

Integers



1.1 PROPERTIES OF ADDITION AND SUBTRACTION OF INTEGERS

We have learnt about whole numbers and integers in Class VI. We have also learnt about addition and subtraction of integers.

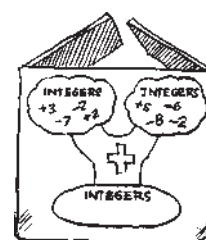
1.1.1 Closure under Addition

We have learnt that sum of two whole numbers is again a whole number. For example, $17 + 24 = 41$ which is again a whole number. We know that, this property is known as the closure property for addition of the whole numbers.

Let us see whether this property is true for integers or not.

Following are some pairs of integers. Observe the following table and complete it.

Statement	Observation
(i) $17 + 23 = 40$	Result is an integer
(ii) $(-10) + 3 = \underline{\hspace{2cm}}$	<u> </u>
(iii) $(-75) + 18 = \underline{\hspace{2cm}}$	<u> </u>
(iv) $19 + (-25) = -6$	Result is an integer
(v) $27 + (-27) = \underline{\hspace{2cm}}$	<u> </u>
(vi) $(-20) + 0 = \underline{\hspace{2cm}}$	<u> </u>
(vii) $(-35) + (-10) = \underline{\hspace{2cm}}$	<u> </u>



What do you observe? Is the sum of two integers always an integer?

Did you find a pair of integers whose sum is not an integer?

Since addition of integers gives integers, we say **integers are closed under addition**.

In general, **for any two integers a and b , $a + b$ is an integer.**

What happens when we subtract an integer from another integer? Can we say that their difference is also an integer?

Observe the following table and complete it:

Statement	Observation
(i) $7 - 9 = -2$	Result is an integer
(ii) $17 - (-21) = \underline{\hspace{2cm}}$	<u> </u>
(iii) $(-8) - (-14) = 6$	Result is an integer
(iv) $(-21) - (-10) = \underline{\hspace{2cm}}$	<u> </u>
(v) $32 - (-17) = \underline{\hspace{2cm}}$	<u> </u>
(vi) $(-18) - (-18) = \underline{\hspace{2cm}}$	<u> </u>
(vii) $(-29) - 0 = \underline{\hspace{2cm}}$	<u> </u>

What do you observe? Is there any pair of integers whose difference is not an integer? Can we say integers are closed under subtraction? Yes, we can see that *integers are closed under subtraction*.

Thus, if a and b are two integers then $a - b$ is also an integer. Do the whole numbers satisfy this property?

We know that $3 + 5 = 5 + 3 = 8$, that is, the whole numbers can be added in any order. In other words, addition is commutative for whole numbers.

Can we say the same for integers also?

We have $5 + (-6) = -1$ and $(-6) + 5 = -1$

So, $5 + (-6) = (-6) + 5$

Are the following equal?

- (i) $(-8) + (-9)$ and $(-9) + (-8)$
- (ii) $(-23) + 32$ and $32 + (-23)$
- (iii) $(-45) + 0$ and $0 + (-45)$

Try this with five other pairs of integers. Do you find any pair of integers for which the sums are different when the order is changed? Certainly not. We say that *addition is commutative for integers*.

In general, for any two integers a and b , we can say

$$a + b = b + a$$

- We know that subtraction is not commutative for whole numbers. Is it commutative for integers?

Consider the integers 5 and (-3) .

Is $5 - (-3)$ the same as $(-3) - 5$? No, because $5 - (-3) = 5 + 3 = 8$, and $(-3) - 5 = -3 - 5 = -8$.

Take atleast five different pairs of integers and check this.

We conclude that subtraction is not commutative for integers.

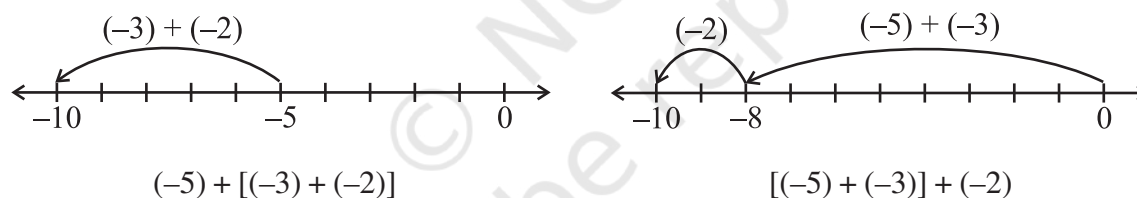
1.1.4 Associative Property

Observe the following examples:

Consider the integers -3 , -2 and -5 .

Look at $(-5) + [(-3) + (-2)]$ and $[(-5) + (-3)] + (-2)$.

In the first sum (-3) and (-2) are grouped together and in the second (-5) and (-3) are grouped together. We will check whether we get different results.



In both the cases, we get -10 .

i.e., $(-5) + [(-3) + (-2)] = [(-5) + (-2)] + (-3)$

Similarly consider -3 , 1 and -7 .

$$(-3) + [1 + (-7)] = -3 + \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$$

$$[(-3) + 1] + (-7) = -2 + \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$$

Is $(-3) + [1 + (-7)]$ same as $[(-3) + 1] + (-7)$?

Take five more such examples. You will not find any example for which the sums are different. *Addition is associative for integers.*

In general for any integers a , b and c , we can say

$$a + (b + c) = (a + b) + c$$

1.1.5 Additive Identity

When we add zero to any whole number, we get the same whole number. Zero is an additive identity for whole numbers. Is it an additive identity again for integers also?

Observe the following and fill in the blanks:

- | | |
|--|--|
| (i) $(-8) + 0 = -8$ | (ii) $0 + (-8) = -8$ |
| (iii) $(-23) + 0 = \underline{\hspace{2cm}}$ | (iv) $0 + (-37) = -37$ |
| (v) $0 + (-59) = \underline{\hspace{2cm}}$ | (vi) $0 + \underline{\hspace{2cm}} = -43$ |
| (vii) $-61 + \underline{\hspace{2cm}} = -61$ | (viii) $\underline{\hspace{2cm}} + 0 = \underline{\hspace{2cm}}$ |

The above examples show that zero is an additive identity for integers.

You can verify it by adding zero to any other five integers.

In general, for any integer a

$$a + 0 = a = 0 + a$$

TRY THESE

1. Write a pair of integers whose sum gives

- | | |
|--|---|
| (a) a negative integer | (b) zero |
| (c) an integer smaller than both the integers. | (d) an integer smaller than only one of the integers. |
| (e) an integer greater than both the integers. | |

2. Write a pair of integers whose difference gives

- | | |
|--|---|
| (a) a negative integer. | (b) zero. |
| (c) an integer smaller than both the integers. | (d) an integer greater than only one of the integers. |
| (e) an integer greater than both the integers. | |



EXAMPLE 1 Write down a pair of integers whose

- | | |
|-----------------------|------------------------|
| (a) sum is -3 | (b) difference is -5 |
| (c) difference is 2 | (d) sum is 0 |

SOLUTION

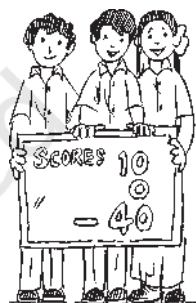
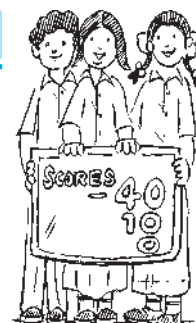
- | | |
|------------------------|--------------------|
| (a) $(-1) + (-2) = -3$ | or $(-5) + 2 = -3$ |
| (b) $(-9) - (-4) = -5$ | or $(-2) - 3 = -5$ |
| (c) $(-7) - (-9) = 2$ | or $1 - (-1) = 2$ |
| (d) $(-10) + 10 = 0$ | or $5 + (-5) = 0$ |



Can you write more pairs in these examples?

EXERCISE 1.1

- Write down a pair of integers whose:
 - sum is -7
 - difference is -10
 - sum is 0
- Write a pair of negative integers whose difference gives 8 .
 - Write a negative integer and a positive integer whose sum is -5 .
 - Write a negative integer and a positive integer whose difference is -3 .
- In a quiz, team A scored $-40, 10, 0$ and team B scored $10, 0, -40$ in three successive rounds. Which team scored more? Can we say that we can add integers in any order?
- Fill in the blanks to make the following statements true:
 - $(-5) + (-8) = (-8) + (\dots\dots\dots)$
 - $-53 + \dots\dots\dots = -53$
 - $17 + \dots\dots\dots = 0$
 - $[13 + (-12)] + (\dots\dots\dots) = 13 + [(-12) + (-7)]$
 - $(-4) + [15 + (-3)] = [-4 + 15] + \dots\dots\dots$



1.2 MULTIPLICATION OF INTEGERS

We can add and subtract integers. Let us now learn how to multiply integers.

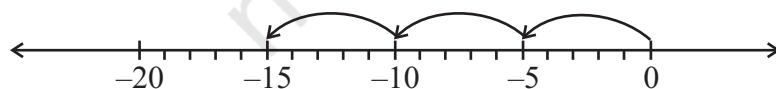
1.2.1 Multiplication of a Positive and a Negative Integer

We know that multiplication of whole numbers is repeated addition. For example,

$$5 + 5 + 5 = 3 \times 5 = 15$$

Can you represent addition of integers in the same way?

We have from the following number line, $(-5) + (-5) + (-5) = -15$



But we can also write

$$(-5) + (-5) + (-5) = 3 \times (-5)$$

Therefore,

$$3 \times (-5) = -15$$

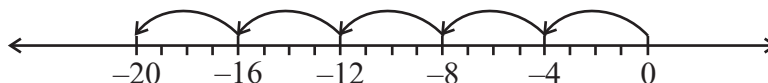
TRY THESE

Find:

- $4 \times (-8),$
- $8 \times (-2),$
- $3 \times (-7),$
- $10 \times (-1)$

using number line.

Similarly $(-4) + (-4) + (-4) + (-4) + (-4) = 5 \times (-4) = -20$



And $(-3) + (-3) + (-3) + (-3) = \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$

Also, $(-7) + (-7) + (-7) = \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$

Let us see how to find the product of a positive integer and a negative integer without using number line.

Let us find $3 \times (-5)$ in a different way. First find 3×5 and then put minus sign $(-)$ before the product obtained. You get -15 . That is we find $-(3 \times 5)$ to get -15 .

Similarly, $5 \times (-4) = -(5 \times 4) = -20$.

Find in a similar way,

$$4 \times (-8) = \underline{\hspace{2cm}} = \underline{\hspace{2cm}}, \quad 3 \times (-7) = \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$$

$$6 \times (-5) = \underline{\hspace{2cm}} = \underline{\hspace{2cm}}, \quad 2 \times (-9) = \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$$

TRY THESE

Find:

- (i) $6 \times (-19)$
- (ii) $12 \times (-32)$
- (iii) $7 \times (-22)$

Using this method we thus have,

$$10 \times (-43) = \underline{\hspace{2cm}} - (10 \times 43) = -430$$

Till now we multiplied integers as (positive integer) \times (negative integer).

Let us now multiply them as (negative integer) \times (positive integer).

We first find -3×5 .

To find this, observe the following pattern:

We have,

$$3 \times 5 = 15$$

$$2 \times 5 = 10 = 15 - 5$$

$$1 \times 5 = 5 = 10 - 5$$

$$0 \times 5 = 0 = 5 - 5$$

So,

$$-1 \times 5 = 0 - 5 = -5$$

$$-2 \times 5 = -5 - 5 = -10$$

$$-3 \times 5 = -10 - 5 = -15$$

We already have

$$3 \times (-5) = -15$$

So we get

$$(-3) \times 5 = -15 = 3 \times (-5)$$

Using such patterns, we also get $(-5) \times 4 = -20 = 5 \times (-4)$

Using patterns, find $(-4) \times 8$, $(-3) \times 7$, $(-6) \times 5$ and $(-2) \times 9$

Check whether, $(-4) \times 8 = 4 \times (-8)$, $(-3) \times 7 = 3 \times (-7)$, $(-6) \times 5 = 6 \times (-5)$



and $(-2) \times 9 = 2 \times (-9)$

Using this we get, $(-33) \times 5 = 33 \times (-5) = -165$

We thus find that while *multiplying a positive integer and a negative integer*, we *multiply them as whole numbers and put a minus sign (-) before the product*. We *thus get a negative integer*.

TRY THESE

- Find: (a) $15 \times (-16)$ (b) $21 \times (-32)$
(c) $(-42) \times 12$ (d) -55×15
- Check if (a) $25 \times (-21) = (-25) \times 21$ (b) $(-23) \times 20 = 23 \times (-20)$

Write five more such examples.



In general, for any two positive integers a and b we can say

$$a \times (-b) = (-a) \times b = -(a \times b)$$

1.2.2 Multiplication of two Negative Integers

Can you find the product $(-3) \times (-2)$?

Observe the following:

$$-3 \times 4 = -12$$

$$-3 \times 3 = -9 = -12 - (-3)$$

$$-3 \times 2 = -6 = -9 - (-3)$$

$$-3 \times 1 = -3 = -6 - (-3)$$

$$-3 \times 0 = 0 = -3 - (-3)$$

$$-3 \times -1 = 0 - (-3) = 0 + 3 = 3$$

$$-3 \times -2 = 3 - (-3) = 3 + 3 = 6$$

Do you see any pattern? Observe how the products change.

Based on this observation, complete the following:

$$-3 \times -3 = \underline{\hspace{2cm}} \quad -3 \times -4 = \underline{\hspace{2cm}}$$

Now observe these products and fill in the blanks:

$$-4 \times 4 = -16$$

$$-4 \times 3 = -12 = -16 + 4$$

$$-4 \times 2 = \underline{\hspace{2cm}} = -12 + 4$$



$$-4 \times 1 = \underline{\hspace{2cm}}$$

$$-4 \times 0 = \underline{\hspace{2cm}}$$

$$-4 \times (-1) = \underline{\hspace{2cm}}$$

$$-4 \times (-2) = \underline{\hspace{2cm}}$$

$$-4 \times (-3) = \underline{\hspace{2cm}}$$

TRY THESE

(i) Starting from $(-5) \times 4$, find $(-5) \times (-6)$

(ii) Starting from $(-6) \times 3$, find $(-6) \times (-7)$

From these patterns we observe that,

$$(-3) \times (-1) = 3 = 3 \times 1$$

$$(-3) \times (-2) = 6 = 3 \times 2$$

$$(-3) \times (-3) = 9 = 3 \times 3$$

and $(-4) \times (-1) = 4 = 4 \times 1$

So, $(-4) \times (-2) = 4 \times 2 = \underline{\hspace{2cm}}$

$$(-4) \times (-3) = \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$$

So observing these products we can say that the *product of two negative integers is a positive integer. We multiply the two negative integers as whole numbers and put the positive sign before the product.*

Thus, we have $(-10) \times (-12) = +120 = 120$

Similarly $(-15) \times (-6) = +90 = 90$

In general, for any two positive integers a and b ,

$$(-a) \times (-b) = a \times b$$

TRY THESE

Find: $(-31) \times (-100)$, $(-25) \times (-72)$, $(-83) \times (-28)$

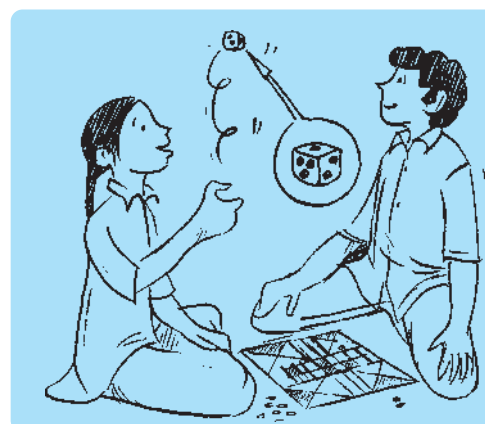
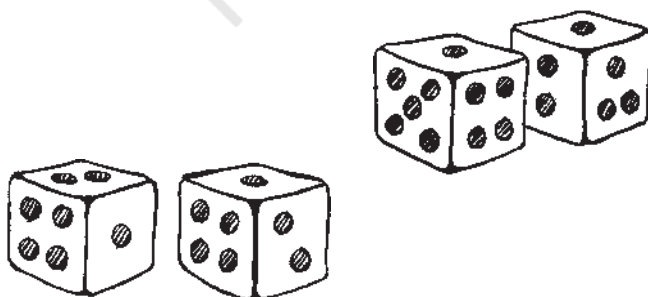
Game 1

- (i) Take a board marked from -104 to 104 as shown in the figure.
- (ii) Take a bag containing two blue and two red dice. Number of dots on the blue dice indicate positive integers and number of dots on the red dice indicate negative integers.
- (iii) Every player will place his/her counter at zero.
- (iv) Each player will take out two dice at a time from the bag and throw them.
- (v) After every throw, the player has to multiply the numbers marked on the dice.

104	103	102	101	100	99	98	97	96	95	94
83	84	85	86	87	88	89	90	91	92	93
82	81	80	79	78	77	76	75	74	73	72
61	62	63	64	65	66	67	68	69	70	71
60	59	58	57	56	55	54	53	52	51	50
39	40	41	42	43	44	45	46	47	48	49
38	37	36	35	34	33	32	31	30	29	28
17	18	19	20	21	22	23	24	25	26	27
16	15	14	13	12	11	10	9	8	7	6
-5	-4	-3	-2	-1	0	1	2	3	4	5
-6	-7	-8	-9	-10	-11	-12	-13	-14	-15	-16
-27	-26	-25	-24	-23	-22	-21	-20	-19	-18	-17
-28	-29	-30	-31	-32	-33	-34	-35	-36	-37	-38
-49	-48	-47	-46	-45	-44	-43	-42	-41	-40	-39
-50	-51	-52	-53	-54	-55	-56	-57	-58	-59	-60
-71	-70	-69	-68	-67	-66	-65	-64	-63	-62	-61
-72	-73	-74	-75	-76	-77	-78	-79	-80	-81	-82
-93	-92	-91	-90	-89	-88	-87	-86	-85	-84	-83
-94	-95	-96	-97	-98	-99	-100	-101	-102	-103	-104



- (vi) If the product is a positive integer then the player will move his counter towards 104; if the product is a negative integer then the player will move his counter towards -104.
- (vii) The player who reaches either -104 or 104 first is the winner.



1.3 PROPERTIES OF MULTIPLICATION OF INTEGERS

1.3.1 Closure under Multiplication

1. Observe the following table and complete it:

Statement	Inference
$(-20) \times (-5) = 100$	Product is an integer
$(-15) \times 17 = -255$	Product is an integer
$(-30) \times 12 = \underline{\hspace{2cm}}$	
$(-15) \times (-23) = \underline{\hspace{2cm}}$	
$(-14) \times (-13) = \underline{\hspace{2cm}}$	
$12 \times (-30) = \underline{\hspace{2cm}}$	

What do you observe? Can you find a pair of integers whose product is not an integer? No. This gives us an idea that the product of two integers is again an integer. So we can say that *integers are closed under multiplication*.

In general,

$a \times b$ is an integer, for all integers a and b .

Find the product of five more pairs of integers and verify the above statement.

1.3.2 Commutativity of Multiplication

We know that multiplication is commutative for whole numbers. Can we say, multiplication is also commutative for integers?

Observe the following table and complete it:

Statement 1	Statement 2	Inference
$3 \times (-4) = -12$	$(-4) \times 3 = -12$	$3 \times (-4) = (-4) \times 3$
$(-30) \times 12 = \underline{\hspace{2cm}}$	$12 \times (-30) = \underline{\hspace{2cm}}$	
$(-15) \times (-10) = 150$	$(-10) \times (-15) = 150$	
$(-35) \times (-12) = \underline{\hspace{2cm}}$	$(-12) \times (-35) = \underline{\hspace{2cm}}$	
$(-17) \times 0 = \underline{\hspace{2cm}}$		
$\underline{\hspace{2cm}} = \underline{\hspace{2cm}}$	$(-1) \times (-15) = \underline{\hspace{2cm}}$	



What are your observations? The above examples suggest *multiplication is commutative for integers*. Write five more such examples and verify.

In general, for any two integers a and b ,

$$a \times b = b \times a$$

1.3.3 Multiplication by Zero

We know that any whole number when multiplied by zero gives zero. Observe the following products of negative integers and zero. These are obtained from the patterns done earlier.

$$(-3) \times 0 = 0$$

$$0 \times (-4) = 0$$

$$-5 \times 0 = \underline{\hspace{2cm}}$$

$$0 \times (-6) = \underline{\hspace{2cm}}$$

This shows that the product of a negative integer and zero is zero.

In general, for any integer a ,

$$a \times 0 = 0 \times a = 0$$

1.3.4 Multiplicative Identity

We know that 1 is the multiplicative identity for whole numbers.

Check that 1 is the multiplicative identity for integers as well. Observe the following products of integers with 1.

$$(-3) \times 1 = -3$$

$$1 \times 5 = 5$$

$$(-4) \times 1 = \underline{\hspace{2cm}}$$

$$1 \times 8 = \underline{\hspace{2cm}}$$

$$1 \times (-5) = \underline{\hspace{2cm}}$$

$$3 \times 1 = \underline{\hspace{2cm}}$$

$$1 \times (-6) = \underline{\hspace{2cm}}$$

$$7 \times 1 = \underline{\hspace{2cm}}$$

This shows that 1 is the multiplicative identity for integers also.

In general, for any integer a we have,

$$a \times 1 = 1 \times a = a$$

What happens when we multiply any integer with -1 ? Complete the following:

$$(-3) \times (-1) = 3$$

$$3 \times (-1) = -3$$

$$(-6) \times (-1) = \underline{\hspace{2cm}}$$

$$(-1) \times 13 = \underline{\hspace{2cm}}$$

$$(-1) \times (-25) = \underline{\hspace{2cm}}$$

$$18 \times (-1) = \underline{\hspace{2cm}}$$

0 is the additive identity whereas 1 is the multiplicative identity for integers. We get additive inverse of an integer a when we multiply (-1) to a , i.e. $a \times (-1) = (-1) \times a = -a$

What do you observe?

Can we say -1 is a multiplicative identity of integers? No.

1.3.5 Associativity for Multiplication

Consider -3 , -2 and 5 .

Look at $[(-3) \times (-2)] \times 5$ and $(-3) \times [(-2) \times 5]$.

In the first case (-3) and (-2) are grouped together and in the second (-2) and 5 are grouped together.

We see that $[(-3) \times (-2)] \times 5 = 6 \times 5 = 30$

and $(-3) \times [(-2) \times 5] = (-3) \times (-10) = 30$

So, we get the same answer in both the cases.

Thus, $[(-3) \times (-2)] \times 5 = (-3) \times [(-2) \times 5]$

Look at this and complete the products:

$$[(7) \times (-6)] \times 4 = \underline{\hspace{2cm}} \times 4 = \underline{\hspace{2cm}}$$

$$7 \times [(-6) \times 4] = 7 \times \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$$

$$\text{Is } [7 \times (-6)] \times 4 = 7 \times [(-6) \times 4]?$$

Does the grouping of integers affect the product of integers? No.

In general, for any three integers a , b and c

$$(a \times b) \times c = a \times (b \times c)$$

Take any five values for a , b and c each and verify this property.

Thus, like whole numbers, *the product of three integers does not depend upon the grouping of integers and this is called the associative property for multiplication of integers.*

1.3.6 Distributive Property

We know

$$16 \times (10 + 2) = (16 \times 10) + (16 \times 2) \quad [\text{Distributivity of multiplication over addition}]$$

Let us check if this is true for integers also.

Observe the following:

$$(a) \quad (-2) \times (3 + 5) = -2 \times 8 = -16$$

$$\text{and} \quad [(-2) \times 3] + [(-2) \times 5] = (-6) + (-10) = -16$$

$$\text{So,} \quad (-2) \times (3 + 5) = [(-2) \times 3] + [(-2) \times 5]$$

$$(b) \quad (-4) \times [(-2) + 7] = (-4) \times 5 = -20$$

$$\text{and} \quad [(-4) \times (-2)] + [(-4) \times 7] = 8 + (-28) = -20$$

$$\text{So,} \quad (-4) \times [(-2) + 7] = [(-4) \times (-2)] + [(-4) \times 7]$$

$$(c) \quad (-8) \times [(-2) + (-1)] = (-8) \times (-3) = 24$$

$$\text{and} \quad [(-8) \times (-2)] + [(-8) \times (-1)] = 16 + 8 = 24$$

$$\text{So,} \quad (-8) \times [(-2) + (-1)] = [(-8) \times (-2)] + [(-8) \times (-1)]$$



Can we say that the distributivity of multiplication over addition is true for integers also? Yes.

In general, for any integers a , b and c ,

$$a \times (b + c) = a \times b + a \times c$$

Take atleast five different values for each of a , b and c and verify the above Distributive property.

TRY THESE

- (i) Is $10 \times [(6 + (-2))] = 10 \times 6 + 10 \times (-2)$?
- (ii) Is $(-15) \times [(-7) + (-1)] = (-15) \times (-7) + (-15) \times (-1)$?



Now consider the following:

Can we say $4 \times (3 - 8) = 4 \times 3 - 4 \times 8$?

Let us check:

$$4 \times (3 - 8) = 4 \times (-5) = -20$$

$$4 \times 3 - 4 \times 8 = 12 - 32 = -20$$

So, $4 \times (3 - 8) = 4 \times 3 - 4 \times 8$.

Look at the following:

$$(-5) \times [(-4) - (-6)] = (-5) \times 2 = -10$$

$$[(-5) \times (-4)] - [(-5) \times (-6)] = 20 - 30 = -10$$

So, $(-5) \times [(-4) - (-6)] = [(-5) \times (-4)] - [(-5) \times (-6)]$

Check this for $(-9) \times [10 - (-3)]$ and $[(-9) \times 10] - [(-9) \times (-3)]$

You will find that these are also equal.

In general, for any three integers a , b and c ,

$$a \times (b - c) = a \times b - a \times c$$

Take atleast five different values for each of a , b and c and verify this property.

TRY THESE

- (i) Is $10 \times (6 - (-2)) = 10 \times 6 - 10 \times (-2)$?
- (ii) Is $(-15) \times [(-7) - (-1)] = (-15) \times (-7) - (-15) \times (-1)$?



EXERCISE 1.2

1. Find each of the following products:

- | | |
|---|--|
| (a) $3 \times (-1)$ | (b) $(-1) \times 225$ |
| (c) $(-21) \times (-30)$ | (d) $(-316) \times (-1)$ |
| (e) $(-15) \times 0 \times (-18)$ | (f) $(-12) \times (-11) \times (10)$ |
| (g) $9 \times (-3) \times (-6)$ | (h) $(-18) \times (-5) \times (-4)$ |
| (i) $(-1) \times (-2) \times (-3) \times 4$ | (j) $(-3) \times (-6) \times (-2) \times (-1)$ |



2. Verify the following:

- (a) $18 \times [7 + (-3)] = [18 \times 7] + [18 \times (-3)]$
 (b) $(-21) \times [(-4) + (-6)] = [(-21) \times (-4)] + [(-21) \times (-6)]$

3. (i) For any integer a , what is $(-1) \times a$ equal to?

(ii) Determine the integer whose product with (-1) is

- (a) -22 (b) 37 (c) 0

4. Starting from $(-1) \times 5$, write various products showing some pattern to show $(-1) \times (-1) = 1$.

1.4 DIVISION OF INTEGERS

We know that division is the inverse operation of multiplication. Let us see an example for whole numbers.

Since $3 \times 5 = 15$

So $15 \div 5 = 3$ and $15 \div 3 = 5$

Similarly, $4 \times 3 = 12$ gives $12 \div 4 = 3$ and $12 \div 3 = 4$

We can say for each multiplication statement of whole numbers there are two division statements.

Can you write multiplication statement and its corresponding division statements for integers?

● Observe the following and complete it.

Multiplication Statement	Corresponding Division Statements
$2 \times (-6) = (-12)$	$(-12) \div (-6) = 2$, $(-12) \div 2 = (-6)$
$(-4) \times 5 = (-20)$	$(-20) \div 5 = (-4)$, $(-20) \div (-4) = 5$
$(-8) \times (-9) = 72$	$72 \div \underline{\hspace{1cm}} = \underline{\hspace{1cm}}$, $72 \div \underline{\hspace{1cm}} = \underline{\hspace{1cm}}$
$(-3) \times (-7) = \underline{\hspace{1cm}}$	$\underline{\hspace{1cm}} \div (-3) = \underline{\hspace{1cm}}$, $\underline{\hspace{1cm}}$
$(-8) \times 4 = \underline{\hspace{1cm}}$	$\underline{\hspace{1cm}}$, $\underline{\hspace{1cm}}$
$5 \times (-9) = \underline{\hspace{1cm}}$	$\underline{\hspace{1cm}}$, $\underline{\hspace{1cm}}$
$(-10) \times (-5) = \underline{\hspace{1cm}}$	$\underline{\hspace{1cm}}$, $\underline{\hspace{1cm}}$

From the above we observe that :

$$(-12) \div 2 = (-6)$$

$$(-20) \div 5 = (-4)$$

$$(-32) \div 4 = (-8)$$

$$(-45) \div 5 = (-9)$$

We observe that *when we divide a negative integer by a positive integer, we divide them as whole numbers and then put a minus sign (-) before the quotient.*

- We also observe that:

$$72 \div (-8) = -9 \quad \text{and} \quad 50 \div (-10) = -5$$

$$72 \div (-9) = -8 \quad \quad \quad 50 \div (-5) = -10$$

So we can say that *when we divide a positive integer by a negative integer, we first divide them as whole numbers and then put a minus sign (-) before the quotient.*

In general, for any two positive integers a and b

$$a \div (-b) = (-a) \div b \quad \text{where } b \neq 0$$

TRY THESE

Find:

- (a) $(-100) \div 5$ (b) $(-81) \div 9$
(c) $(-75) \div 5$ (d) $(-32) \div 2$

Can we say that

$$(-48) \div 8 = 48 \div (-8)?$$

Let us check. We know that

$$(-48) \div 8 = -6$$

$$\text{and } 48 \div (-8) = -6$$

$$\text{So } (-48) \div 8 = 48 \div (-8)$$

Check this for

- (i) $90 \div (-45)$ and $(-90) \div 45$
(ii) $(-136) \div 4$ and $136 \div (-4)$

TRY THESE

Find: (a) $125 \div (-25)$ (b) $80 \div (-5)$ (c) $64 \div (-16)$

- Lastly, we observe that

$$(-12) \div (-6) = 2; (-20) \div (-4) = 5; (-32) \div (-8) = 4; (-45) \div (-9) = 5$$

So, we can say that *when we divide a negative integer by a negative integer, we first divide them as whole numbers and then put a positive sign (+).*

In general, for any two positive integers a and b

$$(-a) \div (-b) = a \div b \quad \text{where } b \neq 0$$



TRY THESE

Find: (a) $(-36) \div (-4)$ (b) $(-201) \div (-3)$ (c) $(-325) \div (-13)$

1.5 PROPERTIES OF DIVISION OF INTEGERS

Observe the following table and complete it:

What do you observe? We observe that integers are not closed under division.



Statement	Inference	Statement	Inference
$(-8) \div (-4) = 2$	Result is an integer	$(-8) \div 3 = \frac{-8}{3}$	_____
$(-4) \div (-8) = \frac{-4}{-8}$	Result is not an integer	$3 \div (-8) = \frac{3}{-8}$	_____

Justify it by taking five more examples of your own.

- We know that division is not commutative for whole numbers. Let us check it for integers also.

You can see from the table that $(-8) \div (-4) \neq (-4) \div (-8)$.

Is $(-9) \div 3$ the same as $3 \div (-9)$?

Is $(-30) \div (-6)$ the same as $(-6) \div (-30)$?

Can we say that division is commutative for integers? No.

You can verify it by taking five more pairs of integers.

- Like whole numbers, any integer divided by zero is meaningless and zero divided by an integer other than zero is equal to zero i.e., *for any integer a , $a \div 0$ is not defined but $0 \div a = 0$ for $a \neq 0$.*
- When we divide a whole number by 1 it gives the same whole number. Let us check whether it is true for negative integers also.

Observe the following :

$$\begin{array}{lll} (-8) \div 1 = (-8) & (-11) \div 1 = -11 & (-13) \div 1 = -13 \\ (-25) \div 1 = \underline{\hspace{1cm}} & (-37) \div 1 = \underline{\hspace{1cm}} & (-48) \div 1 = \underline{\hspace{1cm}} \end{array}$$

This shows that negative integer divided by 1 gives the same negative integer. So, *any integer divided by 1 gives the same integer.*

In general, for any integer a ,

$$a \div 1 = a$$

- What happens when we divide any integer by (-1) ? Complete the following table

$$\begin{array}{lll} (-8) \div (-1) = 8 & 11 \div (-1) = -11 & 13 \div (-1) = \underline{\hspace{1cm}} \\ (-25) \div (-1) = \underline{\hspace{1cm}} & (-37) \div (-1) = \underline{\hspace{1cm}} & -48 \div (-1) = \underline{\hspace{1cm}} \end{array}$$

What do you observe?

We can say that if any integer is divided by (-1) it does not give the same integer.

- Can we say $[(-16) \div 4] \div (-2)$ is the same as $(-16) \div [4 \div (-2)]$?

We know that $[(-16) \div 4] \div (-2) = (-4) \div (-2) = 2$

and $(-16) \div [4 \div (-2)] = (-16) \div (-2) = 8$

So $[(-16) \div 4] \div (-2) \neq (-16) \div [4 \div (-2)]$

Can you say that division is associative for integers? No.

Verify it by taking five more examples of your own.



TRY THESE

Is (i) $1 \div a = 1$?

(ii) $a \div (-1) = -a$? for any integer a .

Take different values of a and check.

EXAMPLE 2 In a test (+5) marks are given for every correct answer and (−2) marks are given for every incorrect answer. (i) Radhika answered all the questions and scored 30 marks though she got 10 correct answers. (ii) Jay also answered all the questions and scored (−12) marks though he got 4 correct answers. How many incorrect answers had they attempted?

SOLUTION

- (i) Marks given for one correct answer = 5

So, marks given for 10 correct answers = $5 \times 10 = 50$

Radhika's score = 30

Marks obtained for incorrect answers = $30 - 50 = -20$

Marks given for one incorrect answer = (−2)

Therefore, number of incorrect answers = $(-20) \div (-2) = 10$

- (ii) Marks given for 4 correct answers = $5 \times 4 = 20$

Jay's score = −12

Marks obtained for incorrect answers = $-12 - 20 = -32$

Marks given for one incorrect answer = (−2)

Therefore number of incorrect answers = $(-32) \div (-2) = 16$



EXAMPLE 3 A shopkeeper earns a profit of ₹ 1 by selling one pen and incurs a loss of 40 paise per pencil while selling pencils of her old stock.

- (i) In a particular month she incurs a loss of ₹ 5. In this period, she sold 45 pens. How many pencils did she sell in this period?
- (ii) In the next month she earns neither profit nor loss. If she sold 70 pens, how many pencils did she sell?

SOLUTION

- (i) Profit earned by selling one pen = ₹ 1

Profit earned by selling 45 pens = ₹ 45, which we denote by + ₹ 45

Total loss given = ₹ 5, which we denote by − ₹ 5

Profit earned + Loss incurred = Total loss

Therefore, Loss incurred = Total Loss − Profit earned

= ₹ (− 5 − 45) = ₹ (−50) = −5000 paise

Loss incurred by selling one pencil = 40 paise which we write as −40 paise

So, number of pencils sold = $(-5000) \div (-40) = 125$



- (ii) In the next month there is neither profit nor loss.

So, Profit earned + Loss incurred = 0

i.e., Profit earned = - Loss incurred.

Now, profit earned by selling 70 pens = ₹ 70

Hence, loss incurred by selling pencils = ₹ 70 which we indicate by - ₹ 70 or - 7,000 paise.

Total number of pencils sold = $(-7000) \div (-40) = 175$ pencils.

EXERCISE 1.3

- Evaluate each of the following:
 - $(-30) \div 10$
 - $50 \div (-5)$
 - $(-36) \div (-9)$
 - $(-49) \div (49)$
 - $13 \div [(-2) + 1]$
 - $0 \div (-12)$
 - $(-31) \div [(-30) + (-1)]$
 - $[(-36) \div 12] \div 3$
 - $[(-6) + 5] \div [(-2) + 1]$
- Verify that $a \div (b + c) \neq (a \div b) + (a \div c)$ for each of the following values of a, b and c .
 - $a = 12, b = -4, c = 2$
 - $a = (-10), b = 1, c = 1$
- Fill in the blanks:
 - $369 \div \underline{\hspace{2cm}} = 369$
 - $(-75) \div \underline{\hspace{2cm}} = -1$
 - $(-206) \div \underline{\hspace{2cm}} = 1$
 - $-87 \div \underline{\hspace{2cm}} = 87$
 - $\underline{\hspace{2cm}} \div 1 = -87$
 - $\underline{\hspace{2cm}} \div 48 = -1$
 - $20 \div \underline{\hspace{2cm}} = -2$
 - $\underline{\hspace{2cm}} \div (4) = -3$
- Write five pairs of integers (a, b) such that $a \div b = -3$. One such pair is $(6, -2)$ because $6 \div (-2) = (-3)$.
- The temperature at 12 noon was 10°C above zero. If it decreases at the rate of 2°C per hour until midnight, at what time would the temperature be 8°C below zero? What would be the temperature at mid-night?
- In a class test (+ 3) marks are given for every correct answer and (-2) marks are given for every incorrect answer and no marks for not attempting any question. (i) Radhika scored 20 marks. If she has got 12 correct answers, how many questions has she attempted incorrectly? (ii) Mohini scores -5 marks in this test, though she has got 7 correct answers. How many questions has she attempted incorrectly?
- An elevator descends into a mine shaft at the rate of 6 m/min. If the descent starts from 10 m above the ground level, how long will it take to reach - 350 m.



WHAT HAVE WE DISCUSSED?

1. We now study the properties satisfied by addition and subtraction.
 - (a) Integers are closed for addition and subtraction both. That is, $a + b$ and $a - b$ are again integers, where a and b are any integers.
 - (b) Addition is commutative for integers, i.e., $a + b = b + a$ for all integers a and b .
 - (c) Addition is associative for integers, i.e., $(a + b) + c = a + (b + c)$ for all integers a , b and c .
 - (d) Integer 0 is the identity under addition. That is, $a + 0 = 0 + a = a$ for every integer a .
2. We studied, how integers could be multiplied, and found that product of a positive and a negative integer is a negative integer, whereas the product of two negative integers is a positive integer. For example, $-2 \times 7 = -14$ and $-3 \times -8 = 24$.
3. Product of even number of negative integers is positive, whereas the product of odd number of negative integers is negative.
4. Integers show some properties under multiplication.
 - (a) Integers are closed under multiplication. That is, $a \times b$ is an integer for any two integers a and b .
 - (b) Multiplication is commutative for integers. That is, $a \times b = b \times a$ for any integers a and b .
 - (c) The integer 1 is the identity under multiplication, i.e., $1 \times a = a \times 1 = a$ for any integer a .
 - (d) Multiplication is associative for integers, i.e., $(a \times b) \times c = a \times (b \times c)$ for any three integers a , b and c .
5. Under addition and multiplication, integers show a property called distributive property. That is, $a \times (b + c) = a \times b + a \times c$ for any three integers a , b and c .
6. The properties of commutativity, associativity under addition and multiplication, and the distributive property help us to make our calculations easier.
7. We also learnt how to divide integers. We found that,
 - (a) When a positive integer is divided by a negative integer, the quotient obtained is negative and vice-versa.
 - (b) Division of a negative integer by another negative integer gives positive as quotient.
8. For any integer a , we have
 - (a) $a \div 0$ is not defined
 - (b) $a \div 1 = a$

