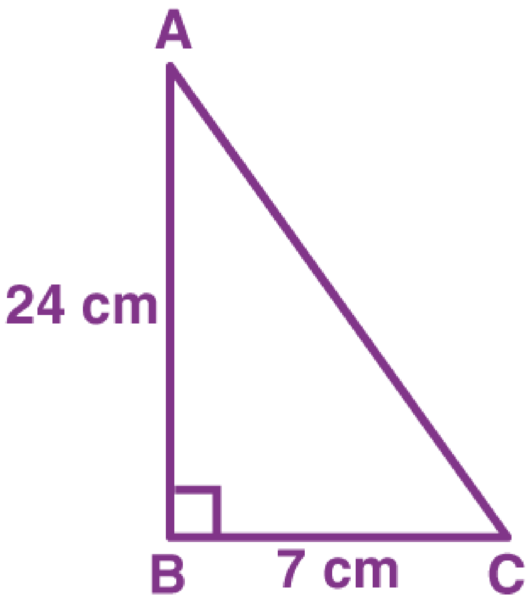
Important Questions for Class 10 Maths Chapter 8- Introduction to Trigonometry

Question. 1 : In Δ ABC, right-angled at B, AB = 24 cm, BC = 7 cm.



Determine:
(i) sin A, cos A
(ii) sin C, cos C

Solution:

In a given triangle ABC, right-angled at B = $\angle B = 90°$

Given: AB = 24 cm and BC = 7 cm

That means, AC = Hypotenuse

According to the Pythagoras Theorem,

In a right-angled triangle, the squares of the hypotenuse side are equal to the sum of the squares of the other two sides.

By applying the Pythagoras theorem, we get

$AC^2 = AB^2 + BC^2$

$AC^2 = (24)^2 + 7^2$

$AC^2 = (576 + 49)$

$AC^2 = 625\ cm^2$

Therefore, AC = 25 cm

(i) We need to find Sin A and Cos A.

As we know, sine of the angle is equal to the ratio of the length of the opposite side and hypotenuse side. Therefore,

Sin A = BC/AC = 7/25

Again, the cosine of an angle is equal to the ratio of the adjacent side and hypotenuse side. Therefore,

cos A = AB/AC = 24/25

(ii) We need to find Sin C and Cos C.

Sin C = AB/AC = 24/25

Cos C = BC/AC = 7/25

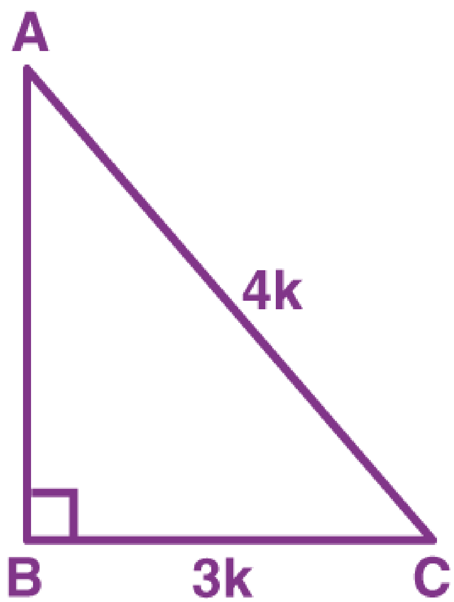
Question 2: If Sin A = 3/4, Calculate cos A and tan A.

Solution: Let us say, ABC is a right-angled triangle, right-angled at B.

Sin A = 3/4

As we know,

Sin A = Opposite Side/Hypotenuse Side = 3/4



Now, let BC be 3k and AC will be 4k.

where k is the positive real number.

As per the Pythagoras theorem, we know;

$\text{Hypotenuse}^2 = \text{Perpendicular}^2 + \text{Base}^2$

$AC^2 = AB^2 + BC^2$

Substitute the value of AC and BC in the above expression to get;

$(4k)^2 = (AB)^2 + (3k)^2$

$16k^2 - 9k^2 = AB^2$

$AB^2 = 7k^2$

Hence, $AB = \sqrt{7}\ k$

Now, as per the question, we need to find the value of cos A and tan A.

cos A = Adjacent Side/Hypotenuse side = AB/AC

$\cos A = \sqrt{7}\ k/4k = \sqrt{7}/4$

And,

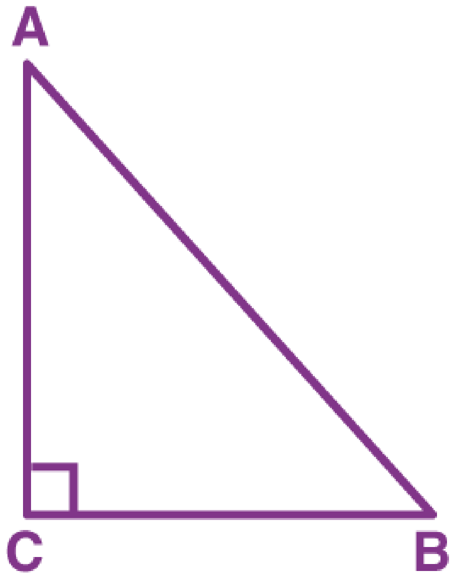
tan A = Opposite side/Adjacent side = BC/AB

$\tan A = 3k/\sqrt{7}\ k = 3/\sqrt{7}$

Question.3: If ∠A and ∠B are acute angles such that cos A = cos B, then show that ∠ A = ∠ B.

Solution:

Suppose a triangle ABC, right-angled at C.



Now, we know the trigonometric ratios,

cos A = AC/AB

cos B = BC/AB

Since, it is given,

cos A = cos B

AC/AB = BC/AB

AC = BC

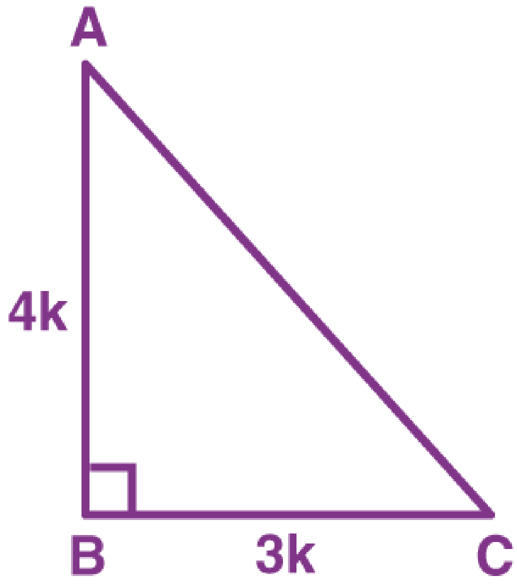
We know that by isosceles triangle theorem, the angles opposite to the equal sides are equal.

Therefore, ∠A = ∠B

Question 4: If 3 cot A = 4, check whether $(1 - \tan^2 A)/(1 + \tan^2 A) = \cos^2 A - \sin^2 A$ or not.

Solution:

Let us consider a triangle ABC, right-angled at B.



Given,

3 cot A = 4

cot A = 4/3

Since, tan A = 1/cot A

tan A = 1/(4/3) = 3/4

BC/AB = 3/4

Let BC = 3k and AB = 4k

By using Pythagoras theorem, we get;

$Hypotenuse^2 = Perpendicular^2 + Base^2$

$AC^2 = AB^2 + BC^2$

$AC^2 = (4k)^2 + (3k)^2$

$AC^2 = 16k^2 + 9k^2$

$AC = \sqrt{25k^2} = 5k$

sin A = Opposite side/Hypotenuse

= BC/AC

=3k/5k

=3/5

In the same way,

cos A = Adjacent side/hypotenuse

= AB/AC

= 4k/5k

= 4/5

To check: $(1-\tan^2 A)/(1+\tan^2 A) = \cos^2 A - \sin^2 A$ or not

Let us take L.H.S. first;

$(1-\tan^2 A)/(1+\tan^2 A) = [1 - (3/4)^2]/ [1 + (3/4)^2]$

$= [1 - (9/16)]/[1 + (9/16)] = 7/25$

R.H.S. $= \cos^2 A - \sin^2 A = (4/5)^2 - (3/5)^2$

$= (16/25) - (9/25) = 7/25$

Since,

L.H.S. = R.H.S.

Hence, proved.

Question 5: In triangle PQR, right-angled at Q, PR + QR = 25 cm and PQ = 5 cm. Determine the values of sin P, cos P and tan P.

Solution: Given,

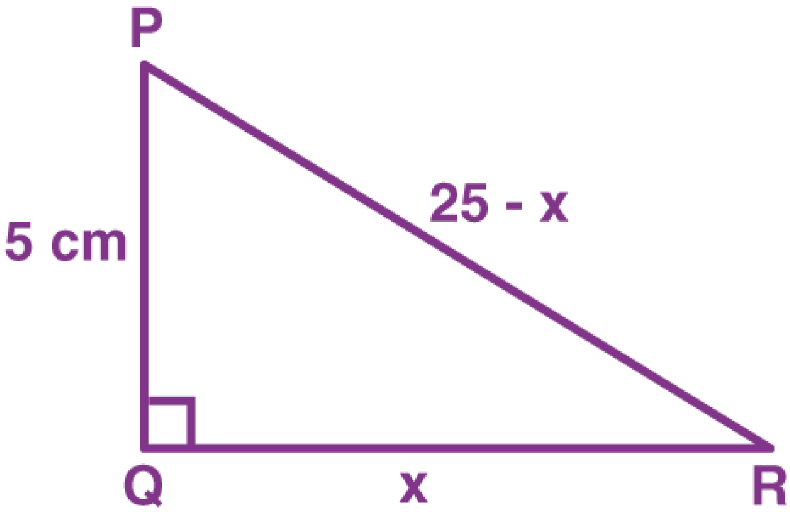
In triangle PQR,

PQ = 5 cm

PR + QR = 25 cm

Let us say, QR = x

Then, PR = 25 – QR = 25 – x



Using Pythagoras theorem:

$PR^2 = PQ^2 + QR^2$

Now, substituting the value of PR, PQ and QR, we get;

$(25 - x)^2 = (5)^2 + (x)^2$

$25^2 + x^2 - 50x = 25 + x^2$

$625 - 50x = 25$

$50x = 600$

$x = 12$

So, QR = 12 cm

PR = 25 – QR = 25 – 12 = 13 cm

Therefore,

sin P = QR/PR = 12/13

cos P = PQ/PR = 5/13

tan P = QR/PQ = 12/5

Question 6: Evaluate 2 $\tan^2 45° + \cos^2 30° - \sin^2 60°$.

Solution: Since we know,

tan 45° = 1

cos 30° = $\sqrt{3}/2$

$\sin 60° = \sqrt{3}/2$

Therefore, putting these values in the given equation:

$2(1)^2 + (\sqrt{3}/2)^2 - (\sqrt{3}/2)^2$

$= 2 + 0$

$= 2$

Question 7: If tan (A + B) =√3 and tan (A – B) =1/√3, 0° < A + B ≤ 90°; A > B, find A and B.

Solution: Given,

$\tan(A + B) = \sqrt{3}$

As we know, $\tan 60° = \sqrt{3}$

Thus, we can write;

$\Rightarrow \tan(A + B) = \tan 60°$

$\Rightarrow (A + B) = 60°$ ...... (i)

Now again given;

$\tan(A - B) = 1/\sqrt{3}$

Since, $\tan 30° = 1/\sqrt{3}$

Thus, we can write;

$\Rightarrow \tan(A - B) = \tan 30°$

$\Rightarrow (A - B) = 30°$ ..... (ii)

Adding the equation (i) and (ii), we get;

$A + B + A - B = 60° + 30°$

$2A = 90°$

$A = 45°$

Now, put the value of A in eq. (i) to find the value of B;

$45° + B = 60°$

$B = 60° - 45°$

$B = 15°$

Therefore A = 45° and B = 15°

Question 8: Show that :

(i) tan 48° tan 23° tan 42° tan 67° = 1

(ii) cos 38° cos 52° – sin 38° sin 52° = 0

Solution:

(i) tan 48° tan 23° tan 42° tan 67°

We can also write the above given tan functions in terms of cot functions, such as;

tan 48° = tan (90° – 42°) = cot 42°

tan 23° = tan (90° – 67°) = cot 67°

Hence, substituting these values, we get

= cot 42° cot 67° tan 42° tan 67°

= (cot 42° tan 42°) (cot 67° tan 67°)

= 1 × 1 [since cot A.tan A = 1]

= 1

(ii) cos 38° cos 52° – sin 38° sin 52°

We can also write the given cos functions in terms of sin functions.

cos 38° = cos (90° – 52°) = sin 52°

cos 52°= cos (90° – 38°) = sin 38°

Hence, putting these values in the given equation, we get;

sin 52° sin 38° – sin 38° sin 52° = 0

Question 9: If tan 2A = cot (A – 18°), where 2A is an acute angle, find the value of A.

Solution: Given,

tan 2A = cot (A – 18°)

As we know by trigonometric identities,

tan 2A = cot (90° – 2A)

Substituting the above equation in the given equation, we get;

$\Rightarrow \cot(90° - 2A) = \cot(A - 18°)$

Therefore,

$\Rightarrow 90° - 2A = A - 18°$

$\Rightarrow 108° = 3A$

$A = 108° / 3$

Hence, the value of $A = 36°$

Question 10: If A, B and C are interior angles of a triangle ABC, then show that sin [(B + C)/2] = cos A/2.

Solution:

As we know, for any given triangle, the sum of all its interior angles is equals to 180°.

Thus,

$A + B + C = 180°$ ....(1)

Now we can write the above equation as;

$\Rightarrow B + C = 180° - A$

Dividing by 2 on both the sides;

$\Rightarrow (B + C)/2 = (180° - A)/2$

$\Rightarrow (B + C)/2 = 90° - A/2$

Now, put sin function on both sides.

$\Rightarrow \sin(B + C)/2 = \sin(90° - A/2)$

Since,

$\sin(90° - A/2) = \cos A/2$

Therefore,

$\sin(B + C)/2 = \cos A/2$

Question 11: Prove the identities:

(i) $\sqrt{[1 + \sin A/1 - \sin A]} = \sec A + \tan A$

(ii) $(1 + \tan^2 A/1 + \cot^2 A) = (1 - \tan A/1 - \cot A)^2 = \tan^2 A$

Solution:

(i) Given: $\sqrt{[1 + \sin A/1 - \sin A]} = \sec A + \tan A$

L.H.S = $\sqrt{\frac{1+\sin A}{1-\sin A}}$

First divide the numerator and denominator of L.H.S. by cos A,

$$= \sqrt{\frac{\frac{1}{\cos A}+\frac{\sin A}{\cos A}}{\frac{1}{\cos A}-\frac{\sin A}{\cos A}}}$$

We know that 1/cos A = sec A and sin A/ cos A = tan A and it becomes,

$$= \sqrt{\frac{\sec A + \tan A}{\text{Sec } A - \tan A}}$$

Now using rationalization, we get

$$= \sqrt{\frac{\sec A+\tan A}{\text{Sec } A-\tan A}} \times \sqrt{\frac{\sec A+\tan A}{\text{Sec } A+\tan A}}$$

$$= \sqrt{\frac{(\sec A + \tan A)^2}{\sec^2 A - \tan^2 A}}$$

= (sec A + tan A)/1

= sec A + tan A = R.H.S

Hence proved

(ii) Given: $(1 + \tan^2 A/1 + \cot^2 A) = (1 - \tan A/1 - \cot A)^2 = \tan^2 A$

LHS:

$= (1+\tan^2 A) / (1+\cot^2 A)$

Using the trigonometric identities we know that $1+\tan^2 A = \sec^2 A$ and $1+\cot^2 A= \text{cosec}^2 A$

$= \sec^2 A/ \text{cosec}^2 A$

On taking the reciprocals we get

$= \sin^2 A/\cos^2 A$

$= \tan^2 A$

RHS:

$=(1-\tan A)^2/(1-\cot A)^2$

Substituting the reciprocal value of tan A and cot A we get,

$= (1-\sin A/\cos A)^2/(1-\cos A/\sin A)^2$

$= [(\cos A-\sin A)/\cos A]^2/ [(\sin A-\cos)/\sin A)^2] = [(\cos A-\sin A)^2\times\sin^2 A] /[\cos^2 A. /(\sin A-\cos A)^2] = \sin^2 A/\cos^2 A$

$= \tan^2 A$

The values of LHS and RHS are the same.

Hence proved.

Question 12: If $\sin \theta + \cos \theta = \sqrt{3}$, then prove that $\tan \theta + \cot \theta = 1$.

Solution:

Given,

$\sin \theta + \cos \theta = \sqrt{3}$

Squaring on both sides,

$(\sin \theta + \cos \theta)^2 = (\sqrt{3})^2$

$\sin^2\theta + \cos^2\theta + 2 \sin \theta \cos \theta = 3$

Using the identity $\sin^2 A + \cos^2 A = 1$,

$1 + 2 \sin \theta \cos \theta = 3$

$2 \sin \theta \cos \theta = 3 - 1$

$2 \sin \theta \cos \theta = 2$

$\sin \theta \cos \theta = 1$

$\sin \theta \cos \theta = \sin^2\theta + \cos^2\theta$

$\Rightarrow (\sin^2\theta + \cos^2\theta)/(\sin \theta \cos \theta) = 1$

$\Rightarrow [\sin^2\theta/(\sin \theta \cos \theta)] + [\cos^2\theta/(\sin \theta \cos \theta)] = 1$

$\Rightarrow (\sin \theta/\cos \theta) + (\cos \theta/\sin \theta) = 1$

$\Rightarrow \tan \theta + \cot \theta = 1$

Hence proved.

Question 13: Express cot 85° + cos 75° in terms of trigonometric ratios of angles between 0° and 45°.

Solution:

$\cot 85° + \cos 75°$

$= \cot (90° - 5°) + \cos (90° - 15°)$

We know that $\cos(90° - A) = \sin A$ and $\cot(90° - A) = \tan A$

$= \tan 5° + \sin 15°$

Question 14: What is the value of ($\cos^2 67° - \sin^2 23°$)?

Solution:

$(\cos^2 67° - \sin^2 23°) = \cos^2(90° - 23°) - \sin^2 23°$

We know that $\cos(90° - A) = \sin A$

$= \sin^2 23° - \sin^2 23°$

$= 0$

Therefore, $(\cos^2 67° - \sin^2 23°) = 1$.

Question 15: Prove that $(\sin A - 2 \sin^3 A)/(2 \cos^3 A - \cos A) = \tan A$.

Solution:

LHS $= (\sin A - 2 \sin^3 A)/(2 \cos^3 A - \cos A)$

$= [\sin A(1 - 2 \sin^2 A)]/ [\cos A(2 \cos^2 A - 1]$

Using the identity $\sin^2\theta + \cos^2\theta = 1$,

$= [\sin A(\sin^2 A + \cos^2 A - 2 \sin^2 A)]/ [\cos A(2 \cos^2 A - \sin^2 A - \cos^2 A]$

$= [\sin A(\cos^2 A - \sin^2 A)]/ [\cos A(\cos^2 A - \sin^2 A)]$

$= \sin A/\cos A$

$= \tan A$

= RHS

Hence proved.