Important Questions Class 10 Maths Chapter 1 Real Numbers

Q.1: Use Euclid's division lemma to show that the square of any positive integer is either of form 3m or 3m + 1 for some integer m.

Solution:

Let x be any positive integer and y = 3.

By Euclid's division algorithm;

$x = 3q + r$ (for some integer $q \geq 0$ and $r = 0, 1, 2$ as $r \geq 0$ and $r < 3$)

Therefore,

$x = 3q$, $3q + 1$ and $3q + 2$

As per the given question, if we take the square on both the sides, we get;

$x^2 = (3q)^2 = 9q^2 = 3.3q^2$

Let $3q^2 = m$

Therefore,

$x^2 = 3m$ ………………….(1)

$x^2 = (3q + 1)^2$

$= (3q)^2 + 1^2 + 2 \times 3q \times 1$

$= 9q^2 + 1 + 6q$

$= 3(3q^2 + 2q) + 1$

Substituting $3q^2+2q = m$ we get,

$x^2 = 3m + 1$ ……………………………. (2)

$x^2 = (3q + 2)^2$

$= (3q)^2 + 2^2 + 2 \times 3q \times 2$

$= 9q^2 + 4 + 12q$

$= 3(3q^2 + 4q + 1) + 1$

Again, substituting $3q^2 + 4q + 1 = m$, we get,

$x^2 = 3m + 1$................................ (3)

Hence, from eq. 1, 2 and 3, we conclude that the square of any positive integer is either of form 3m or 3m + 1 for some integer m.

Q.2: Express each number as a product of its prime factors:
(i) 140
(ii) 156
(iii) 3825
(iv) 5005
(v) 7429
Solution:
(i) 140
Using the division of a number by prime numbers method, we can get the product of prime factors of 140.
Therefore, $140 = 2 \times 2 \times 5 \times 7 \times 1 = 2^2 \times 5 \times 7$

(ii) 156
Using the division of a number by prime numbers method, we can get the product of prime factors of 156.

Hence, $156 = 2 \times 2 \times 13 \times 3 = 2^2 \times 13 \times 3$

(iii) 3825
Using the division of a number by prime numbers method, we can get the product of prime factors of 3825.

Hence, $3825 = 3 \times 3 \times 5 \times 5 \times 17 = 3^2 \times 5^2 \times 17$

(iv) 5005
Using the division of a number by prime numbers method, we can get the product of prime factors of 5005.

Hence, $5005 = 5 \times 7 \times 11 \times 13 = 5 \times 7 \times 11 \times 13$

(v) 7429
Using the division of a number by prime numbers method, we can get the product of prime factors of 7429.

Hence, $7429 = 17 \times 19 \times 23 = 17 \times 19 \times 23$

Q.3: Given that HCF (306, 657) = 9, find LCM (306, 657).

Solution:

As we know that,

HCF × LCM = Product of the two given numbers

So,

9 × LCM = 306 × 657

LCM = (306 × 657)/9 = 22338

Therefore, LCM(306,657) = 22338

Q.4: Prove that 3 + 2√5 is irrational.

Solution:

Let 3 + 2√5 be a rational number.

Then the co-primes x and y of the given rational number where (y ≠ 0) is such that:

3 + 2√5 = x/y

Rearranging, we get,

2√5 = (x/y) – 3

√5 = 1/2[(x/y) – 3]

Since x and y are integers, thus, 1/2[(x/y) – 3] is a rational number.

Therefore, √5 is also a rational number. But this confronts the fact that √5 is irrational.

Thus, our assumption that 3 + 2√5 is a rational number is wrong.

Hence, 3 + 2√5 is irrational.

Q.5: Without actually performing the long division, state whether the following rational numbers will have a terminating decimal expansion or a non-terminating repeating decimal expansion:
(i) 13/3125 (ii) 17/8 (iii) 64/455 (iv) 15/1600

Solution:

Note: If the denominator has only factors of 2 and 5 or in the form of $2^m \times 5^n$ then it has a terminating decimal expansion.
If the denominator has factors other than 2 and 5 then it has a non-terminating repeating decimal expansion.

(i) 13/3125

Factoring the denominator, we get,

$3125 = 5 \times 5 \times 5 \times 5 \times 5 = 5^5$

Or

$= 2^0 \times 5^5$

Since the denominator is of the form $2^m \times 5^n$ then, 13/3125 has a terminating decimal expansion.

(ii) 17/8

Factoring the denominator, we get,

$8 = 2\times 2 \times 2 = 2^3$

Or

$= = 2^3 \times 5^0$

Since the denominator is of the form $2^m \times 5^n$ then, 17/8 has a terminating decimal expansion.

(iii) 64/455

Factoring the denominator, we get,

$455 = 5 \times 7 \times 13$

Since the denominator is not in the form of $2^m \times 5^n$, therefore 64/455 has a non-terminating repeating decimal expansion.

(iv) 15/1600

Factoring the denominator, we get,

$1600 = 2^6 \times 5^2$

Since the denominator is in the form of $2^m \times 5^n$, 15/1600 has a terminating decimal expansion.

Q.6: The following real numbers have decimal expansions as given below. In each case, decide whether they are rational or not. If they are rational, and of the form, p/q what can you say about the prime factors of q?

(i) 43.123456789

(ii) 0.120120012000120000. . .

Solution:

(i) 43.123456789
Since it has a terminating decimal expansion, it is a rational number in the form of p/q and q has factors of 2 and 5 only.

(ii) 0.120120012000120000. . .
Since it has a non-terminating and non-repeating decimal expansion, it is an irrational number.

Q.7: Check whether 6^n can end with the digit 0 for any natural number n.

Solution:

If the number 6n ends with the digit zero (0), then it should be divisible by 5, as we know any number with a unit place as 0 or 5 is divisible by 5.

Prime factorization of $6^n = (2 \times 3)^n$

Therefore, the prime factorization of 6^n doesn't contain the prime number 5.

Hence, it is clear that for any natural number n, 6^n is not divisible by 5 and thus it proves that 6^n cannot end with the digit 0 for any natural number n.

Q.8: What is the HCF of the smallest prime number and the smallest composite number?

Solution:

The smallest prime number = 2

The smallest composite number = 4

Prime factorisation of 2 = 2

Prime factorisation of 4 = 2 × 2

HCF(2, 4) = 2

Therefore, the HCF of the smallest prime number and the smallest composite number is 2.

Q.9: Using Euclid's Algorithm, find the HCF of 2048 and 960.

Solution:

2048 > 960

Using Euclid's division algorithm,

2048 = 960 × 2 + 128

960 = 128 × 7 + 64

128 = 64 × 2 + 0

Therefore, the HCF of 2048 and 960 is 64.

Q.10: Find HCF and LCM of 404 and 96 and verify that HCF × LCM = Product of the two given numbers.

Solution:

Prime factorisation of 404 = 2 × 2 × 101

Prime factorisation of 96 = 2 × 2 × 2 × 2 × 2 × 3 = 25 × 3

HCF = 2 × 2 = 4

LCM = 25 × 3 × 101 = 9696

HCF × LCM = 4 × 9696 = 38784

Product of the given two numbers = 404 × 96 = 38784

Hence, verified that LCM × HCF = Product of the given two numbers.