Important Questions Class 10 Maths Chapter 4 Quadratic Equations

Q.1: Represent the following situations in the form of quadratic equations:

(i) The area of a rectangular plot is 528 m². The length of the plot (in metres) is one more than twice its breadth. We need to find the length and breadth of the plot.

(ii) A train travels a distance of 480 km at a uniform speed. If the speed had been 8 km/h less, then it would have taken 3 hours more to cover the same distance. What is the speed of the train?

Solution:

(i) Let us consider,
The breadth of the rectangular plot is x m.

Thus, the length of the plot = (2x + 1) m

As we know,

Area of rectangle = length × breadth = 528 m^2
Putting the value of length and breadth of the plot in the formula, we get,

$(2x + 1) \times x = 528$

$\Rightarrow 2x^2 + x = 528$

$\Rightarrow 2x^2 + x - 528 = 0$

Hence, $2x^2 + x - 528 = 0$, is the required equation which represents the given situation.

(ii) Let us consider,
speed of train = x km/h
And
Time taken to travel 480 km = 480 (x) km/h
As per second situation, the speed of train = (x – 8) km/h

As given, the train will take 3 hours more to cover the same distance.
Therefore, time taken to travel 480 km = (480/x) + 3 km/h
As we know,
Speed × Time = Distance
Therefore,
$(x - 8)[(480/x) + 3] = 480$

$\Rightarrow 480 + 3x - (3840/x) - 24 = 480$
$\Rightarrow 3x - (3840/x) = 24$
$\Rightarrow 3x^2 - 24x - 3840 = 0$

$\Rightarrow x^2 - 8x - 1280 = 0$

Hence, $x^2 - 8x - 1280 = 0$ is the required representation of the problem mathematically.

Q.2: Find the roots of quadratic equations by factorisation:

(i) $\sqrt{2}\,x^2 + 7x + 5\sqrt{2}=0$

(ii) $100x^2 - 20x + 1 = 0$

Solution:

(i) $\sqrt{2}\,x^2 + 7x + 5\sqrt{2}=0$
Considering the L.H.S. first,

$\Rightarrow \sqrt{2}\,x^2 + 5x + 2x + 5\sqrt{2}$

$\Rightarrow x(\sqrt{2}x + 5) + \sqrt{2}(\sqrt{2}x + 5)= (\sqrt{2}x + 5)(x + \sqrt{2})$
The roots of this equation, $\sqrt{2}\,x^2 + 7x + 5\sqrt{2}=0$ are the values of x for which $(\sqrt{2}x + 5)(x + \sqrt{2}) = 0$
Therefore, $\sqrt{2}x + 5 = 0$ or $x + \sqrt{2} = 0$
$\Rightarrow x = -5/\sqrt{2}$ or $x = -\sqrt{2}$

(ii) Given, $100x^2 - 20x + 1=0$
Considering the L.H.S. first,
$\Rightarrow 100x^2 - 10x - 10x + 1$
$\Rightarrow 10x(10x - 1) -1(10x - 1)$
$\Rightarrow (10x - 1)^2$
The roots of this equation, $100x^2 - 20x + 1=0$, are the values of x for which $(10x - 1)^2= 0$
Therefore,

$(10x - 1) = 0$

or $(10x - 1) = 0$
$\Rightarrow x =1/10$ or $x =1/10$

Q.3: Find two consecutive positive integers, the sum of whose squares is 365.

Solution:

Let us say, the two consecutive positive integers be x and x + 1.
Therefore, as per the given statement,
$x^2 + (x + 1)^2 = 365$
$\Rightarrow x^2 + x^2 + 1 + 2x = 365$
$\Rightarrow 2x^2 + 2x - 364 = 0$

$\Rightarrow x^2 + x - 182 = 0$
$\Rightarrow x^2 + 14x - 13x - 182 = 0$
$\Rightarrow x(x + 14) -13(x + 14) = 0$
$\Rightarrow (x + 14)(x - 13) = 0$
Thus, either, $x + 14 = 0$ or $x - 13 = 0$,
$\Rightarrow x = -14$ or $x = 13$
since, the integers are positive, so x can be 13, only.

So, $x + 1 = 13 + 1 = 14$
Therefore, the two consecutive positive integers will be 13 and 14.

Q.4: Find the roots of the following quadratic equations, if they exist, by the method of completing the square:

(i) $2x^2 - 7x + 3 = 0$

(ii) $2x^2 + x - 4 = 0$

Solution:

(i) $2x^2 - 7x + 3 = 0$
$\Rightarrow 2x^2 - 7x = -3$
Dividing by 2 on both sides, we get
$\Rightarrow x^2 - 7x/2 = -3/2$
$\Rightarrow x^2 - 2 \times x \times 7/4 = -3/2$
On adding $(7/4)^2$ to both sides of above equation, we get
$\Rightarrow (x)^2 - 2 \times x \times 7/4 + (7/4)^2 = (7/4)^2 - (3/2)$
$\Rightarrow (x - 7/4)^2 = (49/16) - (3/2)$
$\Rightarrow (x - 7/4)^2 = 25/16$
$\Rightarrow (x - 7/4) = \pm 5/4$
$\Rightarrow x = 7/4 \pm 5/4$
$\Rightarrow x = 7/4 + 5/4$ or $x = 7/4 - 5/4$

$\Rightarrow x = 12/4$ or $x = 2/4$
$\Rightarrow x = 3$ or $1/2$

(ii) $2x^2 + x - 4 = 0$
$\Rightarrow 2x^2 + x = 4$
Dividing both sides of the above equation by 2, we get
$\Rightarrow x^2 + x/2 = 2$

$\Rightarrow (x)^2 + 2 \times x \times 1/4 = 2$
Now on adding $(1/4)^2$ to both sides of the equation, we get,

$\Rightarrow (x)^2 + 2 \times x \times 1/4 + (1/4)^2 = 2 + (1/4)^2$
$\Rightarrow (x + 1/4)^2 = 33/16$
$\Rightarrow x + 1/4 = \pm \sqrt{33}/4$

$\Rightarrow x = \pm \sqrt{33}/4 - 1/4$
$\Rightarrow x = \pm \sqrt{33} - 1/4$

Therefore, either $x = \sqrt{33}-1/4$ or $x = -\sqrt{33}-1/4$.

Q.5: The diagonal of a rectangular field is 60 metres more than the shorter side. If the longer side is 30 metres more than the shorter side, find the sides of the field.

Solution:

Let us say, the shorter side of the rectangle be x m.
Then, larger side of the rectangle = (x + 30) m
Diagonal of the rectangle = $\sqrt{[x^2+(x+30)^2]}$ As given, the length of the diagonal is = x + 60 m
$\Rightarrow x^2 + (x + 30)^2 = (x + 60)^2$
$\Rightarrow x^2 + x^2 + 900 + 60x = x^2 + 3600 + 120x$
$\Rightarrow x^2 - 60x - 2700 = 0$
$\Rightarrow x^2 - 90x + 30x - 2700 = 0$
$\Rightarrow x(x - 90) + 30(x -90)$
$\Rightarrow (x - 90)(x + 30) = 0$
$\Rightarrow x = 90, -30$

Q.6 : Solve the quadratic equation $2x^2 - 7x + 3 = 0$ by using quadratic formula.

Solution:

$2x^2 - 7x + 3 = 0$

On comparing the given equation with $ax^2 + bx + c = 0$, we get,

a = 2, b = -7 and c = 3
By using quadratic formula, we get,

$x = [-b\pm\sqrt{(b^2 - 4ac)}]/2a$

$\Rightarrow x = [7\pm\sqrt{(49 - 24)}]/4$
$\Rightarrow x = [7\pm\sqrt{25}]/4$
$\Rightarrow x = [7\pm5]/4$

Therefore,
$\Rightarrow x = 7+5/4$ or $x = 7-5/4$
$\Rightarrow x = 12/4$ or $2/4$
$\therefore x = 3$ or ½

Q.7: The sum of the areas of two squares is 468 m². If the difference of their perimeters is 24 m, find the sides of the two squares.

Solution:

Sum of the areas of two squares is 468 m^2.
$\because x^2 + y^2 = 468$ . ...(1) [$\because$ area of square = $side^2$]

→ The difference of their perimeters is 24 m.
$\because 4x - 4y = 24$ [$\because$ Perimeter of square = 4 × side] $\Rightarrow 4(x - y) = 24$
$\Rightarrow x - y = 24/4$
$\Rightarrow x - y = 6$
$\therefore y = x - 6$ ....(2)
From equation (1) and (2),
$\because x^2 + (x - 6)^2 = 468$
$\Rightarrow x^2 + x^2 - 12x + 36 = 468$
$\Rightarrow 2x^2 - 12x + 36 - 468 = 0$
$\Rightarrow 2x^2 - 12x - 432 = 0$
$\Rightarrow 2(x^2 - 6x - 216) = 0$
$\Rightarrow x^2 - 6x - 216 = 0$
$\Rightarrow x^2 - 18x + 12x - 216 = 0$
$\Rightarrow x(x - 18) + 12(x - 18) = 0$
$\Rightarrow (x + 12) (x - 18) = 0$
$\Rightarrow x + 12 = 0$ and $x - 18 = 0$
$\Rightarrow x = - 12m$ [rejected] and $x = 18m$
$\therefore x = 18$ m
Put the value of 'x' in equation (2),
$\because y = x - 6$
$\Rightarrow y = 18 - 6$
$\therefore y = 12$ m
Hence, sides of two squares are 18m and 12m respectively.

Q.8: Find the values of k for each of the following quadratic equations, so that they have two equal roots.
(i) $2x^2 + kx + 3 = 0$
(ii) $kx (x - 2) + 6 = 0$

Solution:

(i) $2x^2 + kx + 3 = 0$

Comparing the given equation with $ax^2 + bx + c = 0$, we get,

$a = 2$, $b = k$ and $c = 3$

As we know, Discriminant = $b^2 - 4ac$

$= (k)^2 - 4(2) (3)$

$= k^2 - 24$

For equal roots, we know,

Discriminant = 0

$k^2 - 24 = 0$

$k^2 = 24$

$k = \pm\sqrt{24} = \pm 2\sqrt{6}$

(ii) kx(x – 2) + 6 = 0

or $kx^2 - 2kx + 6 = 0$

Comparing the given equation with $ax^2 + bx + c = 0$, we get

$a = k$, $b = -2k$ and $c = 6$

We know, Discriminant = $b^2 - 4ac$

$= (-2k)^2 - 4(k)(6)$

$= 4k^2 - 24k$

For equal roots, we know,

$b^2 - 4ac = 0$

$4k^2 - 24k = 0$

$4k(k - 6) = 0$

Either $4k = 0$ or $k = 6 = 0$

$k = 0$ or $k = 6$

However, if k = 0, then the equation will not have the terms 'x^2' and 'x'.

Therefore, if this equation has two equal roots, k should be 6 only.

Q.9: Is it possible to design a rectangular park of perimeter 80 and area 400 sq.m.? If so find its length and breadth.

Solution:

Let the length and breadth of the park be L and B.

Perimeter of the rectangular park = 2 (L + B) = 80

So, L + B = 40

Or, B = 40 – L

Area of the rectangular park = $L \times B = L(40 - L) = 40L - L^2 = 400$

$L^2 - 40L + 400 = 0$,

which is a quadratic equation.

Comparing the equation with $ax^2 + bx + c = 0$, we get

$a = 1$, $b = -40$, $c = 400$

Since, Discriminant = $b^2 - 4ac$

$\Rightarrow (-40)^2 - 4 \times 400$

$\Rightarrow 1600 - 1600$

$= 0$

Thus, $b^2 - 4ac = 0$

Therefore, this equation has equal real roots. Hence, the situation is possible.

Root of the equation,

$L = -b/2a$

$L = (40)/2(1) = 40/2 = 20$

Therefore, length of rectangular park, L = 20 m

And breadth of the park, B = 40 – L = 40 – 20 = 20 m.

Q.10: Find the discriminant of the equation $3x^2 - 2x + 1/3 = 0$ and hence find the nature of its roots. Find them, if they are real.
Solution:

Given,

$3x^2 - 2x + 1/3 = 0$

Here, $a = 3$, $b = -2$ and $c = 1/3$
Since, Discriminant = $b^2 - 4ac$

$= (-2)2 - 4 \times 3 \times 1/3$
$= 4 - 4 = 0$.
Hence, the given quadratic equation has two equal real roots.
The roots are $-b/2a$ and $-b/2a$.

2/6 and 2/6

or

1/3, 1/3

Q.11: In a flight of 600 km, an aircraft was slowed due to bad weather. Its average speed for the trip was reduced by 200 km/hr and the time of flight increased by 30 minutes. Find the original duration of the flight.

Solution:

Let the duration of the flight be x hours.

According to the given,

$(600/x) – [600/(x + 1/2) = 200$

$(600/x) – [1200/(2x + 1)] = 200$

$[600(2x + 1) – 1200x]/ [x(2x + 1)] = 200$

$(1200x + 600 – 1200x)/ [x(2x + 1)] = 200$

$600 = 200x(2x + 1)$

$x(2x + 1) = 3$

$2x^2 + x – 3 = 0$

$2x^2 + 3x – 2x – 3 = 0$

$x(2x + 3) – 1(2x + 3) = 0$

$(2x + 3)(x – 1) = 0$

$2x + 3 = 0, x – 1 = 0$

$x = -3/2, x = 1$

Time cannot be negative.

Therefore, $x = 1$

Hence, the original duration of the flight is 1 hr.

Q.12: If $x = 3$ is one root of the quadratic equation $x^2 – 2kx – 6 = 0$, then find the value of k.

Solution:

Given that $x = 3$ is one root of the quadratic equation $x^2 – 2kx – 6 = 0$.

$\Rightarrow (3)^2 – 2k(3) – 6 = 0$

$\Rightarrow 9 - 6k - 6 = 0$

$\Rightarrow 3 - 6k = 0$

$\Rightarrow 6k = 3$

$\Rightarrow k = 1/2$

Therefore, the value of k is ½.

Q.13: Find the value of p, for which one root of the quadratic equation $px^2 - 14x + 8 = 0$ is 6 times the other.

Solution:

Given quadratic equation is:

$px^2 - 14x + 8 = 0$

Let α and 6α be the roots of the given quadratic equation.

Sum of the roots = -coefficient of x/coefficient of x^2

$\alpha + 6\alpha = -(-14)/p$

$7\alpha = 14/p$

$\alpha = 2/p$....(i)

Product of roots = constant term/coefficient of x^2

$(\alpha)(6\alpha) = 8/p$

$6\alpha^2 = 8/p$

Substituting $\alpha = 2/p$ from (i),

$6 \times (2/p)^2 = 8/p$

$24/p^2 = 8/p$

$3/p = 1$

$p = 3$

Therefore, the value of p is 3.

Q.14: Solve for x: $[1/(x + 1)] + [3/(5x + 1)] = 5/(x + 4)$; $x \neq -1, -1/5, -4$

Solution:

Given,

$[1/(x + 1)] + [3/(5x + 1)] = 5/(x + 4); x \neq -1, -1/5, -4$

Let us take the LCM of denominators and cross multiply the terms.

$[1(5x + 1) + 3(x + 1)]/ [(x + 1)(5x + 1)] = 5/(x + 4)$

$[5x + 1 + 3x + 3]/ [5x^2 + x + 5x + 1] = 5/(x + 4)$

$(8x + 4)(x + 4) = 5(5x^2 + 6x + 1)$

$8x^2 + 32x + 4x + 16 = 25x^2 + 30x + 5$

$25x^2 + 30x + 5 – 8x^2 – 36x – 16 = 0$

$17x^2 – 6x – 11 = 0$

$17x^2 – 17x + 11x – 11 = 0$

$17x(x – 1) + 11(x – 1) = 0$

$(17x + 11)(x – 1) = 0$

$17x + 11 = 0, x – 1 = 0$

$x = -11/17, x = 1$

Q.15: If -5 is a root of the quadratic equation $2x^2 + px – 15 = 0$ and the quadratic equation $p(x^2 + x) + k = 0$ has equal roots, find the value of k.

Solution:

Given that -5 is a root of the quadratic equation $2x^2 + px – 15 = 0$.

$\Rightarrow 2(-5)^2 + p(-5) – 15 = 0$

$\Rightarrow 50 – 5p – 15 = 0$

$\Rightarrow 35 – 5p = 0$

$\Rightarrow 5p = 35$

$\Rightarrow p = 7$

Also, the quadratic equation $p(x^2 + x) + k = 0$ has equal roots.

Substituting p = 7 in $p(x^2 + x) + k = 0$,

$7(x^2 + x) + k = 0$

$7x^2 + 7x + k = 0$

Comparing with the standard form $ax^2 + bx + c = 0$,

$a = 7$, $b = 7$, $c = k$

For equal roots, discriminant is equal to 0.

$b^2 - 4ac = 0$

$(7)^2 - 4(7)(k) = 0$

$49 - 28k = 0$

$28k = 49$

$k = 7/4$

Therefore, the value of k is 7/4.