Important Questions Class 10 Maths Chapter 12 Areas Related to Circles

Short Answer Type Questions

1. If the radius of a circle is 4.2 cm, compute its area and circumference.

Solution:

Area of a circle = πr^2

So, area = $\pi(4.2)^2$ = 55.44 cm^2

Circumference of a circle = $2\pi r$

So, circumference = $2\pi(4.2)$ = 26.4 cm

2. What is the area of a circle whose circumference is 44 cm?

Solution:

Circumference of a circle = $2\pi r$

From the question,

$2\pi r = 44$

Or, $r = 22/\pi$

Now, area of circle = $\pi r^2 = \pi \times (22/\pi)^2$

So, area of circle = $(22\times22)/\pi$ = 154 cm^2

3. Calculate the area of a sector of angle 60°. Given, the circle has a radius of 6 cm.

Solution:

Given,

The angle of the sector = 60°

Using the formula,

The area of sector = $(\theta/360°)\times\pi r^2$

= $(60°/360°) \times \pi r^2$ cm^2

Or, area of the sector = $6 \times 22/7 \text{ cm}^2 = 132/7 \text{ cm}^2$

4. A chord subtends an angle of 90°at the centre of a circle whose radius is 20 cm. Compute the area of the corresponding major segment of the circle.

Solution:

Point to note:

Area of the sector = $\theta/360 \times \pi \times r^2$

Base and height of the triangle formed will be = radius of the circle

Area of the minor segment = area of the sector – area of the triangle formed

Area of the major segment = area of the circle – area of the minor segment

Now,

Radius of circle = r = 20 cm and

Angle subtended = $\theta = 90°$

Area of the sector = $\theta/360 \times \pi \times r^2 = 90/360 \times 22/7 \times 20^2$

Or, area of the sector = 314.2 cm^2

Area of the triangle = $½ \times$ base $\times$ height = $½ \times 20 \times 20 = 200 \text{ cm}^2$

Area of the minor segment = $314.2 - 200 = 114.2 \text{ cm}^2$

Area of the circle = $\pi \times r^2 = (22/7) \times 20^2 = 1257.14$

Area of the major segment = $1257.14 - 114.2 = 1142 .94 \text{ cm}^2$

So, the area of the corresponding major segment of the circle = $1142 .94 \text{ cm}^2$

5. A square is inscribed in a circle. Calculate the ratio of the area of the circle and the square.

Solution:

As the square is inscribed in a circle, a diagonal of the square will be = the diameter of the circle.

Let "r" be the radius of the circle and "d" be the length of each diagonal of the square.

We know,

Length of the diagonal of a square = side (s) $\times \sqrt{2}$

So,

$d = 2r$

And, $s \times \sqrt{2} = 2r$

Or, $s = \sqrt{2}r$

We know, the area of the square = s^2

Thus, the area of the square = $(\sqrt{2}r)^2 = 2r^2$

Now, the area of the circle = $\pi \times r^2$

$\therefore$ Area of the circle : area of the square = $\pi \times r^2 : 2r^2 = \pi : 2$

So, the ratio of the area of the circle and the square is $\pi : 2$.

6. Find the area of the sector of a circle with a radius of 4cm and of angle 30°. Also, find the area of the corresponding major sector.

Solution:

Radius = r = 4 cm, θ=30°
Area of sector = $[\theta/360]\times\pi r^2$
$= 30/360\times3.14\times(4)^2$
$= 1/12\times3.14\times4\times4$
$= 1/3\times3.14\times4$
$= 12.56/3 \text{ cm}^2$
$= 4.19 \text{ cm}^2$
Area of major sector = $((360 - \theta)/360)\times\pi r^2$
$= ((360 - 30))/360\times3.14\times(4)^2$
$= 330/360\times3.14\times4\times4$
$= 11/12\times3.14\times4\times4$
$= 46.05 \text{ cm}^2$

7. Calculate the perimeter of an equilateral triangle if it inscribes a circle whose area is 154 cm²

Solution:

Here, as the equilateral triangle is inscribed in a circle, the circle is an incircle.

Now, the radius of the incircle is given by,

r = Area of triangle/semi-perimeter

In the question, it is given that area of the incircle = 154 cm^2

So, $\pi \times r^2 = 154$

Or, $r = 7$ cm

Now, assume the length of each arm of the equilateral triangle to be "x" cm

So, the semi-perimeter of the equilateral triangle = $(3x/2)$ cm

And, the area of the equilateral triangle = $(\sqrt{3}/4) \times x^2$

We know, r = Area of triangle/semi-perimeter

So, $r = [x^2(\sqrt{3}/4)/ (3x/2)]$

$\Rightarrow 7 = \sqrt{3}x/6$

Or, $x = 42/\sqrt{3}$

Multiply both numerator and denominator by $\sqrt{3}$

So, $x = 42\sqrt{3}/3 = 14\sqrt{3}$ cm

Now, the perimeter of an equilateral triangle will be $= 3x = 3 \times 14\sqrt{3} = 72.7$ cm.

Long Answer Type Questions

Q.1: The cost of fencing a circular field at the rate of Rs. 24 per metre is Rs. 5280. The field is to be ploughed at the rate of Rs. 0.50 per m^2. Find the cost of ploughing the field (Take $\pi = 22/7$).

Solution:

Length of the fence (in metres) = Total cost/Rate = 5280/24 = 220

So, the circumference of the field = 220 m

If r metres is the radius of the field, then $2\pi r = 220$

$2 \times (22/7) \times r = 220$

$r = (220 \times 7)/ (2 \times 22)$

$r = 35$

Hence, the radius of the field = 35 m

Area of the field = πr^2

$= (22/7) \times 35 \times 35$

$= 22 \times 5 \times 35\ m^2$

= 3850 sq. m.

Cost of ploughing 1 m^2 of the field = Rs. 0.50

So, the total cost of ploughing the field = 3850 × Rs. 0.50 = Rs. 1925

Q.2: The wheels of a car are of diameter 80 cm each. How many complete revolutions does each wheel make in 10 minutes when the car is travelling at a speed of 66 km per hour?

Solution:

The radius of car's wheel = 80/2 = 40 cm (as D = 80 cm)

So, the circumference of wheels = $2\pi r$ = 80 π cm

Now, in one revolution, the distance covered = circumference of the wheel = 80 π cm

It is given that the distance covered by the car in 1 hr = 66km

Converting km into cm we get,

Distance covered by the car in 1hr = (66×10^5) cm

In 10 minutes, the distance covered will be = $(66 \times 10^5 \times 10)/60$ = 1100000 cm/s

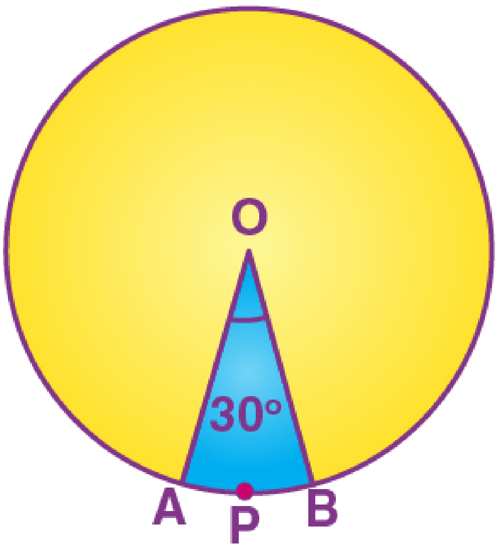
∴ Distance covered by car = 11×10^5 cm

Now, the no. of revolutions of the wheels = (Distance covered by the car/Circumference of the wheels) = 11 × 105 /80 π = 4375.

Q.3: Find the area of the sector of a circle with a radius of 4 cm and of angle 30°. Also, find the area of the corresponding major sector (Use π = 3.14)

Solution:

Let OAPB be the sector.



Area of the major sector = [(360 – θ)/ 360] × πr^2

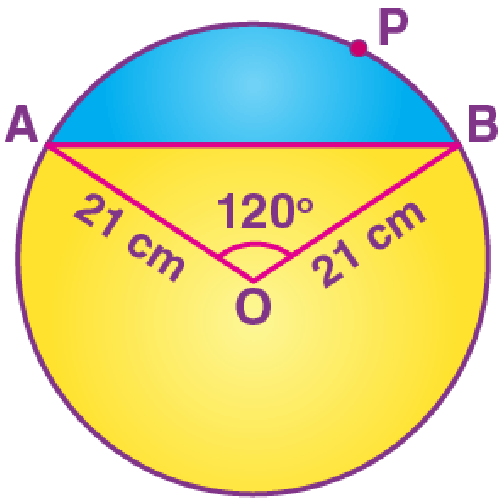
=[(360 – 30)/360] × 3.14 × 4 × 4

= (330/360) × 3.14 × 16

= 46.05 cm^2

= 46.1 cm^2 (approx)

Q.4: Find the area of the segment AYB shown in the figure, if the radius of the circle is 21 cm and ∠ AOB = 120°. (Use π = 22/7).

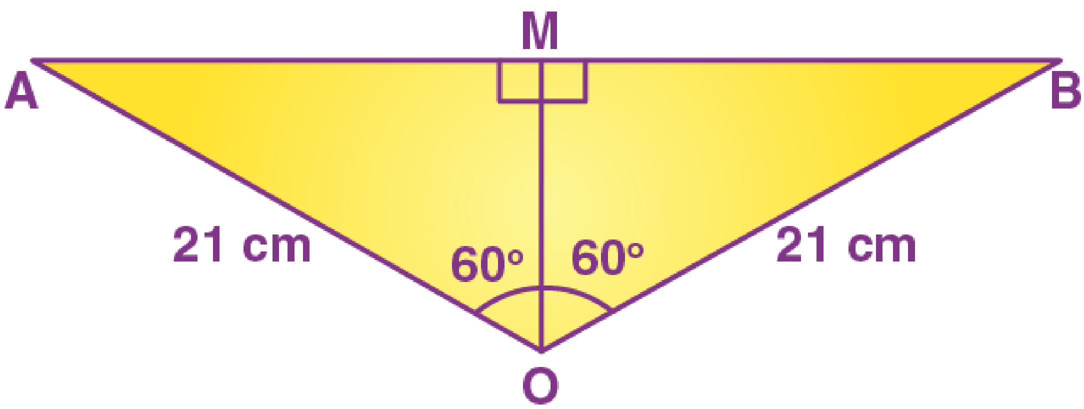


Solution:

Area of the segment AYB = Area of sector OAYB – Area of Δ OAB .....(1)

Area of the sector OAYB = (120/360) × (22/7) × 21 × 21 = 462 cm^2 ......(2)

Draw OM ⊥ AB.



OA = OB (radius)

Therefore, by RHS congruence, Δ AMO ≅ Δ BMO.

M is the mid-point of AB and $\angle$ AOM = $\angle$ BOM = $(1/2) \times 120° = 60°$

Let OM = x cm

In triangle OMA,

OM/OA = cos 60°

$x/21 = ½$

$x = 21/2$

OM = 21/2 cm

Similarly,

AM/OA = sin 60°

$AM/21 = \sqrt{3}/2$

$AM = 21\sqrt{3}/2$ cm

$AB = 2\ AM = 2\ (21\sqrt{3}/2) = 21\sqrt{3}$ cm

Area of triangle OAB = $(½) \times AB \times OM$

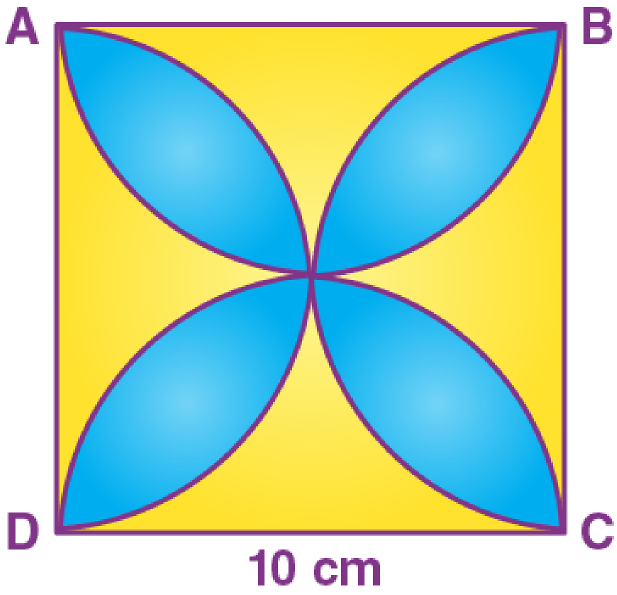
$= (½) \times 21\sqrt{3} \times (21/2)$

$= (441/4)\sqrt{3}\ cm^2$ ....(3)

From (1), (2) and (3),

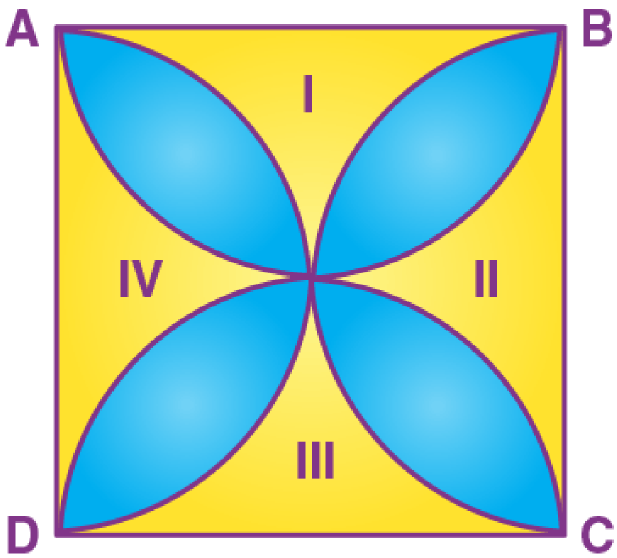
Area of the segment AYB = $[462 – (441/4)\sqrt{3}]\ cm^2$

Q.5: Find the area of the shaded design in the given figure, where ABCD is a square of 10 cm and semicircles are drawn with each side of the square as diameter. (Use π = 3.14).



Solution:

Let us assign I, II, III and IV for the unshaded regions.



Give that, side of square ABCD = 10 cm

The sides of a square are also the diameters of semicircles.

The radius of semicircle = 10/2 = 5 cm

Now, area of the region I +III = Area of square ABCD – Area of two semicircles of radius 5 cm

$= (10)^2 - 2 \times (½)\ \pi \times (5)^2$

= 100 – 3.14 × 25

= 100 – 78.5

= 21.5 cm^2

Similarly,

Area of the region II + Iv = 21.5 cm^2

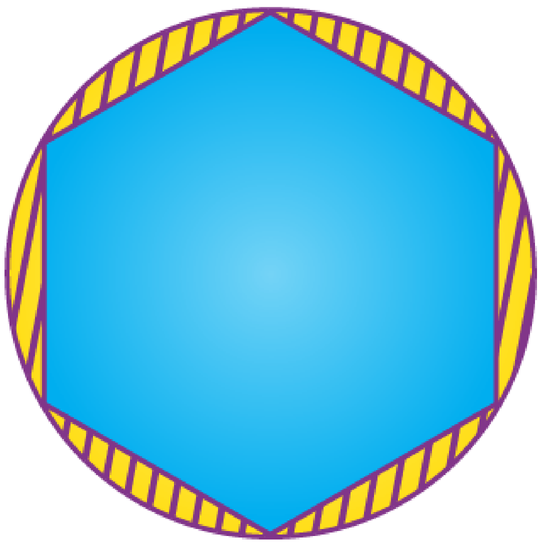
Area of the shaded region = Area of square ABCD – Area of the region (I + II + III + IV)

= 100 – 2× 21.5

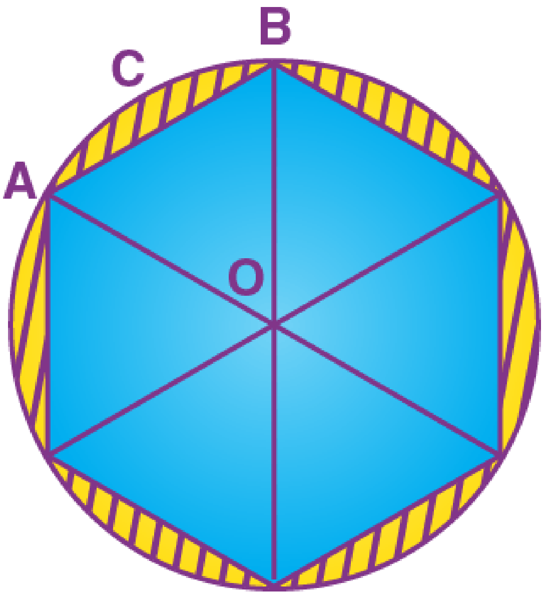
= 100 – 43

= 57 cm^2

Q.6: A round table cover has six equal designs as shown in the figure. If the radius of the cover is 28 cm, find the cost of making the designs at the rate of Rs. 0.35 per cm^2. (Use 3 = 1.7)



Solution:



Total number of equal designs = 6

$\angle AOB = 360°/6 = 60°$

The radius of the cover = 28 cm

Cost of making design = Rs. 0.35 per cm^2

Since the two arms of the triangle are the radii of the circle and thus are equal, and one angle is 60°, ΔAOB is an equilateral triangle. So, its area will be $\sqrt{3}/4 \times a^2$

Here, a = OA

$\therefore$ Area of equilateral $\Delta AOB = \sqrt{3}/4 \times 28^2 = 333.2\ cm^2$

Area of sector ACB $= (60°/360°) \times \pi\, r^2\ cm^2$

$= 410.66\ cm^2$

Area of a single design = area of sector ACB – Area of ΔAOB

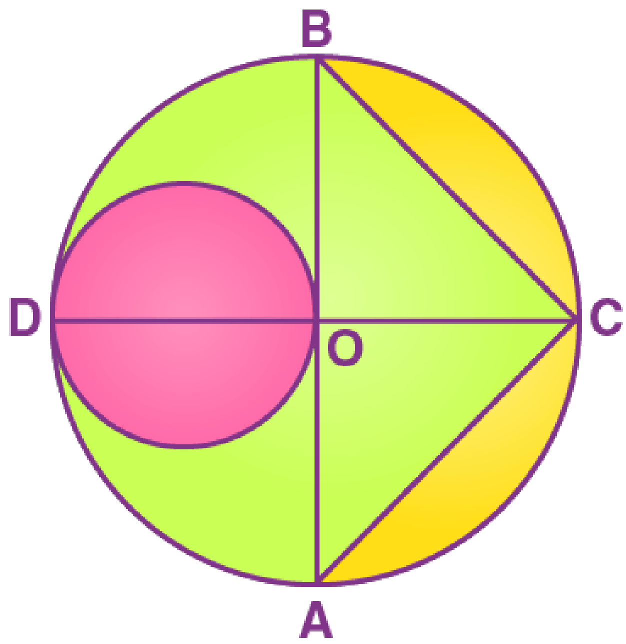
$= 410.66\ cm^2 - 333.2\ cm^2 = 77.46\ cm^2$

$\therefore$ Area of 6 designs $= 6 \times 77.46\ cm^2 = 464.76\ cm^2$

So, the total cost of making design $= 464.76\ cm^2 \times$ Rs. 0.35 per cm^2

= Rs. 162.66

Q.7: In the figure, AB and CD are two diameters of a circle (with centre O) perpendicular to each other and OD is the diameter of the smaller circle. If OA = 7 cm, find the area of the shaded region (pink and yellow regions together).



Solution:

Radius of larger circle, R = 7 cm

Radius of smaller circle, r = 7/2 cm

Height of ΔBCA = OC = 7 cm

Base of ΔBCA = AB = 14 cm

Area of ΔBCA = 1/2 × AB × OC = 1/2 × 7 × 14 = 49 cm^2

Area of larger circle = πR^2 = 22/7 × 7^2 = 154 cm^2

Area of larger semicircle = 154/2 cm2 = 77 cm^2

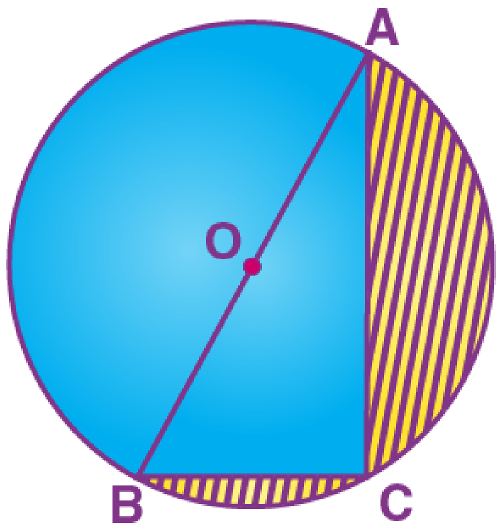
Area of smaller circle = πr^2 = 22/7 × 7/2 × 7/2 = 77/2 cm^2

Area of the shaded region = Area of the larger circle – Area of a triangle – Area of larger semicircle + Area of the smaller circle

Area of the shaded region = (154 – 49 – 77 + 77/2) cm^2

= 66.5 cm^2

Q.8: In the figure, O is the centre of a circle such that diameter AB = 13 cm and AC = 12 cm. BC is joined. Find the area of the shaded region. (take π = 3.14)



Solution:

We know that the angle in the semicircle is the right angle.

Thus, $\angle ACB = 9°$

By Pythagoras theorem,

$BC^2 + AC^2 = AB^2$

$BC^2 = AB^2 – AC^2$

$= (13)^2 – (12)^2$

$= 169 – 144$

$= 25$

$\Rightarrow BC = 5$ cm

From the given,

Diameter of circle = AB = 13 cm

Radius of semicircle = AB/2 = 13/2 cm

Area of the shaded region = Area of the semicircle – Area of right triangle ABC

$= (1/2)\pi r^2 – (1/2) \times BC \times AC$

$= (1/2) \times 3.14 \times (13/2) \times (13/2) – (1/2) \times 5 \times 12$

$= 66.33 – 30$

$= 36.33$ cm^2