Important Questions Class 10 Maths Chapter 2 Polynomials

Q.1: Find the value of "p" from the polynomial $x^2 + 3x + p$, if one of the zeroes of the polynomial is 2.

Solution:

As 2 is the zero of the polynomial.

We know that if α is a zero of the polynomial p(x), then $p(\alpha) = 0$

Substituting $x = 2$ in $x^2 + 3x + p$,

$\Rightarrow 2^2 + 3(2) + p = 0$

$\Rightarrow 4 + 6 + p = 0$

$\Rightarrow 10 + p = 0$

$\Rightarrow p = -10$

Q.2: Does the polynomial $a^4 + 4a^2 + 5$ have real zeroes?

Solution:

In the aforementioned polynomial, let $a^2 = x$.

Now, the polynomial becomes,

$x^2 + 4x + 5$

Comparing with $ax^2 + bx + c$,

Here, $b^2 - 4ac = 4^2 - 4(1)(5) = 16 - 20 = -4$

So, $D = b^2 - 4ac < 0$

As the discriminant (D) is negative, the given polynomial does not have real roots or zeroes.

Q.3: Compute the zeroes of the polynomial $4x^2 - 4x - 8$. Also, establish a relationship between the zeroes and coefficients.

Solution:

Let the given polynomial be $p(x) = 4x^2 - 4x - 8$

To find the zeroes, take $p(x) = 0$

Now, factorise the equation $4x^2 - 4x - 8 = 0$

$4x^2 - 4x - 8 = 0$

$4(x^2 - x - 2) = 0$

$x^2 - x - 2 = 0$

$x^2 - 2x + x - 2 = 0$

$x(x - 2) + 1(x - 2) = 0$

$(x - 2)(x + 1) = 0$

$x = 2, x = -1$

So, the roots of $4x^2 - 4x - 8$ are -1 and 2.

Relation between the sum of zeroes and coefficients:

$-1 + 2 = 1 = -(-4)/4$ i.e. (- coefficient of x/ coefficient of x^2)

Relation between the product of zeroes and coefficients:

$(-1) \times 2 = -2 = -8/4$ i.e (constant/coefficient of x^2)

Q.4: Find the quadratic polynomial if its zeroes are 0, $\sqrt{5}$.

Solution:

A quadratic polynomial can be written using the sum and product of its zeroes as:

$x^2 - (\alpha + \beta)x + \alpha\beta$

Where α and β are the roots of the polynomial.

Here, $\alpha = 0$ and $\beta = \sqrt{5}$

So, the polynomial will be:

$x^2 - (0 + \sqrt{5})x + 0(\sqrt{5})$

$= x^2 - \sqrt{5}x$

Q.5: Find the value of "x" in the polynomial $2a^2 + 2xa + 5a + 10$ if $(a + x)$ is one of its factors.

Solution:

Let $f(a) = 2a^2 + 2xa + 5a + 10$

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Since, $(a + x)$ is a factor of $2a^2 + 2xa + 5a + 10$, $f(-x) = 0$

So, $f(-x) = 2x^2 – 2x^2 – 5x + 10 = 0$

$-5x + 10 = 0$

$5x = 10$

$x = 10/5$

Therefore, $x = 2$

Q.6: How many zeros does the polynomial $(x – 3)^2 – 4$ have? Also, find its zeroes.

Solution:

Given polynomial is $(x – 3)^2 – 4$

Now, expand this expression.

$\Rightarrow x^2 + 9 – 6x – 4$

$= x^2 – 6x + 5$

As the polynomial has a degree of 2, the number of zeroes will be 2.

Now, solve $x^2 – 6x + 5 = 0$ to get the roots.

So, $x^2 – x – 5x + 5 = 0$

$\Rightarrow x(x – 1) -5(x – 1) = 0$

$\Rightarrow (x – 1)(x – 5) = 0$

$x = 1, x = 5$

So, the roots are 1 and 5.

Q.7: α and β are zeroes of the quadratic polynomial $x^2 – 6x + y$. Find the value of 'y' if $3\alpha + 2\beta = 20$.

Solution:

Let, $f(x) = x^2 – 6x + y$

From the given,

$3\alpha + 2\beta = 20$————————(i)

From f(x),

$\alpha + \beta = 6$————————(ii)

And,

$\alpha\beta = y$————————(iii)

Multiply equation (ii) by 2. Then, subtract the whole equation from equation (i),

=> $\alpha = 20 - 12 = 8$

Now, substitute this value in equation (ii),

=> $\beta = 6 - 8 = -2$

Substitute the values of α and β in equation (iii) to get the value of y, such as;

$y = \alpha\beta = (8)(-2) = -16$

Q.8: If the zeroes of the polynomial $x^3 - 3x^2 + x + 1$ are a – b, a, a + b, then find the value of a and b.

Solution:

Let the given polynomial be:

$p(x) = x^3 - 3x^2 + x + 1$

Given,

The zeroes of the p(x) are a – b, a, and a + b.

Now, compare the given polynomial equation with general expression.

$px^3 + qx^2 + rx + s = x^3 - 3x^2 + x + 1$

Here, p = 1, q = -3, r = 1 and s = 1

For sum of zeroes:

Sum of zeroes will be = a – b + a + a + b

$-q/p = 3a$

Substitute the values q and p.

$-(-3)/1 = 3a$

a = 1

So, the zeroes are 1 – b, 1, 1 + b.

For the product of zeroes:

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Product of zeroes = 1(1 – b)(1 + b)

-s/p = $1 - b^2$

=> -1/1 = $1 - b^2$

Or, $b^2 = 1 + 1 = 2$

So, b = √2

Thus, 1 – √2, 1, 1 + √2 are the zeroes of equation $x^3 - 3x^2 + x + 1$.

Q.9: Find a quadratic polynomial each with the given numbers as the sum and product of its zeroes, respectively.

(i) 1/4, -1

(ii) 1, 1

(iii) 4, 1

Solution:

(i) From the formulas of sum and product of zeroes, we know,
Sum of zeroes = $\alpha + \beta$
Product of zeroes = $\alpha\beta$

Given,

Sum of zeroes = 1/4
Product of zeroes = -1

Therefore, if α and β are zeroes of any quadratic polynomial, then the polynomial can be written as:-
$x^2 - (\alpha + \beta)x + \alpha\beta$

$= x^2 - (1/4)x + (-1)$

$= 4x^2 - x - 4$

Thus, $4x^2 - x - 4$ is the required quadratic polynomial.

(ii) Given,
Sum of zeroes = 1 = $\alpha + \beta$
Product of zeroes = 1 = $\alpha\beta$

Therefore, if α and β are zeroes of any quadratic polynomial, then the polynomial can be written as:-

$x^2 - (\alpha + \beta)x + \alpha\beta$

$= x^2 - x + 1$

Thus, $x^2 - x + 1$ is the quadratic polynomial.

(iii) Given,
Sum of zeroes, $\alpha + \beta = 4$
Product of zeroes, $\alpha\beta = 1$

Therefore, if α and β are zeroes of any quadratic polynomial, then the polynomial can be written as:-

$x^2 - (\alpha + \beta)x + \alpha\beta$

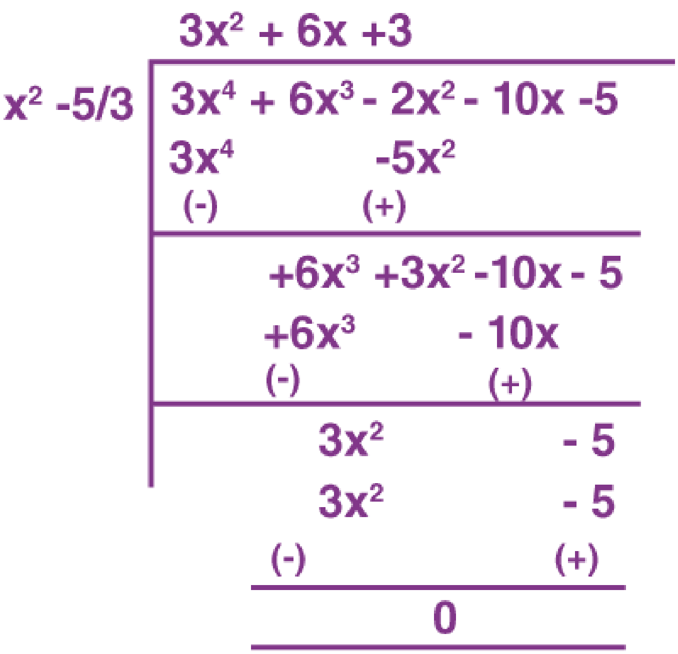
$= x^2 - 4x + 1$

Thus, $x^2 - 4x + 1$ is the quadratic polynomial.

Q.10: Obtain all other zeroes of $3x^4 + 6x^3 - 2x^2 - 10x - 5$, if two of its zeroes are $\sqrt{(5/3)}$ and $-\sqrt{(5/3)}$.

Solution: Since this is a polynomial of degree 4, hence there will be a total of 4 roots.

$\sqrt{(5/3)}$ and $-\sqrt{(5/3)}$ are zeroes of polynomial f(x).
$\therefore [x-\sqrt{(5/3)}]\,[x+\sqrt{(5/3)}] = x^2-(5/3)$



Therefore, $3x^2 + 6x + 3 = 3x(x + 1) + 3(x + 1)$
$= (3x + 3)(x + 1)$
$= 3(x + 1)(x + 1)$
$= 3(x + 1)(x + 1)$

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Hence, $x + 1 = 0$ i.e. $x = -1$, -1 is a zero of p(x).
So, its zeroes are given by: $x = -1$ and $x = -1$.
Therefore, all four zeroes of the given polynomial are:
$\sqrt{(5/3)}$ and $-\sqrt{(5/3)}$, -1 and -1.

Q.11: Find a quadratic polynomial whose zeroes are reciprocals of the zeroes of the polynomial $f(x) = ax^2 + bx + c$, $a \neq 0$, $c \neq 0$.

Solution:

Let α and β be the zeroes of the polynomial $f(x) = ax^2 + bx + c$.

So, $\alpha + \beta = -b/a$

$\alpha\beta = c/a$

According to the given, $1/\alpha$ and $1/\beta$ are the zeroes of the required quadratic polynomial.

Now, the sum of zeroes $= (1/\alpha) + (1/\beta)$

$= (\alpha + \beta)/\alpha\beta$

$= (-b/a)/ (c/a)$

$= -b/c$

Product of two zeroes $= (1/\alpha) (1/\beta)$

$= 1/\alpha\beta$

$= 1/(c/a)$

$= a/c$

The required quadratic polynomial $= k[x^2 - (\text{sum of zeroes})x + (\text{product of zeroes})]$

$= k[x^2 - (-b/c)x + (a/c)]$

$= k[x^2 + (b/c) + (a/c)]$

Q.12: Divide the polynomial $f(x) = 3x^2 - x^3 - 3x + 5$ by the polynomial $g(x) = x - 1 - x^2$ and verify the division algorithm.

Solution:

Given,

$f(x) = 3x^2 - x^3 - 3x + 5$

$g(x) = x - 1 - x^2$

Dividing f(x) = $3x^2 - x^3 - 3x + 5$ by g(x) = $x - 1 - x^2$

$$
\begin{array}{r|rrrr}
 & x & -2 & & \\
\hline
-x^2 + x - 1 & -x^3 & +3x^2 & -3x & +5 \\
 & - & & & \\
 & -x^3 & +x^2 & -x & \\
\hline
 & & 2x^2 & -2x & +5 \\
 & & - & & \\
 & & 2x^2 & -2x & +2 \\
\hline
 & & & & 3
\end{array}
$$

Here,

Quotient = q(x) = x – 2

Remainder = r(x) = 3

By division algorithm of polynomials,

Dividend = (Quotient × Divisor) + Remainder

So,

[q(x) × g(x)] + r(x) = $(x - 2)(x - 1 - x^2) + 3$

= $x^2 - x - x^3 - 2x + 2 + 2x^2 + 3$

= $3x^2 - x^3 - 3x + 5$

= f(x)

Hence, the division algorithm is verified.

Q.13: For what value of k, is the polynomial f(x) = $3x^4 - 9x^3 + x^2 + 15x + k$ completely divisible by $3x^2 - 5$?

Solution:

Given,

f(x) = $3x^4 - 9x^3 + x^2 + 15x + k$

g(x) = $3x^2 - 5$

Dividing f(x) by g(x),

$$
\begin{array}{r|rrrrr}
 & x^2 & -3x & +2 & & \\
\hline
3x^2-5 & 3x^4 & -9x^3 & +x^2 & +15x & +k \\
 & - & & & & \\
 & 3x^4 & +0x^3 & -5x^2 & & \\
\hline
 & & -9x^3 & +6x^2 & +15x & +k \\
 & & - & & & \\
 & & -9x^3 & +0x^2 & +15x & \\
\hline
 & & & 6x^2 & +0x & +k \\
 & & & - & & \\
 & & & 6x^2 & +0x & -10 \\
\hline
 & & & & & k+10
\end{array}
$$

Given that f(x) is completely divisible by $3x^2 – 5$.

So, the remainder = 0

k + 10 = 0

k = -10

Q.14: If 4 is a zero of the cubic polynomial $x^3 – 3x^2 – 10x + 24$, find its other two zeroes.

Solution:

Given cubic polynomial is $p(x) = x^3 – 3x^2 – 10x + 24$

4 is a zero of p(x).

So, (x – 4) is the factor of p(x).

Let us divide the given polynomial by (x – 4).

Here, the quotient = $x^2 + x – 6$

$= x^2 + 3x – 2x – 6$

$= x(x + 3) – 2(x + 3)$

$= (x – 2)(x + 3)$

Therefore, the other two zeroes of the given cubic polynomial are 2 and -3.

$$
\begin{array}{r|rrrr}
 & x^2 & +x & -6 & \\
\hline
x-4 & x^3 & -3x^2 & -10x & +24 \\
- & & & & \\
 & x^3 & -4x^2 & & \\
\hline
 & & x^2 & -10x & +24 \\
 & - & & & \\
 & & x^2 & -4x & \\
\hline
 & & & -6x & +24 \\
 & & - & & \\
 & & & -6x & +24 \\
\hline
 & & & & 0
\end{array}
$$

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