

Important Questions Class 10 Maths Chapter 3 Linear Equations In Two Variables

Q.1: The cost of 2 kg of apples and 1kg of grapes on a day was found to be Rs.160. After a month, the cost of 4 kg of apples and 2 kg of grapes is Rs.300. Represent the situation algebraically.

Solution:

Let the cost of 1 kg of apples be 'Rs. x'.

And, let the cost of 1 kg of grapes be 'Rs. y'.

According to the question, the algebraic representation is

$$2x + y = 160$$

$$\text{And } 4x + 2y = 300$$

For, $2x + y = 160$ or $y = 160 - 2x$, the solution table is;

x	50	60	70
y	60	40	20

For $4x + 2y = 300$ or $y = (300 - 4x)/2$, the solution table is;

x	70	80	75
y	10	-10	0

Note: Students can also represent these two equations graphically, by using the given points of x-coordinate and y-coordinate.

Q.2: Half the perimeter of a rectangular garden, whose length is 4 m more than its width, is 36 m. Find the dimensions of the garden.

Solution:

Given, half the perimeter of a rectangular garden = 36 m

$$\text{so, } 2(l + b)/2 = 36$$

$$(l + b) = 36 \dots\dots\dots(1)$$

Given, the length is 4 m more than its width.

Let width = x

And length = $x + 4$

Substituting this in eq(1), we get;

$$x + x + 4 = 36$$

$$2x + 4 = 36$$

$$2x = 32$$

$$x = 16$$

Therefore, the width is 16 m and the length is $16 + 4 = 20$ m.

Q.3: On comparing the ratios a_1/a_2 , b_1/b_2 , and c_1/c_2 , find out whether the following pair of linear equations are consistent, or inconsistent.

(i) $3x + 2y = 5$; $2x - 3y = 7$

(ii) $2x - 3y = 8$; $4x - 6y = 9$

Solution:

(i) Given : $3x + 2y = 5$ or $3x + 2y - 5 = 0$
and $2x - 3y = 7$ or $2x - 3y - 7 = 0$

Comparing the above equations with $a_1x + b_1y + c_1 = 0$

And $a_2x + b_2y + c_2 = 0$

We get,

$$a_1 = 3, b_1 = 2, c_1 = -5$$

$$a_2 = 2, b_2 = -3, c_2 = -7$$

$$a_1/a_2 = 3/2, b_1/b_2 = 2/-3, c_1/c_2 = -5/-7 = 5/7$$

Since, $a_1/a_2 \neq b_1/b_2$ the lines intersect each other at a point and have only one possible solution.

Hence, the equations are consistent.

(ii) Given $2x - 3y = 8$ and $4x - 6y = 9$

Therefore,

$$a_1 = 2, b_1 = -3, c_1 = -8$$

$$a_2 = 4, b_2 = -6, c_2 = -9$$

$$a_1/a_2 = 2/4 = 1/2, b_1/b_2 = -3/-6 = 1/2, c_1/c_2 = -8/-9 = 8/9$$

Since, $a_1/a_2 = b_1/b_2 \neq c_1/c_2$

Therefore, the lines are parallel to each other and they have no possible solution. Hence, the equations are inconsistent.

Q.4: Solve the following pair of linear equations by the substitution method.

(i) $x + y = 14$

$x - y = 4$

(ii) $3x - y = 3$

$9x - 3y = 9$

Solution:

(i) Given,

$x + y = 14$ and $x - y = 4$ are the two equations.

From 1st equation, we get,

$x = 14 - y$

Now, put the value of x in second equation to get,

$(14 - y) - y = 4$

$14 - 2y = 4$

$2y = 10$

Or $y = 5$

By the value of y , we can now find the value of x ;

$\therefore x = 14 - y$

$\therefore x = 14 - 5$

Or $x = 9$

Hence, $x = 9$ and $y = 5$.

(ii) Given,

$3x - y = 3$ and $9x - 3y = 9$ are the two equations.

From 1st equation, we get,

$x = (3 + y)/3$

Now, substitute the value of x in the given second equation to get,

$9[(3 + y)/3] - 3y = 9$

$\Rightarrow 3(3+y) - 3y = 9$

$\Rightarrow 9 + 3y - 3y = 9$

$\Rightarrow 9 = 9$

Therefore, y has infinite values and since, $x = (3 + y)/3$, so x also has infinite values.

Q.5: Solve $2x + 3y = 11$ and $2x - 4y = -24$ and hence find the value of ‘ m ’ for which $y = mx + 3$.

Solution:

$2x + 3y = 11$(i)

$2x - 4y = -24$ (ii)

From equation (i), we get;

$x = (11 - 3y)/2$ (iii)

Putting the value of x in equation (ii), we get

$$2[(11 - 3y)/2] - 4y = -24$$

$$11 - 3y - 4y = -24$$

$$-7y = -35$$

$$y = 5 \dots \dots \dots (iv)$$

Putting the value of y in equation (iii), we get;

$$x = (11 - 15)/2 = -4/2 = -2$$

$$\text{Hence, } x = -2, y = 5$$

Also,

$$y = mx + 3$$

$$5 = -2m + 3$$

$$-2m = 2$$

$$m = -1$$

Therefore, the value of m is -1.

Q.6: The coach of a cricket team buys 7 bats and 6 balls for Rs.3800. Later, she buys 3 bats and 5 balls for Rs.1750. Find the cost of each bat and each ball.

Solution:

Let the cost of a bat be x and the cost of a ball be y.

According to the question,

$$7x + 6y = 3800 \dots \dots \dots (i)$$

$$3x + 5y = 1750 \dots \dots \dots (ii)$$

From (i), we get;

$$y = (3800 - 7x)/6 \dots \dots \dots (iii)$$

Substituting (iii) in (ii). we get,

$$3x + 5[(3800 - 7x)/6] = 1750$$

$$\Rightarrow 3x + (9500/3) - (35x/6) = 1750$$

$$3x - (35x/6) = 1750 - (9500/3)$$

$$(18x - 35x)/6 = (5250 - 9500)/3$$

$$\Rightarrow -17x/6 = -4250/3$$

$$\Rightarrow -17x = -8500$$

$$x = 500$$

Putting the value of x in (iii), we get;

$$y = (3800 - 7 \times 500)/6 = 300/6 = 50$$

Hence, the cost of a bat is Rs 500 and the cost of a ball is Rs 50.

Q.7: A fraction becomes 9/11 if 2 is added to both the numerator and the denominator. If, 3 is added to both the numerator and the denominator it becomes 5/6. Find the fraction.

Solution:

Let the fraction be x/y.

According to the question,

$$(x + 2)/(y + 2) = 9/11$$

$$11x + 22 = 9y + 18$$

$$11x - 9y = -4 \dots\dots\dots (1)$$

$$(x + 3)/(y + 3) = 5/6$$

$$6x + 18 = 5y + 15$$

$$6x - 5y = -3 \dots\dots\dots (2)$$

From (1), we get

$$x = (-4 + 9y)/11 \dots\dots\dots (3)$$

Substituting the value of x in (2), we get

$$6[(-4 + 9y)/11] - 5y = -3$$

$$-24 + 54y - 55y = -33$$

$$-y = -9$$

$$y = 9 \dots\dots\dots (4)$$

Substituting the value of y in (3), we get

$$x = (-4 + 81)/11 = 77/11 = 7$$

Hence, the fraction is 7/9.

Q.8 Form the pair of linear equations in the following problems, and find their solutions (if they exist) by the elimination method:

(i) Five years ago, Nuri was thrice as old as Sonu. Ten years later, Nuri will be twice as old as Sonu. How old are Nuri and Sonu?

Solution:

Let us assume, the present age of Nuri be x.

And the present age of Sonu is y.

According to the given condition, we can write as;

$$x - 5 = 3(y - 5)$$

$$x - 3y = -10 \dots\dots\dots (1)$$

Now,

$$x + 10 = 2(y + 10)$$

$$x - 2y = 10 \dots\dots\dots (2)$$

Subtract eq. 1 from 2, to get,

$$y = 20 \dots\dots\dots (3)$$

Substituting the value of y in eq.1, we get,

$$x - 3(20) = -10$$

$$x - 60 = -10$$

$$x = 50$$

Therefore,
The age of Nuri is 50 years
The age of Sonu is 20 years.

(ii) A lending library has a fixed charge for the first three days and an additional charge for each day thereafter. Saritha paid Rs.27 for a book kept for seven days, while Susy paid Rs.21 for the book she kept for five days. Find the fixed charge and the charge for each extra day.

Solution:

Let the fixed charge for the first three days be Rs. A and the charge for each day extra be Rs. B.

According to the information given,

$$A + 4B = 27 \dots\dots\dots (i)$$

$$A + 2B = 21 \dots\dots\dots (ii)$$

When equation (ii) is subtracted from equation (i) we get,

$$2B = 6$$

$$B = 3 \dots\dots\dots(iii)$$

Substituting B = 3 in equation (i) we get,

$$A + 12 = 27$$

$$A = 15$$

Hence, the fixed charge is Rs. 15.

And the Additional charge per day is Rs. 3.

Q.9: Solve the following pair of linear equations by the substitution and cross-multiplication methods:

$$8x + 5y = 9$$

$$3x + 2y = 4$$

Solution:

$$8x + 5y = 9 \dots\dots\dots(1)$$

$$3x + 2y = 4 \dots\dots\dots(2)$$

From equation (2) we get;

$$x = (4 - 2y) / 3 \dots\dots\dots (3)$$

Substituting this value in equation 1, we get

$$8[(4 - 2y)/3] + 5y = 9$$

$$32 - 16y + 15y = 27$$

$$-y = -5$$

$$y = 5 \dots\dots\dots(4)$$

Substituting this value in equation (2), we get

$$3x + 10 = 4$$

$$3x = -6$$

$$x = -2$$

Thus, x = -2 and y = 5.

Now, Using Cross Multiplication method:

$$8x + 5y - 9 = 0$$

$$3x + 2y - 4 = 0$$

$$x/(-20 + 18) = y/(-27 + 32) = 1/(16 - 15)$$

$$-x/2 = y/5 = 1/1$$

$$\therefore x = -2 \text{ and } y = 5.$$

Q.10: Formulate the following problems as a pair of equations, and hence find their solutions:

(i) Ritu can row downstream 20 km in 2 hours, and upstream 4 km in 2 hours. Find her speed of rowing in still water and the speed of the current.

Solution:

(i) Let us consider,

Speed of boat in still water = x km/hr

Speed of current = y km/hr

Now, speed of Ritu, during,

Downstream = $x + y$ km/hr

Upstream = $x - y$ km/hr

As per the question given,

$$2(x + y) = 20$$

$$\text{Or } x + y = 10 \dots\dots\dots(1)$$

$$\text{And, } 2(x - y) = 4$$

$$\text{Or } x - y = 2 \dots\dots\dots(2)$$

Adding both the eq.1 and 2, we get,

$$2x = 12$$

$$x = 6$$

Putting the value of x in eq.1, we get,

$$y = 4$$

Therefore,

Speed of Ritu in still water = 6 km/hr

Speed of current = 4 km/hr

Q.11: Solve the equations $x + 2y - 4 = 0$ and $2x + 4y - 12 = 0$ graphically.

Solution:

Given,

$$x + 2y - 4 = 0 \dots(i)$$

$$2x + 4y - 12 = 0 \dots(ii)$$

From (i),

$$x + 2y = 4$$

$$2y = 4 - x$$

$$y = (4 - x)/2$$

x	0	2	4
y	2	1	0

From (ii),

$$2x + 4y = 12$$

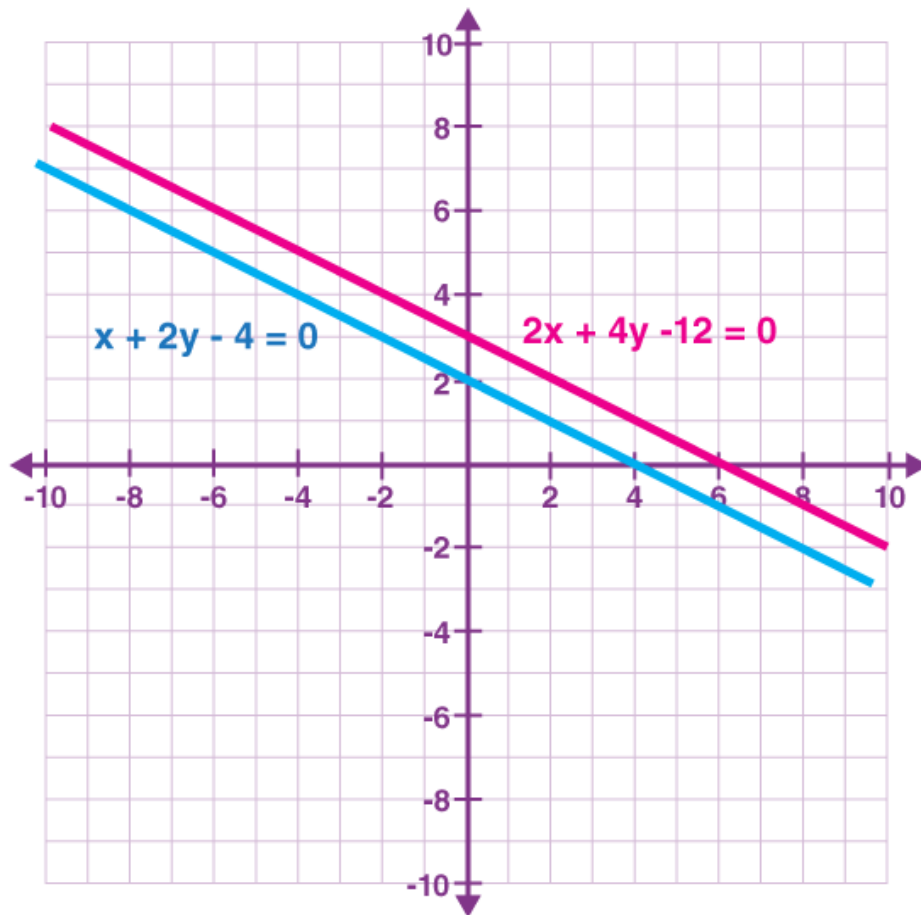
$$x + 2y = 6$$

$$2y = 6 - x$$

$$y = (6 - x)/2$$

x	0	2	4
y	3	2	1

Plotting the points on the graph, we get;



Here, the lines represent the given pair of linear equations are parallel.

Thus, there is no solution to the given pair of linear equations.

Q.12: Find the value(s) of k so that the pair of equations $x + 2y = 5$ and $3x + ky + 15 = 0$ has a unique solution.

Solution:

Given,

$$x + 2y = 5$$

$$3x + ky + 15 = 0$$

Also, given that the pair of equations has a unique solution.

Comparing the given equations with standard form,

$$a_1 = 1, b_1 = 2, c_1 = -5$$

$$a_2 = 3, b_2 = k, c_2 = 15$$

Condition for unique solution is:

$$a_1/a_2 \neq b_1/b_2$$

$$1/3 \neq 2/k$$

$$k \neq (2)(3)$$

$$k \neq 6$$

Thus, for all real values of k except 6, the given pair of equations has a unique solution.

Q.13: Determine graphically the coordinates of vertices of a triangle, the equation of whose sides are given by $2y - x = 8$, $5y - x = 14$ and $y - 2x = 1$.

Solution:

Given,

$$2y - x = 8 \dots (i)$$

$$5y - x = 14 \dots (ii)$$

$$y - 2x = 1 \dots (iii)$$

From (i),

$$2y = x + 8$$

$$y = (x + 8)/2$$

x	-4	0	2
y	2	4	5

From (ii),

$$5y = x + 14$$

$$y = (x + 14)/5$$

x	-4	1	6
y	2	3	4

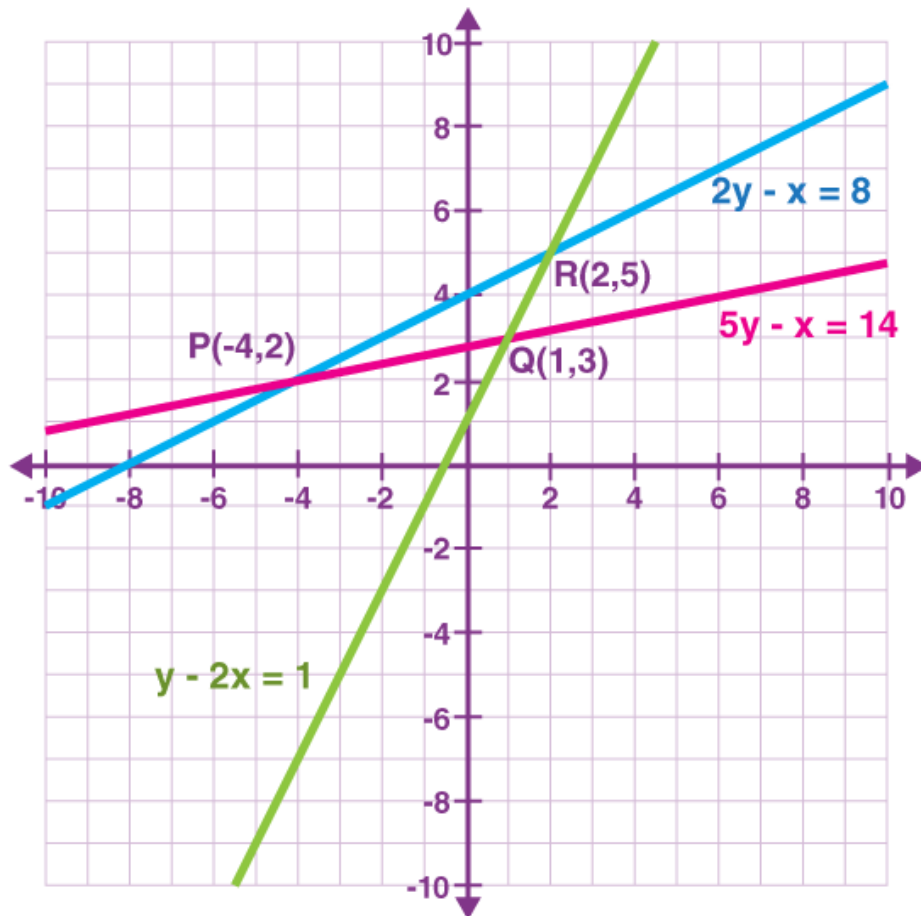
From (iii),

$$y = 2x + 1$$

x	-1	1	2
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y	-1	3	5
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Let us plot all these points on the graph.



From the graph, we can write the coordinates of vertices of triangle formed are:

$P(-4, 2)$, $Q(1, 3)$, and $R(2, 5)$

Q.14: Use elimination method to find all possible solutions of the following pair of linear equation:

$$2x + 3y = 8$$

$$4x + 6y = 7$$

Solution:

Given,

$$2x + 3y = 8 \dots (i)$$

$$4x + 6y = 7 \dots (ii)$$

Multiply Equation (1) by 2 and Equation (2) by 1 to make the coefficients of x equal.

$$4x + 6y = 16 \dots \text{(iii)}$$

$$4x + 6y = 7 \dots \text{(iv)}$$

Subtracting (iv) from (iii),

$$4x + 6y - 4x - 6y = 16 - 7$$

$$0 = 9, \text{ it is not possible}$$

Therefore, the pair of equations has no solution.

Q.15: Solve the following pairs of equations by reducing them to a pair of linear equations:

$$\frac{1}{2}x + \frac{1}{3}y = 2$$

$$\frac{1}{3}x + \frac{1}{2}y = \frac{13}{6}$$

Solution:

Given,

$$\frac{1}{2}x + \frac{1}{3}y = 2$$

$$\frac{1}{3}x + \frac{1}{2}y = \frac{13}{6}$$

Let us assume $\frac{1}{x} = m$ and $\frac{1}{y} = n$, then the equations will change as follows.

$$\frac{m}{2} + \frac{n}{3} = 2$$

$$\Rightarrow 3m + 2n - 12 = 0 \dots \text{(1)}$$

$$\frac{m}{3} + \frac{n}{2} = \frac{13}{6}$$

$$\Rightarrow 2m + 3n - 13 = 0 \dots \text{(2)}$$

Now, using cross-multiplication method, we get,

$$\frac{m}{(-26 - (-36))} = \frac{n}{(-24 - (-39))} = \frac{1}{(9-4)}$$

$$\frac{m}{10} = \frac{n}{15} = \frac{1}{5}$$

$$\frac{m}{10} = \frac{1}{5} \text{ and } \frac{n}{15} = \frac{1}{5}$$

$$\text{So, } m = 2 \text{ and } n = 3$$

$$\frac{1}{x} = 2 \text{ and } \frac{1}{y} = 3$$

$$x = \frac{1}{2} \text{ and } y = \frac{1}{3}$$