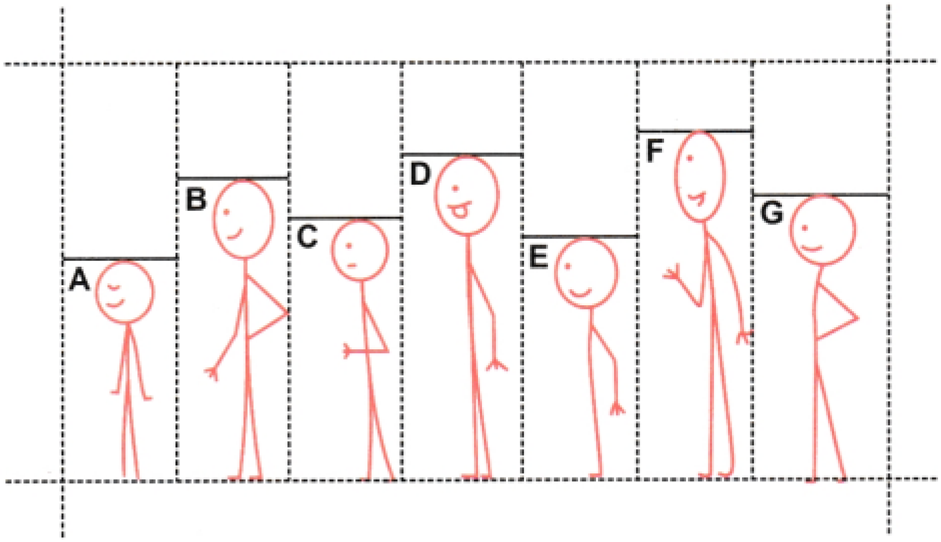
NCERT Class 7 Maths Chapter 6 Number Play Solutions Question Answer

6.1 Numbers Tell Us Things

Figure It Out (Page 128)

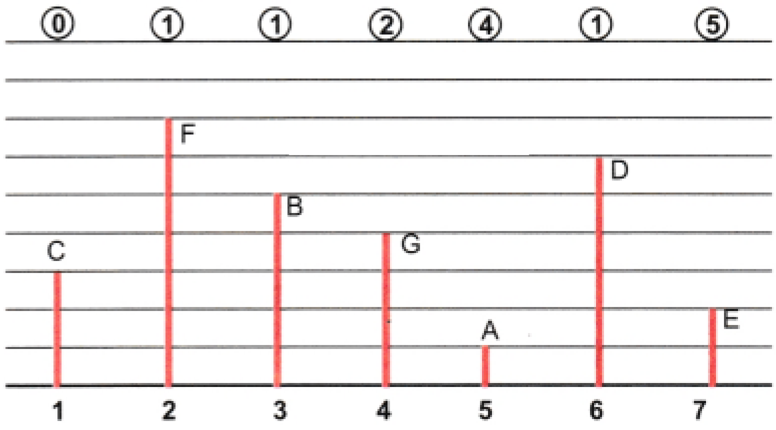
Question 1.
Arrange the stick figure cutouts given below or draw a height arrangement such that the sequence reads:
(a) 0, 1, 1, 2, 4, 1, 5
(b) 0, 0, 0, 0, 0, 0, 0
(c) 0, 1, 2, 3, 4, 5, 6
(d) 0, 1, 0, 1, 0, 1, 0
(e) 0, 1, 1, 1, 1, 1, 1
(f) 0, 0, 0, 3, 3, 3, 3

Solution:
Heights of figure cutouts in increasing order are A, E, C, G, B, D, F.



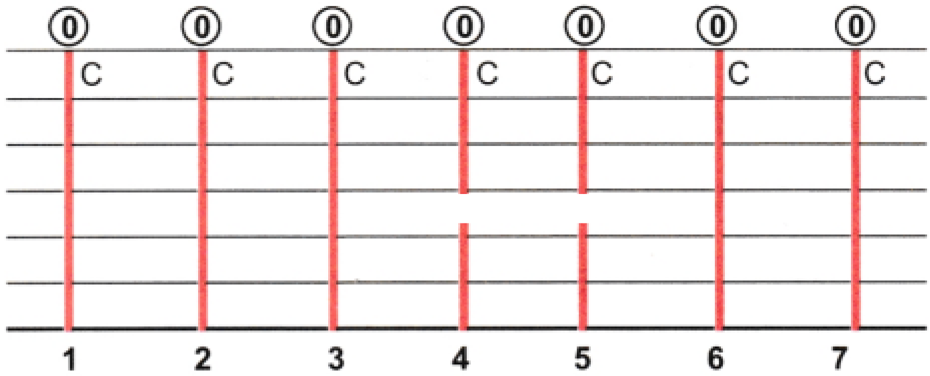
(a) Given sequence is: 0, 1, 1, 2, 4, 1, 5.

The required height arrangement is given below:

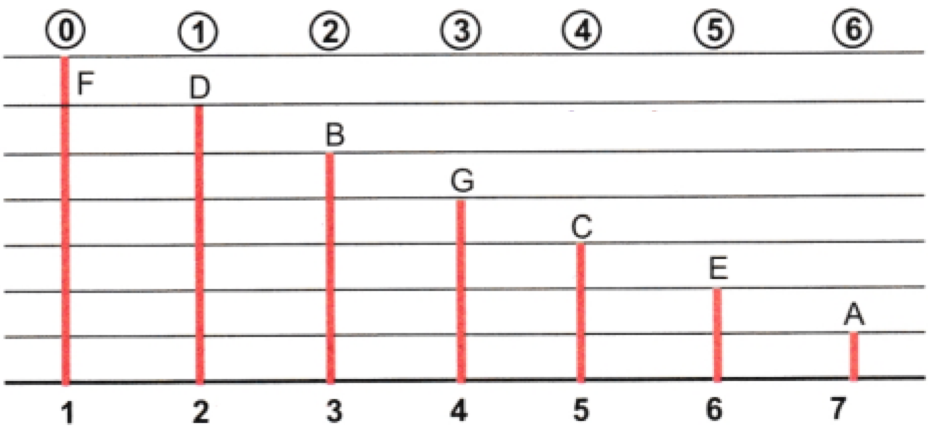


(b) Given sequence is: 0, 0, 0, 0, 0, 0, 0.

The required height arrangement is given below:

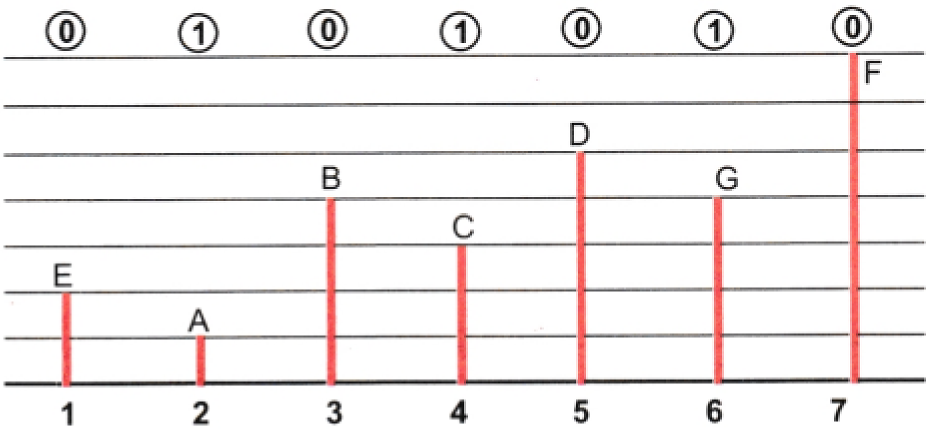


(c) Given sequence is: 0, 1, 2, 3, 4, 5, 6. The required height arrangement is given below:



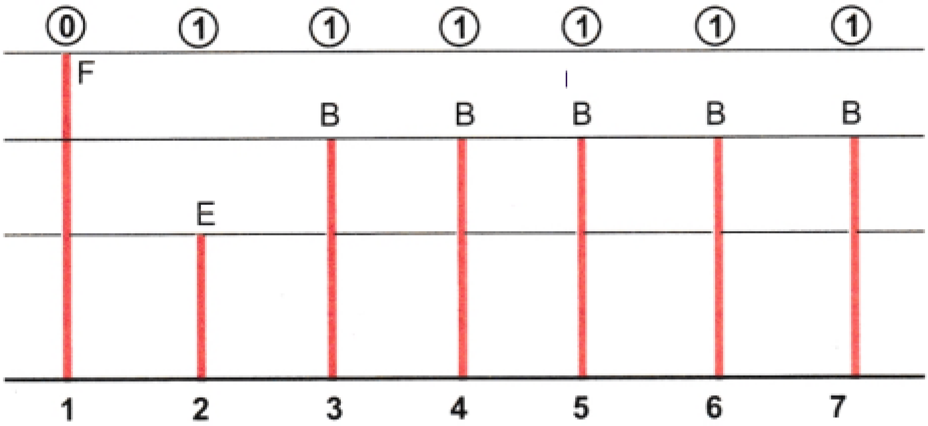
(d) Given sequence is: 0, 1, 0, 1, 0, 1, 0.

The required height arrangement is given below:



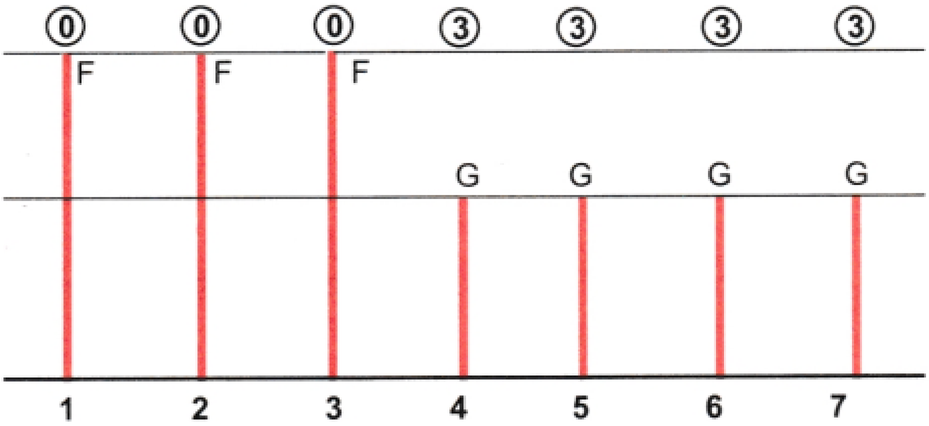
(e) Given sequence is: 0, 1, 1, 1, 1, 1, 1.

The required height arrangement is given below:



(f) Given sequence is: 0, 0, 0, 3, 3, 3, 3.

The required height arrangement is given below:



Question 2.

For each of the statements given below, think and identify if it is Always True, Sometimes True, or Never True. Share your reasoning.

(a) If a person says '0', then he is the tallest in the group.

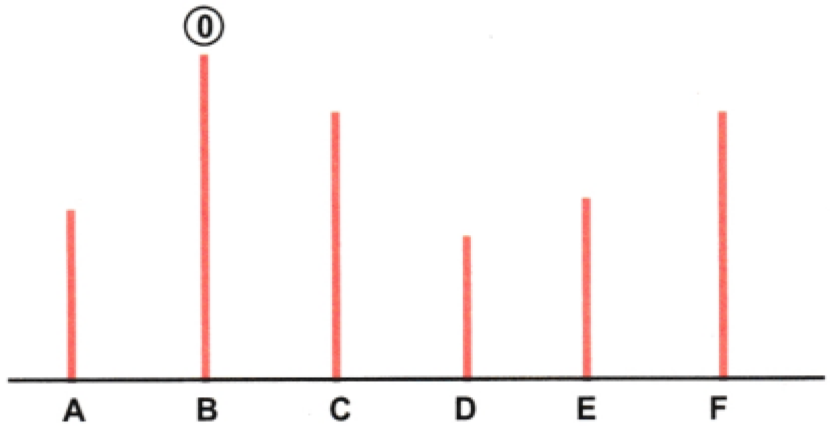
(b) If a person is the tallest, then his number is '0'.

(c) The first person's number is '0'.

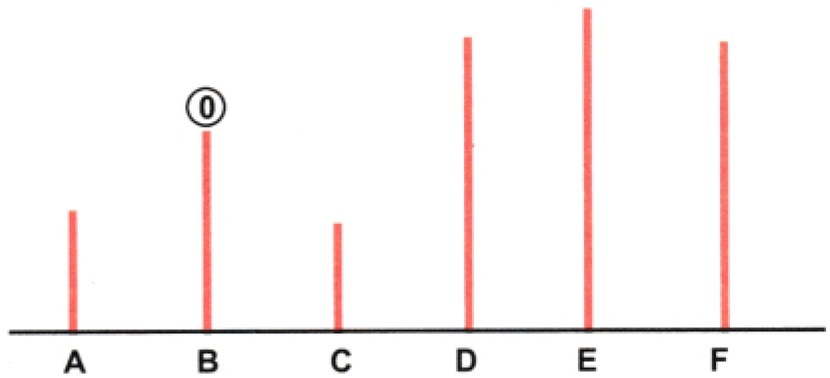
(d) If a person is not first or last in line (i.e., they are standing somewhere in between), then he cannot say '0'.

(e) The person who calls out the largest number is the shortest. (f) What is the largest number possible in a group of 8 people?

Solution:
(a) The given statement is ‘Sometimes True’.
In the Figure, the person says ‘0’ and he is the tallest in the group.



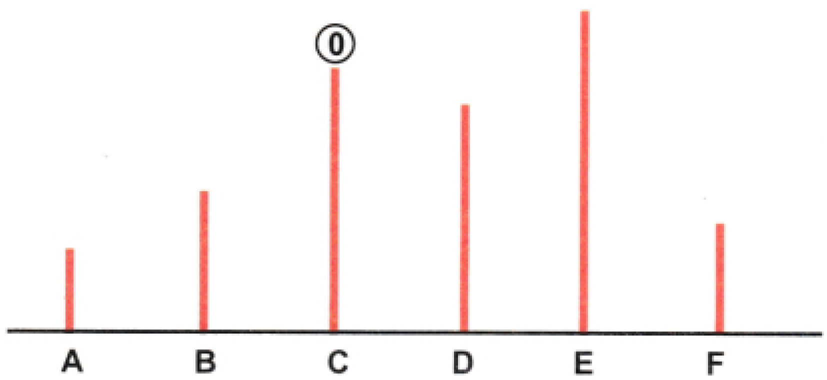
In the Figure, person B says ‘0’ and he is not the tallest in the group.



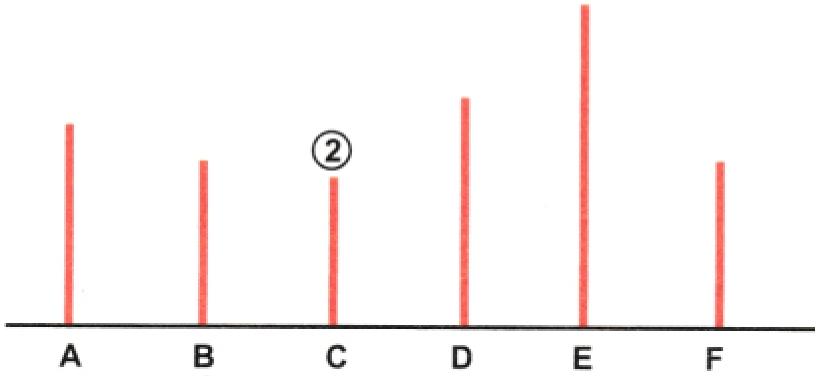
(b) The given statement is ‘Always True’, because if a person is the tallest, then no person before this tallest person can be taller than the tallest person, and hence the number of the tallest person is always ‘0’.

(c) The given statement is ‘Always True’, because no person can be on the left of the first person, and hence the number of taller persons before the first person is always ‘0’.

(d) The given statement is ‘Sometimes True’.
In the Figure, person C is standing in between and he says ‘0’.

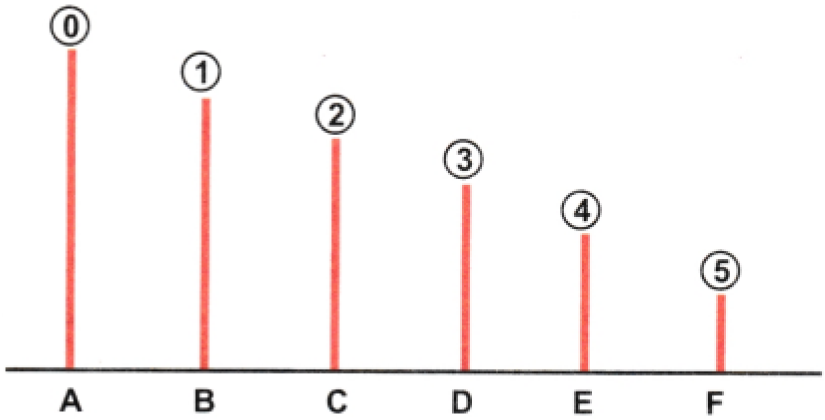


In the Figure, person C is standing in between and not saying ‘0’.

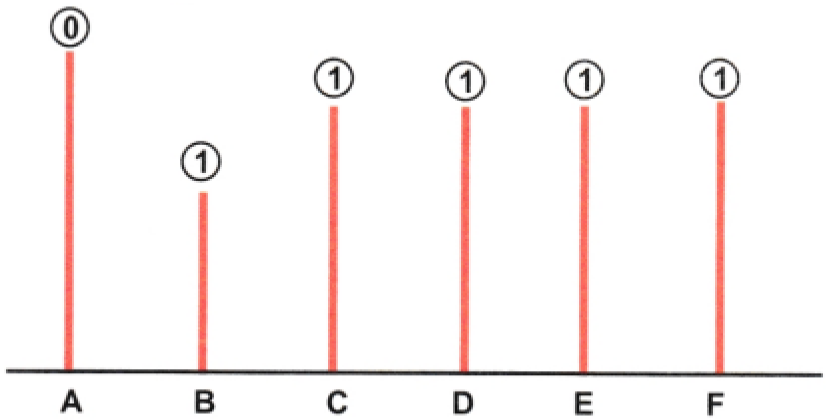


(e) The given statement is ‘Sometimes True’.

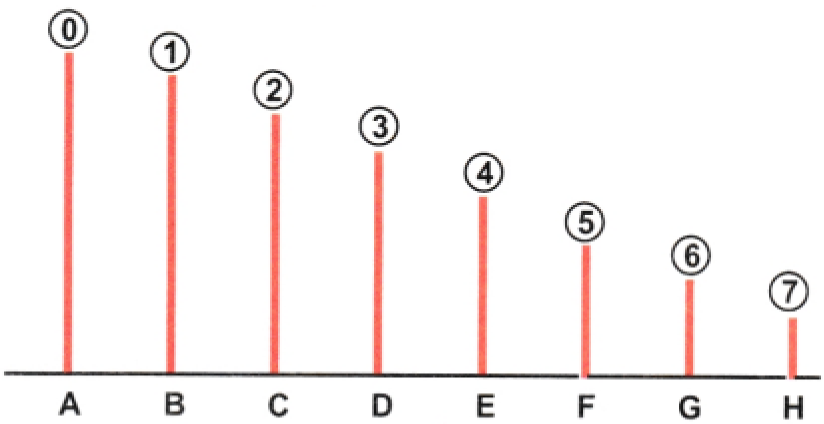
In the Figure, the number of F is the largest number 5, and F is also the shortest in the group.



In the Figure, the number of C is the largest number 1, and C is not the shortest in the group.



(f) The largest possible number in a group of 8 people is 7, as shown in the following Figure.



6.2 Picking Parity

Figure It Out (Page 131)

Question 1.
Using your understanding of the pictorial representation of odd and even numbers, find out the parity of the following sums:
(a) Sum of 2 even numbers and 2 odd numbers (e.g. even + even + odd + odd)
(b) Sum of 2 odd numbers and 3 even numbers
(c) Sum of 5 even numbers
(d) Sum of 8 odd numbers

Solution:
(a) We have 2 even numbers and 2 odd numbers.
The sum of 2 even numbers is an even number.
The sum of 2 odd numbers is an even number.
Also, the sum of two even numbers is an even number.
∴ The required sum is an even number.

(b) We have 2 odd numbers and 3 even numbers.
The sum of 2 odd numbers is an even number.
The sum of 3 even numbers is an even number.
Also, the sum of two even numbers is an even number.
∴ The required sum is an even number.

(c) We have 5 even numbers.
We know that the sum of any number of even numbers is an even number.
∴ The required sum is an even number.

(d) We have 8 odd numbers.
∴ We know that the sum of an even number of odd numbers is an even number.
∴ The required sum is an even number.

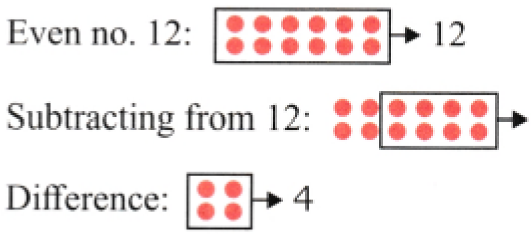
Question 2.
Lakpa has an odd number of ₹ 1 coins, an odd number of ₹ 5 coins and an even number of ₹ 10 coins in his piggy bank. He calculated the total and got ₹ 205. Did he make a mistake? If he did, explain why. If he didn’t, how many coins of each type could he have?

Solution:
Let the number of ₹ 1 coins be an odd number x.
Since 1 and x are both odd, the value of ₹ 1 coins is an odd number.
Let the number of ₹ 5 coins be an odd number y.

Since 5 and y are both odd, the value of ₹ 5 coins is an odd number.
∴ The sum of the values of ₹ 1 coins and ₹ 5 coins is are even number, because the sum of two odd numbers is an even number.
Let the number of ₹ 10 coins be an even number z.
Since 10 and z are both even, the value of ₹ 10 coins is an even number.
∴ The sum of the values of ₹ 1 coins, ₹ 5 coins, and ₹ 10 coins is an even number, because the sum of two even numbers is an even number.
Since 205 is an odd number, the sum of the values of all coins can not be 205.
∴ Lakpa made a mistake in calculating his total money.

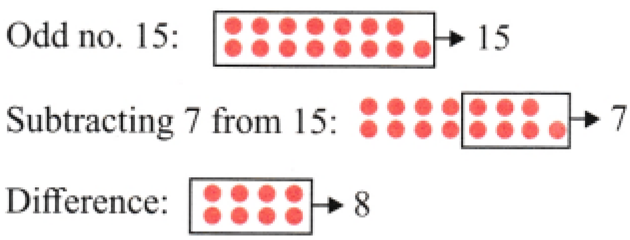
Question 3.
We know that:
(a) even + even = even
(b) odd + odd = even
(c) even + odd = odd.
Similarly, find out the parity for the scenarios below.
(d) even – even = __________________
(e) odd – odd = __________________
(f) even – odd = __________________
(g) odd – even = __________________

Solution:
(d) We are to find the parity of the number obtained by subtracting an even number from an even number.
Let us subtract 8 from 12:



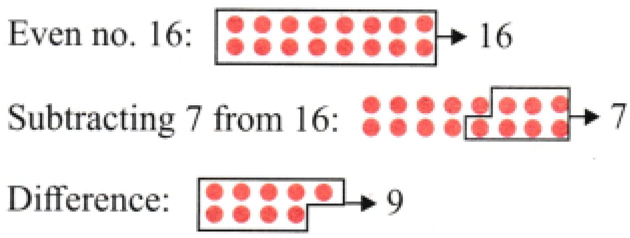
Parity of difference 4 is an even number. In general, parity of ‘even-even’ is an even number.

(e) We are to find the parity of the number obtained by subtracting an odd number from an odd number.
Let us subtract 7 from 15.



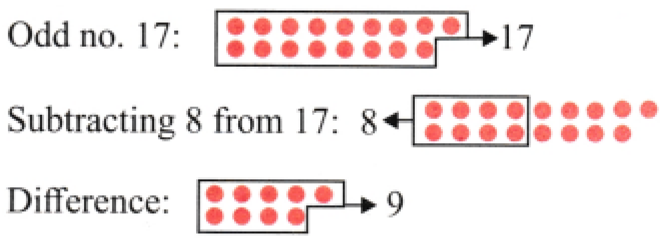
Parity of the difference 8 is an even number. In general, parity of ‘odd-odd’ is an even number.

(f) We are to find the parity of the number obtained by subtracting an odd number from an even number.
Let us subtract 7 from 16.



Parity of the difference 9 is an odd number. In general, parity of ‘even-odd’ is an odd number.

(g) We are to find the parity of the number obtained by subtracting an even number from an odd number.
Let us subtract 8 from 17.



Parity of difference 9 is an odd number. In general, parity of ‘odd-even’ is an odd number.

Question 4.
Find the parity of the number of small squares in these grids:
(a) 27 × 13
(b) 42 × 78
(c) 135 × 654.

Solution:
(a) The given grid is 27 × 13.
Here, 27 and 13 are both odd numbers.
∴ The parity of the number of small squares in this grid is an odd number.

(b) The given grid is 42 × 78.
Here, 42 and 78 are both even numbers.
∴ The parity of the number of small squares in this grid is even number.

(c) The given grid is 135 × 654.
Here, 654 is an even number.

∴ The parity of the number of small squares in this grid is even number.

6.3 Some Explorations in Grids

Figure It Out (Page 136)

Question 1.
How many different magic squares can be made using the numbers 1 – 9?

Solution: 8 different 3×3 magic squares are using the numbers 1 to 9 without repetition. These are given below.

8	1	6
3	5	7
4	9	2

I

6	1	8
7	5	3
2	9	4

II

8	3	4
1	5	9
6	7	2

III

4	3	8
9	5	1
2	7	6

IV

4	9	2
3	5	7
8	1	6

V

6	7	2
1	5	9
8	3	4

VI

2	9	4
7	5	3
6	1	8

VII

2	7	6
9	5	1
4	3	8

VIII

These 8 magic squares can be thought of as reflections and rotations of any one of the above 8 magic squares.

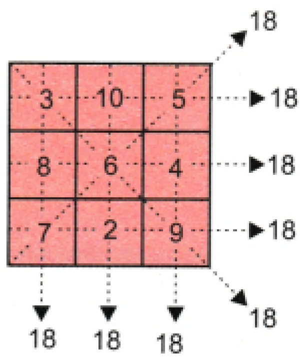
Question 2.
Create a magic square using the numbers 2 – 10. What strategy would you use for

this? Compare it with the magic squares made using 1 – 9.

Solution: We are to use numbers 2, 3, 4, 5, 6, 7, 8, 9, 10.
These numbers are one more than the numbers 1, 2, 3, 4, 5, 6, 7, 8, 9.
The following is a magic square using the numbers 1, 2, 3, 4, 5, 6, 7, 8, 9.

2	9	4
7	5	3
6	1	8

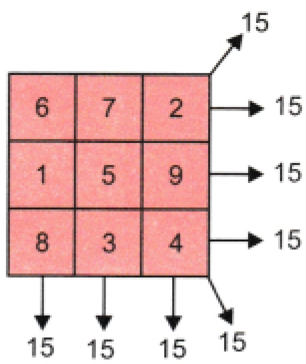
We prepare a new 3 × 3 grid by adding 1 to each number in the above magic square as given below.



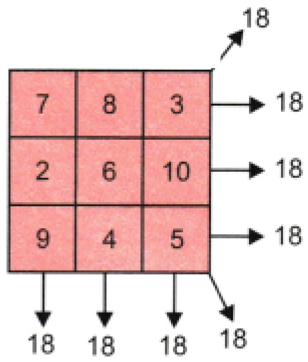
This is a magic square using the numbers 2 to 10.

Question 3.
Take a magic square and:
(a) Increase each number by 1
(a) Double each number.
In each case, is the resulting grid also a magic square? How do the magic sums change in each case?

Solution:
Consider the following 3 × 3 grid using numbers 1 – 9 without repetition:

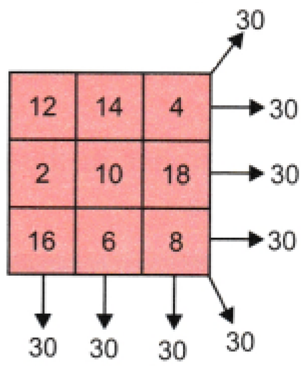


This is a magic square with a magic sum of 15.
(a) We form a new grid by increasing each number by 1.
The new grid is given below:



We see that the new grid is also a magic square and the magic sum is 18, which is 3 more than the magic sum 15.

(b) We form a new grid by doubling each number. The new grid is given below:



We see that the new grid is a magic square and the magic sum is 30, which is also the double of the magic sum 15.

Question 4.
What other operations can be performed on a magic square to yield another magic square?

Solution:
The following operations can be performed upon a magic square to yield another magic square.

- Subtract the same number from each number in the magic square.
- Dividing the numbers in the magic square by the same common factor of all numbers in the magic square.

Question 5.
Discuss ways of creating a magic square using any set of 9 consecutive numbers (like 2 – 10, 3 – 11, 9 – 17, etc.).

Solution: 1 – 9 is a set of 9 consecutive numbers.
The following is a magic square of numbers 1 – 9.

4	9	2
3	5	7
8	1	6

Here, the magic sum is 15.
The numbers 2 – 10 are 1 more than the numbers 1 – 9.
A magic square using numbers 2 – 10 can be obtained by adding 1 to each number in the above magic square of numbers 1 – 9, as shown below:

5	10	3
4	6	8
9	2	7

The numbers 3 – 11 are 2 more than the numbers 1 – 9.
A magic square using numbers 3 – 11 can be obtained by adding 2 to each number in the above magic square of numbers 1 – 9 as shown below:

6	11	4
5	7	9
10	3	8

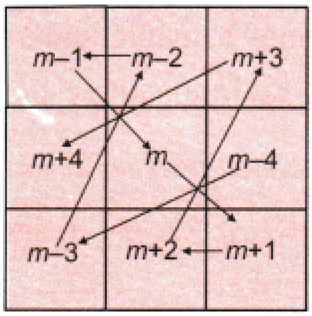
The numbers 9 – 17 are 8 more than the numbers 1 – 9.
A magic square using numbers 9 – 17 can be obtained by adding 8 to each number in the above magic square of numbers 1 – 9, as shown below:

12	17	10
11	13	15
16	9	14

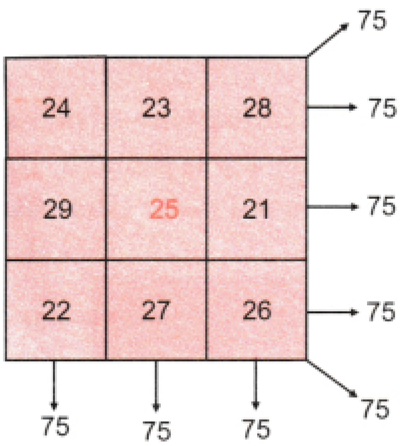
Figure It Out (Page 137)

Question 1.
Using this generalised form, find a magic square if the centre number is 25.

Solution:
We know that if the centre number is m, then the magic square is as follows:



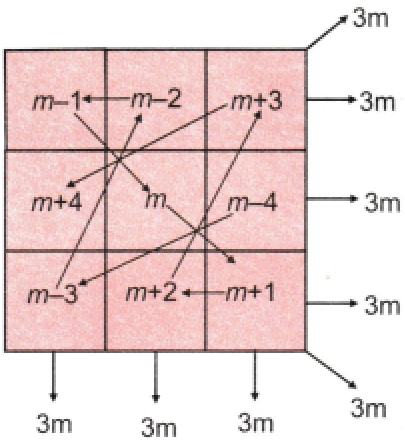
Let m = 25
∴ m – 4 = 21, m – 3 = 22, m – 2 = 23
m – 1 = 24, m + 1 = 26, m + 2 = 27
m + 3 = 28, m + 4 = 29.
Replacing these values in the above magic square, we get



This is the required magic square with centre number 25.

Question 2.
What is the expression obtained by adding the 3 terms of any row, column, or diagonal?

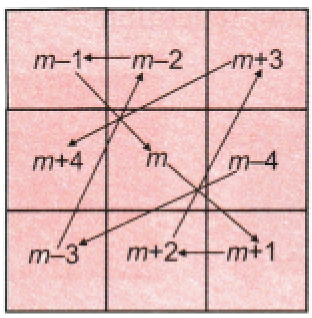
Solution:
If the number in the centre of a magic square is m, then the generalised magic square is the following:



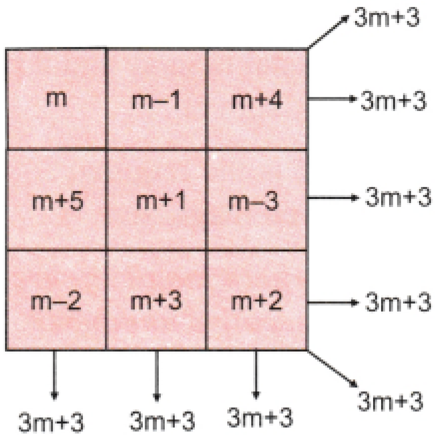
Here, the sum of terms in any row = 3m
Sum of terms in any column = 3m
and, sum of terms along any diagonal = 3m.

Question 3.
Write the result obtained by:
(a) Adding 1 to every term in the generalised form.
(b) Doubling every term in the generalised form.

Solution:
Let the number in the centre of a magic square be m.
For this, the generalised magic square is shown in Figure.

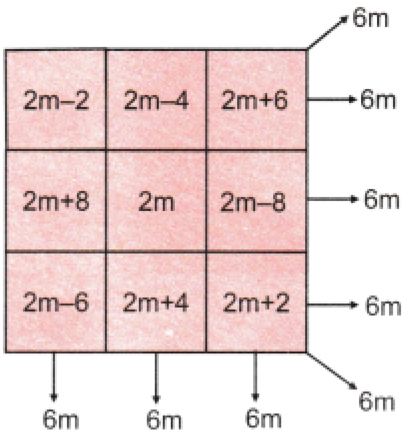


(a) We add 1 to every term in the magic square in Figure.
The new grid is:



This is a magic square with a magic sum of 3m + 3.

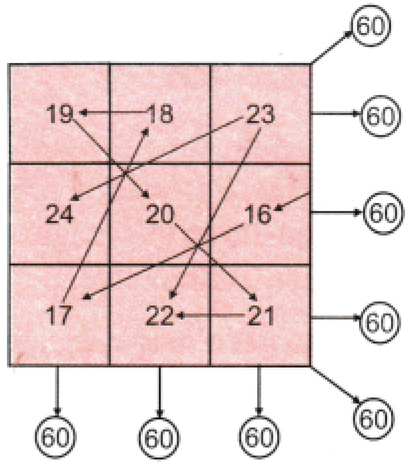
(b) We double every term in the magic square in Figure. The new grid is:



This is a magic square with a magic sum of 6m.

Question 4.
Create a magic square whose magic sum is 60.

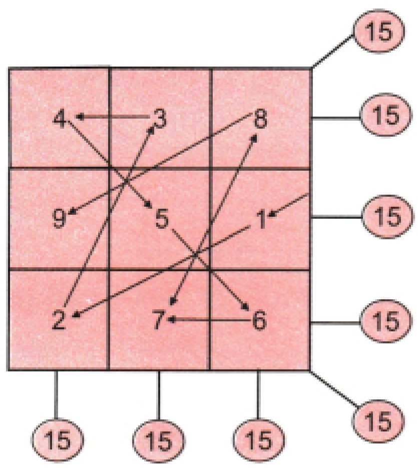
Solution:
Step 1: 60 is a multiple of 3, and 60 ÷ 3 = 20.
Step 2: Consider the 9 consecutive numbers:
20 – 4, 20 – 3, 20 – 2, 20 – 1, 20, 20 + 1, 20 + 2, 20 + 3, 20 + 4, i.e., 16, 17, 18, 19, 20, 21, 22, 23, 24.
Step 3: Starting with 16, fill these 9 numbers in 9 squares along the arrows as shown below:



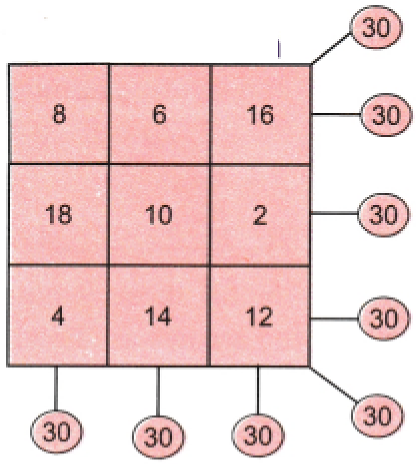
Step 4: In the above grid, there are 9 numbers in 3 rows and 3 columns, and the sum of numbers in any row or any column or along any diagonal is the same and it is equal to the given magic sum 60.
Step 5: The above grid is the required 3 × 3 magic square with given magic sum 60.

Question 5.
Is it possible to get a magic square by filling nine non-consecutive numbers?

Solution:
Consider 9 consecutive numbers: 1, 2, 3, 4, 5, 6, 7, 8, and 9. Starting with 1, fill these 9 numbers in 9 squares along the arrows as shown below:



The grid in the Figure is a magic square with a magic sum of 15.
We multiply each number in the above magic square by 2 as shown in the Figure.



The grid in the Figure is also a magic square with a magic sum of 30. In this magic square, the numbers are non-consecutive.
∴ It is possible to get a magic square by filling in 9 non-consecutive numbers.

6.4 Nature's Favourite Sequence: The Virahanka-Fibonacci Numbers!, 6.5 Digits in Disguise

Figure It Out (Pages 143 – 144)

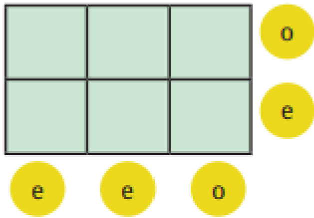
Question 1.
A light bulb is ON. Dorjee toggles its switch 77 times. Will the bulb be lighted or not? Why?

Solution:
The switch is toggled 77 times.
We know that toggling a switch 2 times, 4 times, 6 times…….. i.e. an even number of times does not change the status of the bulb. ∴ Toggling 76 times, the states of the bulb is not changed i.e., the bulb is ON. ∴ Switching the bulb 77th times, the bulb would be OFF.

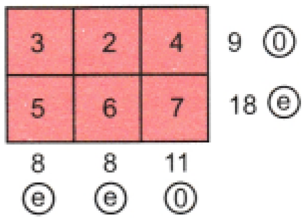
Question 2.
Liswini has a large old encyclopaedia. When she opened it, several loose pages fell out of it. She counted 50 sheets in total, each printed on both sides. Can the sum of the page numbers of the loose sheets be 6000? Why or why not?

Solution:
We know that the page numbers on any two sided sheet in a book are consecutive numbers.
Also, two consecutive numbers are one even number and one odd number.
∴ Sum of page numbers of a two sided sheet of a book is an odd number.
Since, the number of loose sheets is 50, which is an even number, the sum of page numbers of 50 sheets is an even number.
∴ Sum of pages of loose sheets can be 6000.

Question 3.
Here is a 2 × 3 grid. For each row and column, the parity of the sum is written in the circle; 'e' for even and 'o' for odd. Fill the 6 boxes with 3 odd numbers ('o') and 3 even numbers ('e') to satisfy the parity of the row and column sums.



Solution:
We fill 3 odd numbers 3, 5, 7 and 3 even numbers 2, 4, 6 in the given 2 × 3 grid as given below.



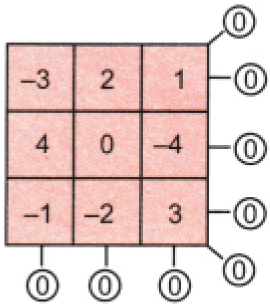
This filled 2 × 3 grid satisfy the required conditions.

Question 4.
Make a 3 × 3 magic square with 0 as the magic sum. All numbers cannot be zero. Use negative numbers, as needed.

Solution:
We consider a magic square of order 3 × 3 with numbers 1 – 9 without repetition as shown below:

2	7	6
9	5	1
4	3	8

This is a magic square with magic sum 15.
To get a magic square with magic sum 0, we subtract 5 from each number in small boxes.
The new 3 × 3 grid is as follows.



This is a 3 × 3 magic square with 0 as the magic sum and all numbers are not zero.

Question 5.
Fill in the following blanks with ‘odd’ or ‘even’:
a) Sum of an odd number of even numbers is ________________
(b) Sum of an even number of odd numbers is ________________
(c) Sum of an even number of even numbers is ________________
(d) Sum of an odd number of odd numbers is ________________

Solution:
(a) even
(b) even
(c) even
(d) odd

Question 6.
What is the parity of the sum of the numbers from 1 to 100?

Solution:
Sum of numbers from 1 to 100 = (1 + 2) + (3 + 4) + (5 + 6) + ….. + (99 + 100)
In the above sum, there are 50 brackets containing pairs of an even number and an odd number.
Since odd + even is odd, the number in each bracket is odd.
Since, the number of brackets is 50 and this is an even number, so the sum of all numbers from 1 to 100 is an even number.

Question 7.
Two consecutive numbers in the Virahahka sequence are 987 and 1597. What are the next 2 numbers in the sequence? What are the previous 2 numbers in the sequence?

Solution:
Given consecutive numbers in the Virahahka sequence are 987 and 1597.
The Virahahka sequence is 1, 2, 3, 5, 8, …, 987, 1597,…
Let Tn = 987
∴ Tn+1 = 1597

∴ $T_{n+2} = T_n + T_{n+1}$
= 987 + 1597 = 2584
and $T_{n+3} = T_{n+1} + T_{n+2}$
= 1597 + 2584 = 4181
∴ Next 2 numbers are 2584 and 4181.
Also, $T_{n+1} = T_{n-1} + T_n$
∴ $1597 = T_{n-1} + 987$
∴ $T_{n-1} = 1597 – 987 = 610$
Also $T_n = T_{n-2} + T_{n-1}$
∴ $987 = T_{n-2} + 610$
∴ $T_{n-2} = 987 – 610 = 377$
∴ Previous 2 numbers are 377 and 610.

Question 8.
Angaan wants to climb an 8-step staircase. His playful rule is that he can take either 1 step or 2 steps at a time. For example, one of his paths is 1, 2, 2, 1, 2. In how many different ways can he reach the top?

Solution:
As per the given playful rule of climbing of Angaan, the total number of ways of climbing the 8-step staircase is given by the eighth term of the Virahanka sequence:
1, 2, 3, 5, 8, 13, 21, …. "
Here, $T_8 = T_6 + T_7$
= 13 + 21
= 34
∴ There are 34 ways of reaching the top.

Question 9.
What is the parity of the 20th term of the Virahahka sequence?

Solution:
The Virahahka sequence is 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144,….. In this sequence, (3n + 2)th term is even for n = 0, 1, 2, 3,……
Let 3n + 2 = 20
∴ 3n = 20 – 2 = 18
∴ n = 18 ÷ 3 = 6
∴ 20th term = (3n + 2)th term, where n is 6.
∴ The 20th term is an even number.

Question 10.
Identify the true statements.
(a) The expression 4m – 1 always gives odd numbers.
(b) All even numbers can be expressed as 6j – 4.

(c) Both expressions 2p + 1 and 2q – 1 describe all odd numbers.
(d) The expression 2f + 3 gives both even and odd numbers.

Solution:
(a) 4m is always even.
∴ 4m – 1 is always an odd number
∴ The given statement is true.

(b) For j = 1, 2, 3, 4,………
The values of 6j are 6, 12, 18, 24,…..
∴ The values of 6j – 4 are 2, 8, 14, 20,…..
The even numbers 4, 6, 10,… are not covered in the above values of 6j – 4.
∴ Not all even numbers are captured in 6j – 4.
∴ The given statement is not true.

(c) For p = 1, 2, 3, 4,…..
The values of 2p + 1 are 3, 5, 7, 9,……
For q = 1, 2, 3, 4,…
The values of 2q – 1 are 1, 3, 5, 7,………
∴ 2p + 1 does not describe all odd numbers.
∴ The given statement is not true.

(d) For f = 1, 2, 3, 4, …
The values of 2f + 3 are 5, 7, 9, 11,……
These are all odd numbers.
∴ The given statement is not true.

Question 11. Solve this Cryptarithm:

$$\begin{array}{r} UT \\ +\ TA \\ \hline TAT \end{array}$$

Solution:
Given Cryptarithm is:

$$\begin{array}{r} UT \\ +\ TA \\ \hline TAT \\ \hline \end{array}$$

We choose U = 9, T = 1 and A = 0.
∴ Cryptarithm becomes

$$\begin{array}{r} 91 \\ +\ 10 \\ \hline 101 \\ \hline \end{array}$$

This is true because 91 + 10 = 101.