from matchsticks. For example, U (⊔), V (\/), triangle (△), square (□) etc. Choose any five and write the rules for making matchstick patterns with them.

11.5 More Examples of Variables

We have used the letter n to show a variable. Raju asks, "Why not m"? There is nothing special about n, any letter can be used.

One may use any letter as m, l, p, x, y, z etc. to show a variable. Remember, a variable is a number which does not have a fixed value. For example, the number 5 or the number 100 or any other given number is not a variable. They have fixed values. Similarly, the number of angles of a triangle has a fixed value i.e. 3. It is not a variable. The number of corners of a quadrilateral (4) is fixed; it is also not a variable. But n in the examples we have looked is a variable. It takes on various values 1, 2, 3, 4, ...



Let us now consider variables in a more familiar situation.

Students went to buy notebooks from the school bookstore. Price of one notebook is ₹ 5. Munnu wants to buy 5 notebooks, Appu wants to buy 7 notebooks, Sara wants to buy 4 notebooks and so on. How much money should a student carry when she or he goes to the bookstore to buy notebooks?

This will depend on how many notebooks the student wants to buy. The students work together to prepare a table.

Table 3

Number of notebooks required	1	2	3	4	5	.....	m	.....
Total cost in rupees	5	10	15	20	25	.....	$5m$	.....

The letter m stands for the number of notebooks a student wants to buy; m is a variable, which can take any value 1, 2, 3, 4, ... The total cost of m notebooks is given by the rule :

The total cost in rupees $= 5 \times$ number of note books required

$= 5m$

If Munnu wants to buy 5 notebooks, then taking $m = 5$, we say that Munnu should carry ₹ 5×5 or ₹ 25 with him to the school bookstore.

Let us take one more example. For the Republic Day celebration in the school, children are going to perform mass drill in the presence of the chief guest. They stand 10 in a row (Fig 11.4). How many children can there be in the drill?

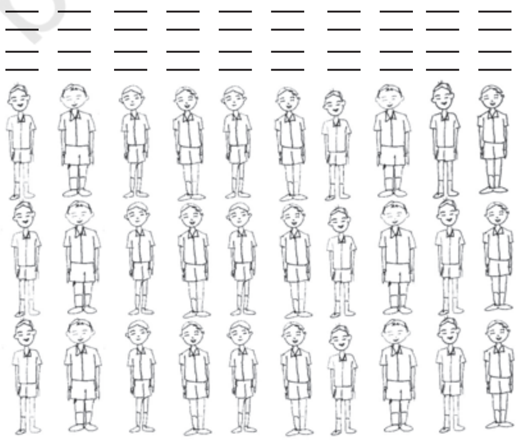


Fig 11.4

The number of children will depend on the number of rows. If there is 1 row, there will be 10 children. If there are 2 rows, there will be 2×10 or 20 children and so on. If there are r rows, there will be $10r$ children in the drill; here, r is a variable which stands for the number of rows and so takes on values 1, 2, 3, 4, ...

In all the examples seen so far, the variable was multiplied by a number. There can be different situations as well in which numbers are added to or subtracted from the variable as seen below.

Sarita says that she has 10 more marbles in her collection than Ameena. If Ameena has 20 marbles, then Sarita has 30. If Ameena has 30 marbles, then Sarita has 40 and so on. We do not know exactly how many marbles Ameena has. She may have any number of marbles.

But we know that, Sarita's marbles = Ameena's marbles + 10.

We shall denote Ameena's marbles by the letter x. Here, x is a variable, which can take any value 1, 2, 3, 4,... ,10,... ,20,... ,30,... . Using x, we write Sarita's marbles = $x + 10$. The expression $(x + 10)$ is read as 'x plus ten'. It means 10 added to x. If x is 20, $(x + 10)$ is 30. If x is 30, $(x + 10)$ is 40 and so on.

The expression $(x + 10)$ cannot be simplified further.
Do not confuse $x + 10$ with $10x$, they are different.
In $10x$, x is multiplied by 10. In $(x + 10)$, 10 is added to x.
We may check this for some values of x.
For example,
If $x = 2$, $10x = 10 \times 2 = 20$ and $x + 10 = 2 + 10 = 12$.
If $x = 10$, $10x = 10 \times 10 = 100$ and $x + 10 = 10 + 10 = 20$.

Raju and Balu are brothers. Balu is younger than Raju by 3 years. When Raju is 12 years old, Balu is 9 years old. When Raju is 15 years old, Balu is 12 years old. We do not know Raju's age exactly. It may have any value. Let x denote Raju's age in years, x is a variable. If Raju's age in years is x, then Balu's age in years is $(x – 3)$. The expression $(x – 3)$ is read as x minus three. As you would expect, when x is 12, $(x – 3)$ is 9 and when x is 15, $(x – 3)$ is 12.

EXERCISE 11.1

1. Find the rule which gives the number of matchsticks required to make the following matchstick patterns. Use a variable to write the rule.
   (a) A pattern of letter T as 
   (b) A pattern of letter Z as 

(c) A pattern of letter U as 

(d) A pattern of letter V as 

(e) A pattern of letter E as 

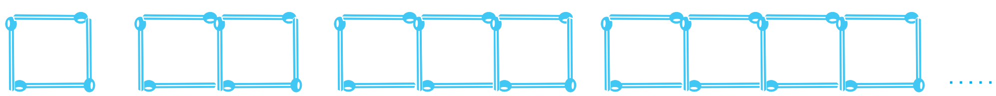
(f) A pattern of letter S as 

(g) A pattern of letter A as 

2. We already know the rule for the pattern of letters L, C and F. Some of the letters from Q.1 (given above) give us the same rule as that given by L. Which are these? Why does this happen?
3. Cadets are marching in a parade. There are 5 cadets in a row. What is the rule which gives the number of cadets, given the number of rows? (Use n for the number of rows.)
4. If there are 50 mangoes in a box, how will you write the total number of mangoes in terms of the number of boxes? (Use b for the number of boxes.)
5. The teacher distributes 5 pencils per student. Can you tell how many pencils are needed, given the number of students? (Use s for the number of students.)
6. A bird flies 1 kilometer in one minute. Can you express the distance covered by the bird in terms of its flying time in minutes? (Use t for flying time in minutes.)
7. Radha is drawing a dot Rangoli (a beautiful pattern of lines joining dots) with chalk powder. She has 9 dots in a row. How many dots will her Rangoli have for r rows? How many dots are there if there are 8 rows? If there are 10 rows?
8. Leela is Radha's younger sister. Leela is 4 years younger than Radha. Can you write Leela's age in terms of Radha's age? Take Radha's age to be x years.

Fig 11.5

9. Mother has made laddus. She gives some laddus to guests and family members; still 5 laddus remain. If the number of laddus mother gave away is l, how many laddus did she make?
10. Oranges are to be transferred from larger boxes into smaller boxes. When a large box is emptied, the oranges from it fill two smaller boxes and still 10 oranges remain outside. If the number of oranges in a small box are taken to be x, what is the number of oranges in the larger box?
11. (a) Look at the following matchstick pattern of squares (Fig 11.6). The squares are not separate. Two neighbouring squares have a common matchstick. Observe the patterns and find the rule that gives the number of matchsticks

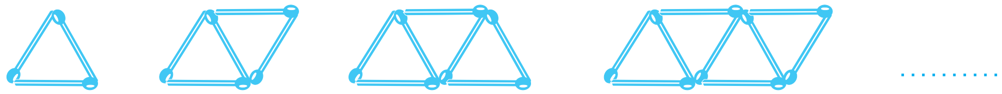


(a) (b) (c) (d)

Fig 11.6

in terms of the number of squares. (Hint : If you remove the vertical stick at the end, you will get a pattern of Cs.)

(b) Fig 11.7 gives a matchstick pattern of triangles. As in Exercise 11 (a) above, find the general rule that gives the number of matchsticks in terms of the number of triangles.



(a) (b) (c) (d)

Fig 11.7

What have we discussed?

1. We looked at patterns of making letters and other shapes using matchsticks. We learnt how to write the general relation between the number of matchsticks required for repeating a given shape. The number of times a given shape is repeated varies; it takes on values 1,2,3,... . It is a variable, denoted by some letter like n.
2. A variable takes on different values, its value is not fixed. The length of a square can have any value. It is a variable. But the number of angles of a triangle has a fixed value 3. It is not a variable.
3. We may use any letter $n, l, m, p, x, y, z,$ etc. to show a variable.
4. A variable allows us to express relations in any practical situation.
5. Variables are numbers, although their value is not fixed. We can do the operations of addition, subtraction, multiplication and division on them just as in the case of fixed numbers. Using different operations we can form expressions with variables like $x - 3$, $x + 3$, $2n$, $5m$, $\frac{p}{3}$, $2y + 3$, $3l - 5$, etc.