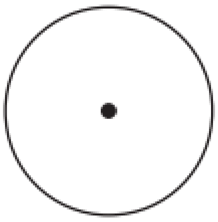
NCERT Class 7 Maths Chapter 7 A Tale of Three Intersecting Lines Solutions Question Answer

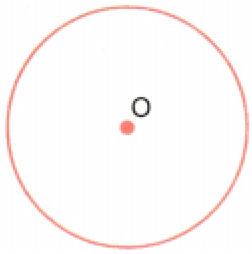
7.1 Equilateral Triangles, 7.2 Constructing a Triangle When its Sides are Given

Figure It Out (Pages 150 – 151)

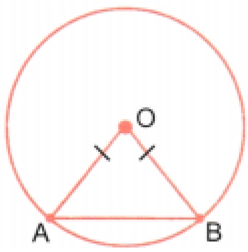
Question 1.
Use the points on the circle and/or the centre to form isosceles triangles.



Solution:
1. Let O be the centre of the circle.

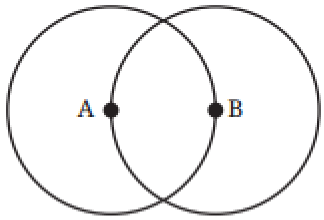


2. Take two points A and B on the circle.
3. Join OA, AB, and BO.

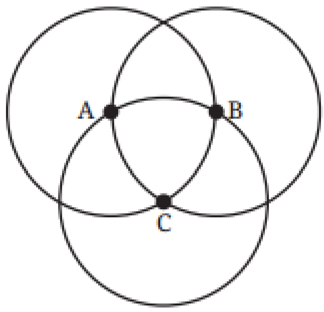


4. Then, △AOB is the required isosceles triangle.

Question 2.
Use the points on the circles and/or their centres to form isosceles and equilateral triangles. The circles are of the same size.



A and B are the centres of circles of the same size

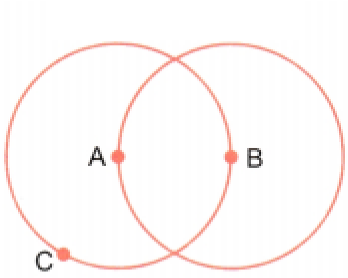


A, B, and C are the centres of circles of the same size

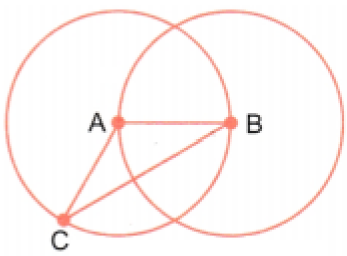
Solution:

Isosceles Triangles

1. In the given figure, take another point C on the first circle.

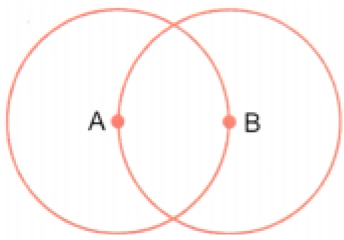


2. Join AB, CB and BA.

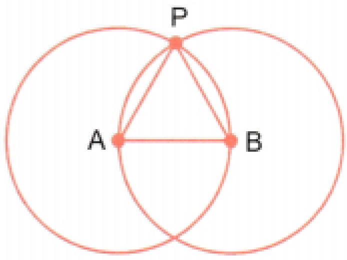


3. Then, ∆ABC is the required isosceles triangle.

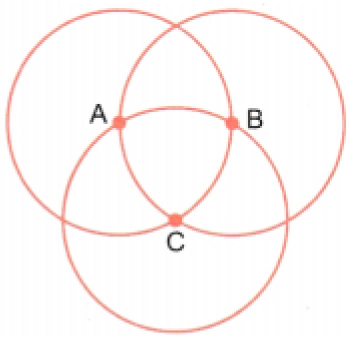
To form an equilateral triangle



1. To form an equilateral triangle, take a point P on one of the points of intersection.



2. Join AP, PB, and BA.
3. Then, △APB is the required equilateral triangle.



To form an equilateral triangle
1. Join AB, BC, and CA.
2. Then, △ABC is required to be an equilateral triangle.

Figure It Out (Page 154)

Question 1.
We checked by construction that there are no triangles with sidelengths of 3 cm, 4 cm, and 8 cm, and 2 cm, 3 cm, and 6 cm. Check if you could have found this without trying to construct the triangle.

Solution:
If the sum of the lengths of any two sides is greater than the third side, a triangle is possible.

Here, 3 + 4 = 7, which is not greater than 8.
Hence triangle is not possible.

Question 2.
Can we say anything about the existence of a triangle for each of the following sets of lengths?
(a) 10 km, 10 km, and 25 km
(b) 5 mm, 10 mm, and 20 mm
(c) 12 cm, 20 cm, and 40 cm
You would have realised that using a rough figure and comparing the direct path lengths with their corresponding roundabout path lengths is the same as comparing each length with the sum of the other two lengths. There are three such comparisons to be made.

Solution:
(a) 10 km, 10 km, and 25 km
Here, $10 + 10 < 25$
Since the sum of the two sides is less than the third side. Therefore, this triangle is not possible.

(b) 5 mm, 10 mm, and 20 mm
Here, $5 + 10 < 20$
$15 < 20$
∵ The sum of the two sides is less than the third side.
Therefore, a triangle is not possible.

(c) 12 cm, 20 cm, and 40 cm
$12 + 20 = 32 < 40$
Since the sum of the two sides is less than the third side. Therefore, a triangle is not possible.

Question 3.
For each set of lengths seen so far, you might have noticed that in at least two of the comparisons, the direct length was less than the sum of the other two (if not, check again!).
For example, for the set of lengths 10 cm, 15 cm, and 30 cm, there are two comparisons where this happens:
$10 < 15 + 30$
$15 < 10 + 30$,
but this doesn't happen for the third length: $30 > 10 + 15$.
(a) Will this always happen? That is, for any set of lengths, will there be at least two comparisons where the direct length is less than the sum of the other two? Explore different sets of lengths.

(b) Further, for a given set of lengths, is it possible to identify which lengths will immediately be less than the sum of the other two, without calculations? [Hint: Consider the direct lengths in the increasing order]

(c) Given three sidelengths, what do we need to compare to check for the existence of a triangle?

Solution:
(a) No, this will not always happen for every set of three lengths. Take lengths 5 cm, 6 cm, and 8 cm.
5 + 6 > 8, 6 + 8 > 5 and 5 + 6 > 6

(b) Yes, it is possible to identify without calculations.
If you arrange the three sides' lengths in increasing order, it becomes easier.
Let the sides be arranged as a < b < c (smallest to largest).
Then you only need to check whether the largest side c is less than the sum of the other two sides, a + b.
If c < a + b, then all three triangle inequalities will automatically be satisfied.
If c > a + b, then the triangle cannot be formed.

(c) To check if a triangle can be formed, we use the triangle inequality rule: Each side must be smaller than the sum of the other two sides. That is, for sides a, b, and c, you must check:
1. a + b > c
2. a + c > b
3. b + c > a

Figure It Out (Pages 156 – 159)

Question 1.
Which of the following lengths can be the sidelengths of a triangle? Explain your answers. Note that for each set, the three lengths have the same unit of measure.
(a) 2, 2, 5
(b) 3, 4, 6
(c) 2, 4, 8
(d) 5, 5, 8
(e) 10, 20, 25
(f) 10, 20, 35
(g) 24, 26, 28

Solution:
(a) Here, 2, 2, 5
∵ 2 + 2 = 4, which is less than 5.
Hence, 2, 2, 5 cannot be the sidelengths of a triangle.

(b) Here, 3, 4, 6

3 + 4 = 7 > 6

3 + 6 = 9 > 4

4 + 6 = 10 > 3

The sum of the two sides is greater than the third side.

Hence, 3, 4, 6 can be the sidelengths of a triangle.

(c) Here, 2, 4, 8

∵ 2 + 4 = 6 < 8

Hence, 2, 4, 8 cannot be the sidelengths of a triangle.

(d) 5, 5, 8 5 + 5 = 10 > 8

5 + 8 = 13 > 5

5 + 8 = 13 > 5

Hence, 5, 5, 8 can be the sidelengths of a triangle.

(e) Here, 10, 20, 25

10 + 20 = 30 > 25

20 + 25 = 45 > 10

10 + 25 = 35 > 20

All conditions are satisfied.

Hence, 10, 20, and 25 cannot be the sidelengths of a triangle.

(f) 10, 20, 35

∵ 10 + 20 = 30 < 35

Hence, 10, 20, 35 cannot be the sidelengths of a triangle.

(g) 24, 26, 28

24 + 26 = 50 > 28

24 + 28 = 52 > 26

26 + 28 = 54 > 24

All conditions are satisfied.

Hence, 24, 26, 28 can be the sidelengths of a triangle.

Will triangles always exist when a set of lengths satisfies the triangle inequality? How can we be sure?

Solution:

Yes, triangles will always exist if the set of three lengths satisfies the triangle inequality.

Can we use this analysis to tell if a triangle exists when the lengths satisfy the triangle inequality?

Solution:
Yes.

Figure It Out (Pago 159)

Question 1.
Check if a triangle exists for each of the following sets of lengths:
(a) 1, 100, 100
(b) 3, 6, 9
(c) 1, 1, 5
(d) 5, 10, 12

Solution:
We use the triangle inequality rule, i.e., the sum of any two sides must be greater than the third side.
(a) 1, 100, 100
1 + 100 = 101 > 100
1 + 100 = 101 > 100
100 + 100 = 200 > 1
Since all the conditions are satisfied.
∴ A triangle exists.

(b) 3, 6, 9
3 + 6 = 9 = 9 (not greater)
3 + 9 = 12 > 6
6 + 9 = 15 > 3
But 3 + 6 = 9 is not greater, only equal.
Thus triangle doesn’t exist.

(c) 1, 1, 5
∵ 1 + 1 = 2 < 5
∴ Thus, a triangle doesn’t exist.

(d) 5, 10, 12 5 + 10 = 15 > 12
5 + 12 = 17 > 10
10 + 12 = 22 > 5
∵ All conditions are satisfied.
∴ A triangle exists.

Question 2.
Does there exist an equilateral triangle with sides 50, 50, 50? In general, does there exist an equilateral triangle of any sidelength? Justify your answer.

Solution:

In an equilateral triangle, all three sides must be equal, and the triangle inequality must also be satisfied.
50 + 50 = 100 > 50 (true for all pairs)
So, yes, an equilateral triangle with sides 50, 50, 50 exists. In general, any set of equal sides that are non-zero can form an equilateral triangle.

Question 3.
For each of the following, give at least 5 possible values for the third length so there exists a triangle having these as sidelengths (decimal values could also be chosen):
(a) 1, 100
(b) 5, 5
(c) 3, 7

Solution:
(a) 1, 100
The third side must be less than the sum of these two sides, i.e., 101. Also, the third side must be greater than the difference between the two sides, that is, 99.
So, possible third sides are 99.1, 99.2, 99.3, 99.4, 99.5, 99.6 upto 100.9.

(b) 5, 5
The third side must be greater than 0 (5 – 5), and less than
10 (5 + 5). So, 1, 2, 3, 4, 5, 6, 7, 8, 9 are the possible third sides.

(c) 3, 7
The third side must be greater than 4 (7 – 3) and less than
10 (3 + 7). So, 5, 6, 7, 8, 9 are the possible third sides.

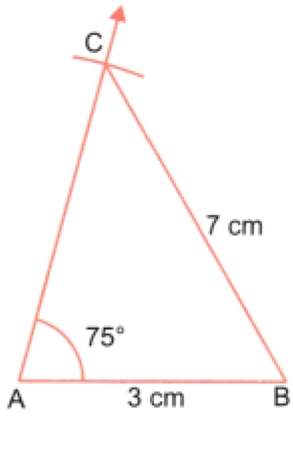
7.3 Construction of Triangles When Some Sides and Angles are Given

Figure It Out (Page 161)

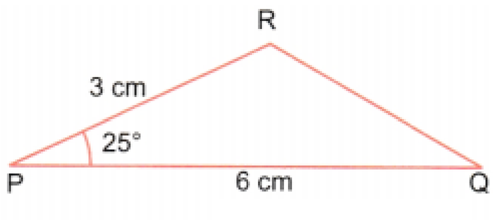
Question 1.
Construct triangles for the following measurements, where the angle is included between the sides:
(a) 3 cm, 75°, 7 cm
(b) 6 cm, 25°, 3 cm
(c) 3 cm, 120°, 8 cm

Solution:
(a) 3 cm, 75°, 7 cm
Construction Steps:
1. Draw a line segment AB of 3 cm.
2. At point A, draw an angle of 75°.

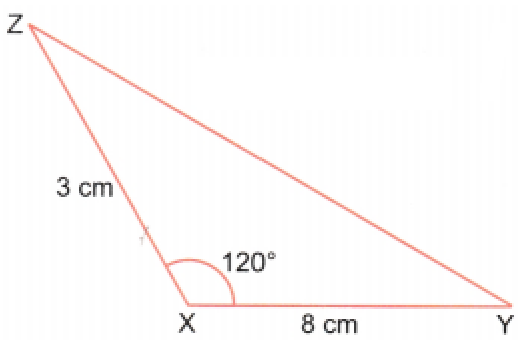
3. From point B, open an arc of 7 cm to intersect the line at point C.
4. Join AC and BC to get the required triangle.



(b) 6 cm, 25°, 3 cm
Construction Steps:
1. Draw a line segment PQ of 6 cm.
2. At point P, construct an angle of 25°.
3. From point P, cut an arc of 3 cm such that PR = 3 cm.
4. Join RQ to get the required triangle.



(c) 3 cm, 120°, 8 cm
Construction Steps:
1. Draw a line segment XY of 8 cm.
2. At point X, draw an angle of 120°.
3. Open an arc of 3 cm and cut the same line such that XZ = 3 cm.
4. Join ZY to get the required triangle.



Math Talk (Page 161)

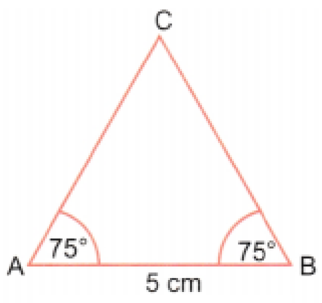
We have seen that triangles do not exist for all sets of sidelengths. Is there a combination of measurements in the case of two sides and the included angle where a triangle is not possible? Justify your answer using what you observe during construction.

Solution:
Yes, it is possible that a triangle is not possible only when the included angle is 0° or 180°.

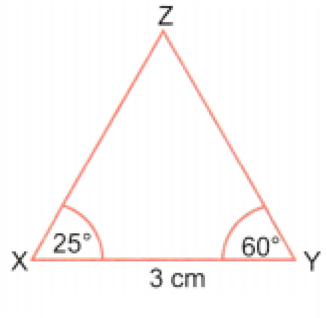
Figure It Out (Page 162)

Question 1.
Construct triangles for the following measurements:
(a) 75°, 5 cm, 75°
(b) 25°, 3 cm, 60°
(c) 120°, 6 cm, 30°
Do triangles always exist?

Solution:
(a) 75°, 5 cm, 75°
Construction Steps:
1. Draw a line segment AB of length 5 cm.
2. Draw 75° at point A.
3. Draw 75° at point B.
4. Let both the lines meet each other at point C. 5. Then ABC is a required triangle.



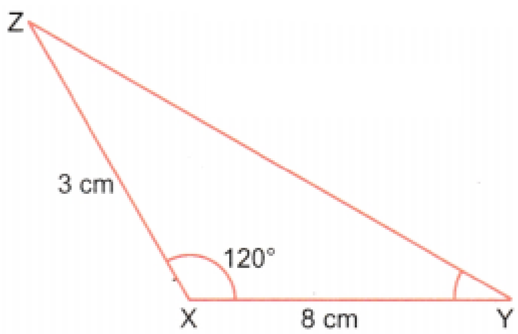
(b) 25°, 3 cm, 60°
Construction Steps:
1. Construct a line segment XY of length 3 cm.
2. Draw ∠25° and ∠60° at points X and Y, respectively.
3. The point of intersection of the two new line segments is Z.
4. XYZ is the required triangle.



(c) 120°, 6 cm, 30°

Construction Steps:

1. Draw a line segment PQ of length 6 cm.
2. Draw ∠120° and ∠30° at points P and Q, respectively.
3. Let both the new lines join each other at point R.
4. PQR is a required triangle.



Math Talk (Page 162)

Do triangles exist for every combination of two angles and their included side? Explore. As in the case when we are given all three sides, it turns out that there is not always a triangle for every combination of two angles and the included side.

Solution:

Triangles don't exist if the given two angles and the included side when the given two angles sum up to 180° or more than 180°, i.e., the two lines can never meet each other.

Find examples of measurements of two angles with the included side where a triangle is not possible.

Solution:

Examples:

- Two angles of 90° and 90° with any included side will not form a triangle.
- If both angles are 100° and 90°. Their sum will be 190°, which is more than 180°.

Figure It Out (Pages 163 – 164)

Question 1.
For each of the following angles, find another angle for which a triangle is
(a) possible,
(b) not possible. Find at least two different angles for each category:
(a) 30°
(b) 70°
(c) 54°
(d) 144°

Solution:
(a) 30°
Possible angles:
(1) 60°: 30°+ 60° = 90° (Still third angle can be added to form a triangle)
(2) 80° : 80° + 30° = 110°
Not possible:
(1) 160° : 30° + 160° = 190° which is already greater than 180°. (2) 150° : 30° + 150° = 180° (As the sum of two angles is already 180°, the third angle can't be added.)

(b) 70°
Possible angles:
(1) 50° : 50° + 70° = 120°
(2) 80° : 80° + 70° = 150°
Since the sum of two angles is less than 180°, the third angle can be added to make it 180°.
Not possible:
(1) 120° : 120° + 70° = 190° (> 180°)
(2) 110° : 110° + 70° = 180° (= 180°)

(c) 54°
Possible angles:
(1) 60° : 54° + 60° = 114°
(2) 80° : 54° + 80° = 134°
(Since, in both cases, the sum is less than 180°, therefore, the third angle can be added to form a triangle.)
Not possible:
(1) 130° : 130° + 54° = 184° (> 180°)
(2) 126° : 126° + 54° = 180° (= 180°)
In both cases, the third angle can't be added as both the angles sum up to 180° already.

(d) 144°
Possible angle:
(1) 30° : 144° + 30° = 174°

(2) 20° : 144° + 20° = 164°
Not possible:
(1) 50° : 144° + 50° = 194° (> 180°)
(2) 40° : 40° + 144° = 184° (> 180°)

Question 2.
Determine which of the following pairs can be the angles of a triangle and which cannot:
(a) 35°, 150°
(b) 70°, 30°
(c) 90°, 85°
(d) 50°, 150°

Solution:
(a) 35°, 150°
Sum = 35° + 150° = 185°
Since the sum is more than 180°.
∴ A triangle is not possible.

(b) 70°, 30°
Sum = 70° + 30°= 100°
∵ Sum < 180°.
∴ A triangle is possible.

(c) 90°, 85°
Sum = 90° + 85° = 175°
∵ Sum < 180°
∴ A triangle is possible.

(d) 50°, 150°
Sum = 50° + 150° = 200°
∵ Sum > 180°
∴ A triangle is not possible.

Like the triangle inequality, can you form a rule that describes the two angles for which a triangle is possible? Can the sum of the two angles be used for framing this rule? When the sum of two given angles is less than 180°, a triangle exists with these angles. If the sum is greater than or equal to 180°, there is no triangle with these angles. Let us take two angles, say 60° and 70°, whose sum is less than 180°. Let the included side be 5 cm. (Page 163)

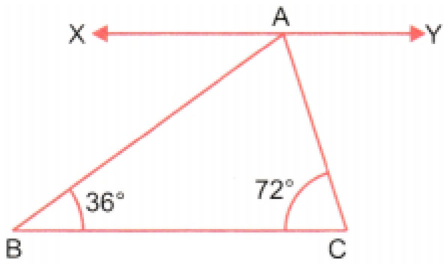
Solution:
The sum of the two angles of a triangle must be less than 180° for a triangle to exist. Given

two angles, 60° and 70°
Sum = 60° + 70° = 130°
Third Angle = 180° – 130° = 50°

Figure It Out (Page 165)

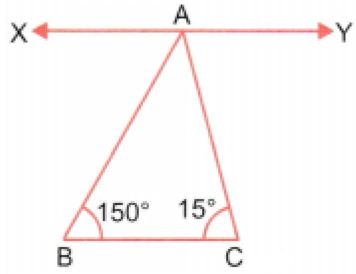
Question 1.
Find the third angle of a triangle (using a parallel line) when two of the angles are:
(a) 36°, 72°
(b) 150°, 15°
(c) 90°, 30°
(d) 75°, 45°

Solution:
(a) 36°, 72°
Draw a line XY such that XY || BC passing through A.



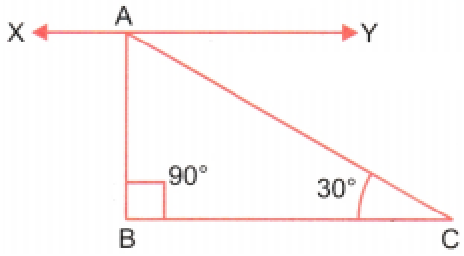
∠XAB = ∠ABC = 36°
∠YAC = ∠ACB = 72° [Alternate interior angles]
∠XAB + ∠BAC + ∠CAY = 180° [Angles in a straight line]
⇒ 36° + ∠BAC + 72° = 180°
⇒ ∠BAC + 108° = 180°
⇒ ∠BAC = 180° – 108°
⇒ ∠BAC = 72°

(b) 150°, 15°
Draw XY || BC such that XY passes through A.
∠XAB = ∠ABC = 150°
∠ACB = ∠YAC = 15° [Alternate interior angles]



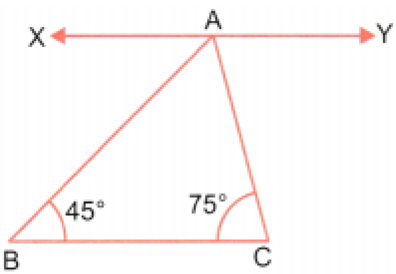
∠XAB + ∠BAC + ∠CAY = 180° [Angles in a straight line]
⇒ 150° + ∠BAC + 15° = 180°
⇒ ∠BAC = 180°- 165°
⇒ ∠BAC = 15°

(c) 90°, 30° Draw XY || BC such that XY passes through A. ∠XAB = ∠ABC = 90°
∠ACB = ∠YAC = 30° [Alternate interior angles]



∠XAB + ∠BAC + ∠CAY = 180° [Angles in a straight line]
⇒ 90° + ∠BAC + 30° = 180°
⇒ ∠BAC + 120° = 180°
⇒ ∠BAC = 180°- 120°
⇒ ∠BAC = 60°

(d) 75°, 45°
∠XAB = ∠ABC = 45°
∠YAC = ∠ACB = 75° [Alternate interior angles]

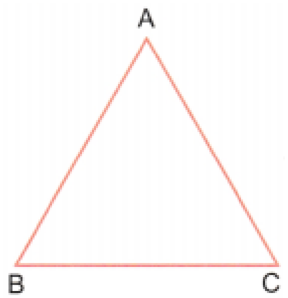


∠XAB + ∠BAC + ∠CAY = 180° [Angles in a straight line]
⇒ 45° + ∠BAC + 75° = 180°
⇒ ∠BAC + 120° = 180°
⇒ ∠BAC = 180° – 120°
⇒ ∠BAC = 60°

Question 2.
Can you construct a triangle all of whose angles are equal to 70°? If two of the angles are 70°, what would the third angle be? If all the angles in a triangle have to be equal, then what must its measure be? Explore and find out.

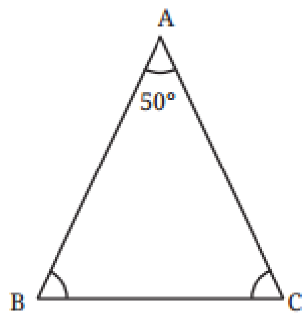
Solution:
No, a triangle can’t be constructed when all the angles are 70°. As 70° + 70° + 70° = 210°

which is greater than 180°, and all the angles must add up to 180° only.
If the angles are 70°,
Let ∠A = ∠B = 70°
In ∆ABC
∠A + ∠B + ∠C = 180° (Angle sum property)
⇒ 70° + 70° + ∠C = 180°
⇒ ∠C = 180° – 140°
⇒ ∠C = 40°
Let all the angles be equal.

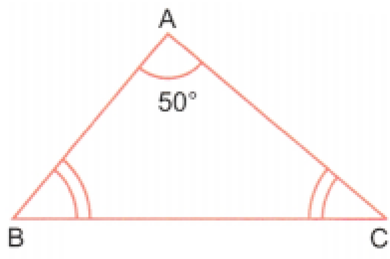


In ∆ABC,
∠A = ∠B = ∠C
∠A + ∠B + ∠C = 180°
⇒ ∠A + ∠A + ∠A = 180°
⇒ 3 × ∠A = 180°
⇒ ∠A = 180/3 = 60°
All the angles must be 60°.

Question 3.
Here is a triangle in which we know ∠B = ∠C and ∠A = 50°. Can you find ∠B and ∠C?



Solution:
Given ∠B = ∠C, ∠A = 50°



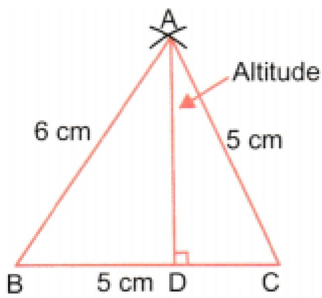
∠A + ∠B + ∠C = 180° (Angle Sum Property)
⇒ 50° + ∠B + ∠B = 180° (∠B = ∠C)
⇒ 2∠B = 130°
⇒ ∠B = 130/2 = 65°
⇒ ∠B = ∠C = 65°

7.4 Constructions Related to Altitudes of Triangles, 7.5 Types of Triangles

Figure It Out (Pages 170 – 171)

Question 1.
Construct a triangle ABC with BC = 5 cm, AB = 6 cm, CA = 5 cm. Construct an altitude from A to BC.

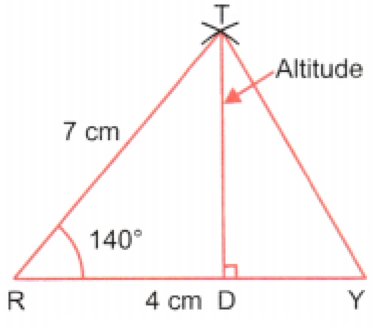
Solution:



1. Draw a line segment BC of 5 cm.
2. Open an arc of 6 cm and place the compass at vertex B, and draw an arc.
3. Now open an arc of 5 cm and place the compass at vertex C, and cut the previous arc.
4. Name the point of intersection as A.
5. ABC is the required triangle.
6. Now, keep the ruler aligned with base BC.
7. Place the set square on the miter such that BC aligns with it. 8. Slide the set square along the ruler till the vertical edge of the set square touches the vertex A.
9. Draw an altitude AD through A using the vertical edge of the set square.

Question 2.
Construct a triangle TRY with RY = 4 cm, TR = 7 cm, ∠R = 140°. Construct an altitude from T to RY.

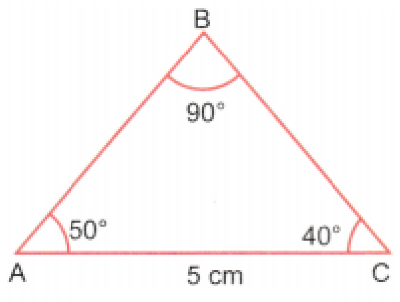
Solution:



1. Draw a line segment RY of 4 cm.
2. At point R, construct an angle of 140°.
3. Open an arc of 7 cm and place the compass at point R, and cut the previous line.
4. Name this point at T.
5. Join T to Y.
6. Now, keep the ruler aligned with RY.
7. Place the set square aligned with the ruler such that it aligns with side RY also.
8. Slide the set square along the ruler so that the vertical edge of the set square touches the vertex T.
9. Draw an altitude TP through T using the vertical edge of the set square.

Question 3.
Construct a right-angled triangle ∆ABC with ∠B = 90°, AC = 5 cm. How many different triangles exist with these measurements?
[Hint: Note that the other measurements can take any values. Take AC as the base. What values can ∠A and ∠C take so that the other angle is 90°?]

Solution:
Since only AC is given 5 cm and ∠B = 90°, multiple triangles are possible by Let us assume ∠A = 40°
∠A + ∠B + ∠C = 180° [Angle Sum Property]
⇒ 40° + 90° + ∠C = 180°
⇒ ∠C = 180° – 130°
⇒ ∠C = 50°



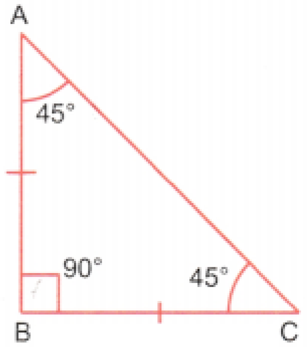
(a) Draw a line segment AC of 5 cm.
(b) From point A, construct an angle of 40° and from point B, draw an angle of 50°.
(c) Let both lines meet each other at point A.
(d) ABC is the required triangle. Likewise, you can assume ∠A as any value and get the value of ∠C such that the sum of both angles is always 90°.

Question 4.
Through construction, explore if it is possible to construct an equilateral triangle that is
(i) right-angled,
(ii) obtuse-angled. Also construct an isosceles triangle that is
(i) right-angled,
(ii) obtuse-angled.

Solution:
(i) A right-angle equilateral triangle is impossible, as all the angles in a triangle can never be 90° as they would add to 270°.
(ii) Obtuse angled equilateral triangle is not possible as all the angles of a triangle can never be greater than 90°.

Isosceles Right-Angled Triangle 1. Draw a base BC of 5 cm.

2. Find the other two angles of a triangle using the angle sum property.
∠A + ∠B + ∠C = 180° [Angle Sum Property]
⇒ ∠A + ∠B + ∠A = 180° [∴ ∠A = ∠C]
⇒ 2∠A + 90° = 180°
⇒ 2∠A = 90°
⇒ ∠A = 45°



3. Construct 90° and 45° angles at points B and C, respectively. 4. Let the lines meet each other at A.
5. Then, ∆ABC is a required triangle.

Isosceles obtuse-angled triangle

1. Draw a base BC of 6 cm.
2. Let us assume ∠B = 120°.
3. Since it is an isosceles triangle, ∠A must be equal to ∠C.
4. By applying the angle sum property, find ∠A and ∠C.

$\angle A + \angle B + \angle C = 180°$

$\Rightarrow \angle A + 120° + \angle C = 180°$

$\Rightarrow 2\angle A = 60°$

$\Rightarrow \angle A = 30°$

5. Through points B and C, draw angles of 120° and 30° respectively.
6. Let the lines meet each other and name it as A.
7. Then, ABC is a required triangle.