

NCERT MOST IMPORTANT QUESTIONS CLASS – 11

Statistics for Economics' CHAPTER – 6 Measures of Dispersion

Question 1

Define dispersion.

Answer: Dispersion is the measure of the extent to which different items tend to disperse away from the central tendency.

Question 2

What is the coefficient of dispersion?

Answer: The coefficient of dispersion shows different data percentage or relative value. The coefficient of dispersion is known as a relative measure of dispersion.

Question 3

Define range.

Answer: Range is the variance between the lowest and highest value in a series. Therefore, Range = Highest value in the series – Lowest value in the series.

Question 4

Explain the interquartile range.

Answer: In a series, the difference between the first Quartile (Q1) and third Quartile (Q3) is known as the interquartile range.

Question 5

What is the quartile deviation?

Answer: Half of the interquartile range is Quartile deviation. It can also be mentioned as semi-inter quartile range.

Question 6

What is the coefficient of quartile deviation formula?

Answer: For calculating the coefficient of quartile deviation, the formula applied is.

$Q_3 - Q_1 / Q_3 + Q_1$

Question 7

Define mean deviation.

Answer: A mathematical average of the deviations of all the principles taken from some average value (mean, median, mode) of the series, ignoring signs (+ or -) of the deviation is mean deviation.

Question 8

What is standard deviation?

Answer: The square root of the arithmetic mean of the squared deviations of the items from their mean value.

Question 9

What is a Lorenz curve?

Answer: Lorenz curve is a curve that shows the actual distribution deviation (of income or wealth) from the line exhibiting equal distribution.

Question 10

Define variance.

Answer: Variance is another measure of dispersion. Variance is the square of the standard deviation.

Question 11.

A measure of dispersion is a good supplement to the central value in understanding a frequency distribution. Comment.

Answer:

Dispersion is the extent to which values in a distribution differ from the average of the distribution. Knowledge of only average is insufficient as it does not reflect the quantum of variation in values.

Measures of dispersion enhance the understanding of a distribution considerably by providing information about how much the actual value of items in a series deviate from the central value, e.g., per capita income gives only the average income but a measure of dispersion can tell you about income inequalities, thereby improving the understanding of the relative living standards of different sections of the society. Through value of dispersion one can better understand the distribution.

Thus a measure of dispersion is a good supplement to the central value in understanding a frequency distribution.

Question 12.**Which measure of dispersion is the best and how?****Answer:**

Standard Deviation is considered to be the best measure of dispersion and is therefore the most widely used measure of dispersion.

- It is based on all values and thus provides information about the complete series. Because of this reason, a change in even one value affects the value of standard deviation.
- It is independent of origin but not of scale.
- It is useful in advanced statistical calculations like comparison of variability in two data sets.
- It can be used in testing of hypothesis.
- It is capable of further algebraic treatment.

Question 13.

Some measures of dispersion depend upon the spread of values whereas some calculate the variation of values from a central value. Do you agree?

Answer:

Yes, it is true that some measures of dispersion depend upon the spread of values, whereas some calculate the variation of values from the central value. Range and Quartile Deviation measure the dispersion by calculating the spread within which the value lie. Mean Deviation and Standard Deviation calculate the extent to which the values differ from the average or the central value.

Question 14.

In town, 25% of the persons earned more than ₹ 45,000 whereas 75% earned more than 18,000. Calculate the absolute and relative values of dispersion.

Answer:

25% of the persons earned more than ₹ 45,000. This implies that upper quartile $Q_3 = 45,000$ 75% earned more than 18,000. This implies that lower quartile $Q_1 = 18,000$

Absolute Measure of Dispersion = $Q_3 - Q_1 = 45,000 - 18,000 = 27,000$

Relative Measure of Dispersion

Co-efficient of Quartile Deviation

$$\begin{aligned}
 &= \frac{Q_3 - Q_1}{Q_3 + Q_1} \quad \text{where } Q_3 = 3\text{rd Quartile, } Q_1 = 1\text{st Quartile} \\
 &= \frac{45,000 - 18,000}{45,000 + 18,000} \\
 &= \frac{27,000}{63,000} = 0.428
 \end{aligned}$$

Question 15.

The yield of wheat and rice per acre for 10 districts of a state is as under

District	1	2	3	4	5	6	7	8	9	10
Wheat	12	10	15	19	21	16	18	9	25	10
Rice	22	29	12	23	18	15	12	34	18	12

Calculate for each crop,

- (i) Range
- (ii) QD
- (iii) Mean Deviation about Mean
- (iv) Mean Deviation about Median
- (v) Standard Deviation
- (vi) Which crop has greater variation?
- (vii) Compare the value of different measures for each crop.

Answer:

(i) Range

(a) Wheat Highest value of distribution (H) = 25

Lowest value of distribution (L) = 9

Range = $H - L = 25 - 9 = 16$

(b) Rice Highest value of distribution (H) = 34

Lowest value of distribution (L) = 12

Range = $H - L = 34 - 12 = 22$

(ii) Quartile Deviation

(a) Wheat Arranging the production of wheat in increasing order 9, 10, 10, 12, 15, 16, 18, 19, 21, 25

$Q_1 = N+14\text{th item} = 10+14\text{th item} = 114\text{th item}$

= 2.75th item

= Size of 2nd item + 0.75 (size of 3rd item – size of 2nd item)

= $10 + 0.75(10 - 10)$

= $10 + 0.75 \times 0$

= 10

$Q_3 = 3(N+1)4\text{th item} = 3(10+1)4\text{th item}$

= 334th item = 8.25th

= Size of 8th item + 0.25 (size of 9th item – size of 8th item)

= $19 + 0.25(21 - 19)$

= $19 + 0.25 \times 2$

= $19 + 0.50 = 19.50$

Quartile Deviation = $Q_3 - Q_1 = 19.50 - 10 = 9.502 = 4.75$

(b) Rice Arranging the data of production of rice

12, 12, 12, 15, 18, 18, 22, 23, 29, 34 item

$Q_1 = N+14\text{th item} = 10+14\text{th item}$

= 2.75 th item

= Size of 2nd item + 0.75 (size of 3rd item – size of 2nd item)

= $12 + 0.75(12 - 12) = 12 + 0.75 \times 0$

= 12

$$\begin{aligned}
 &= 8.25\text{th item} \\
 &= \text{Size of 8th item} + 0.25 (\text{size of 9th item} - \text{size of 8th item}) \\
 &= 23 + 0.25(29 - 23) \\
 &= 23 + 0.25 \times 6 \\
 &= 23 + 1.5 \\
 &= 24.5
 \end{aligned}$$

$$\begin{aligned}
 Q_1 &= \frac{3(N+1)}{4}\text{th item} \\
 &= \frac{3(10+1)}{4}\text{th item} \\
 &= \frac{33}{4}\text{th item}
 \end{aligned}$$

$$\text{Quartile Deviation} = Q_3 - Q_1 = 24.5 - 12.2 = 12.502 = 6.25$$

(iii) Mean Deviation about Mean

(a) Wheat

Wheat Production (X)	$ d = X - \bar{A\bar{X}} $ $\bar{A\bar{X}} = 15$
9	6
10	5
10	5
12	3
15	0
16	1
18	3
19	4
21	6
25	10
$\Sigma X = 155$	$\Sigma d = 43$

(b) Rice

$$\text{Mean} = \frac{\Sigma X}{N} = \frac{155}{10} = 15.5$$

Mean Deviation from Mean

$$\begin{aligned}
 \text{MD}(\bar{X}) &= \frac{\Sigma |d| + (\bar{X} - A\bar{X})(\Sigma f_B - \Sigma f_A)}{N} \\
 &= \frac{43 + (15.5 - 15)(5 - 5)}{10} = \frac{43}{10} = 4.3
 \end{aligned}$$

Rice Production (X)	$ d = X - A\bar{X} $ $A\bar{X} = 18$
12	6
12	6
12	6
15	3
18	0
18	0
22	4
23	5
29	11
34	16
$\Sigma X = 195$	$\Sigma d = 57$

(iv) Mean Deviation about Median

(a) Wheat

Production of Wheat (X)	$ d = X - 15 $
09	6
10	5
10	5
12	3
15	0
16	1
18	3
19	4
21	6
25	10
	$\Sigma d = 43$

$$\begin{aligned} \text{Median} &= \text{size of } \left(\frac{N+1}{2}\right) \text{th item} = \text{Size of } \left(\frac{10+1}{2}\right) \text{th item} \\ &= \text{size of } (5.5) \text{th item} \end{aligned}$$

$$= \frac{\text{Size of 5th item} + \text{Size of 6th item}}{2}$$

$$= \frac{15+16}{2} = 15.5$$

$$\begin{aligned} \text{MD}_{\text{Median}} &= \frac{\Sigma |d| + (\text{Median} - A)(\Sigma f_B - \Sigma f_A)}{n} \\ &= \frac{57 + (15.5 - 18)(6 - 4)}{10} = \frac{57}{10} = 5.7 \end{aligned}$$

(b) Rice

Production of Rice	$d = X - 18$
12	6
12	6
12	6
15	3
18	0
18	0
22	4
23	5
29	11
34	16
	$\Sigma d = 57$

Since n is even.

Therefore, Median = size of $\left(\frac{N+1}{2}\right)$ th item

$$= \text{Size of } \left(\frac{10+1}{2}\right)\text{th Item}$$

$$\Rightarrow \text{Size of (5.5)th item} = \frac{\text{Size of 5th item} + \text{Size of 6th item}}{2}$$

$$= \frac{36}{2}$$

$$= 18$$

$$MD_{\text{Median}} = \frac{\Sigma|d| + (\text{Median} - A)(\Sigma f_B - \Sigma f_A)}{N}$$

$$= \frac{57 + (18 - 18)(6 - 4)}{10}$$

$$= \frac{57}{10} = 5.7$$

(v) Standard Deviation

(a) Wheat

Reduction of Wheat (X)	AX=15 d=X-AX	d ²
9	-6	36
10	-5	25
10	-5	25
12	-3	9
15	0	0
16	1	1
18	3	9
19	4	16
21	6	36
25	10	100
	$\Sigma d = 5$	$\Sigma d^2 = 257$

$$\begin{aligned} \sigma &= \sqrt{\frac{\Sigma d^2}{n} - \left(\frac{\Sigma d}{n}\right)^2} \\ &= \sqrt{\frac{257}{10} - \left(\frac{5}{10}\right)^2} \\ &= \sqrt{\frac{257}{10} - \frac{25}{100}} \\ &= \sqrt{\frac{2570 - 25}{100}} \\ &= 5.04 \end{aligned}$$

(b) Rice

Production of Rice (X)	AX=18 d=X-AX	d ²
12	-6	36
12	-6	36
12	-6	36
15	-3	9
18	0	0
22	4	16
23	5	25
29	11	121
34	16	256
	$\Sigma d = 15$	$\Sigma d^2 = 535$

(vi) Coefficient of Variation

(a) Wheat

$$CV = \frac{\sigma}{\bar{X}} \times 100 = \frac{5.04}{15.5} \times 100 = 32.51$$

(b) Rice

$$CV = \frac{\sigma}{\bar{X}} \times 100 = \frac{7.16}{19.5} \times 100 = 36.71$$

Rice crop has greater variation as the coefficient of variation is higher for rice as compared to that of wheat.

(vii) Rice crop has higher Range, Quartile Deviation, Mean Deviation about Mean, Mean Deviation about Median, Standard Deviation and Coefficient of Variation.

$$\begin{aligned}\sigma &= \sqrt{\frac{\sum d^2}{n} - \left(\frac{\sum d}{n}\right)^2} \\ &= \sqrt{\frac{535}{10} - \left(\frac{15}{10}\right)^2} \\ &= \sqrt{\frac{535}{10} - \frac{225}{100}} \\ &= \sqrt{51.25} \\ &= 7.16\end{aligned}$$

Question 16.

A batsman is to be selected for a cricket team. The choice is between X and Y on the basis of their scores in five previous scores which are

X	25	85	40	80	120
Y	50	70	65	45	80

Which batsman should be selected if we want,

- (i) a higher run-getter, or
- (ii) a more reliable batsman in the team?

Answer:

Batsman X

X	$X - \bar{X} = x$ X - 70	x^2
25	-45	2025
85	+15	225
40	-30	900
80	10	100
120	50	2500
$\Sigma X = 350$		$\Sigma x^2 = 5750$

$$\bar{X} = \frac{\Sigma X}{N} = \frac{350}{5} = 70$$

$$\sigma = \sqrt{\frac{\Sigma X^2}{N}} = \sqrt{\frac{5750}{5}} = 33.91$$

$$CV = \frac{\sigma}{\bar{X}} \times 100 = \frac{33.91}{70} \times 100 = 48.44$$

Batsman Y

Y	Y - \bar{Y} = y	y ²
50	-12	144
70	8	64
65	3	9
45	-17	289
80	18	324
$\Sigma Y = 310$		$\Sigma y^2 = 830$

$$\bar{Y} = \frac{\Sigma Y}{N} = \frac{310}{5} = 62$$

$$\sigma = \sqrt{\frac{\Sigma y^2}{N}} = \sqrt{\frac{830}{5}} = 12.88$$

$$CV = \frac{\sigma}{\bar{Y}} \times 100 = \frac{12.88}{62} \times 100 = 20.78$$

(i) Average of Batsman X is higher than that of Batsman Y, so he should be selected if we want a high scorer.

(ii) The Batsman Y is more reliable than Batsman X. This is because the coefficient of variation of Batsman X is higher than that of Batsman Y.

Question 17.

To check the quality of two brands of light bulbs, their life in burning hours was estimated as under for 100 bulbs of each brand.

Life (in hrs)	Number of Bulbs (Brand A)	Number of Bulbs (Brand B)
0-50	15	2
50-100	20	8
100-150	18	60
150-200	25	25
200-250	22	5
Total	100	100

(i) Which brand gives higher life?

(ii) Which brand is more dependable?

Answer:

For Brand A

Life (in hrs)	Number of Bulbs (f)	M X	A=125 X-A=d	$d = \frac{X-A}{i}$	d^2	fd	fd^2
0-50	15	25	-100	-2	4	-30	60
50-100	20	75	-50	-1	1	-20	20
100-150	18	125	0	0	0	0	0
150-200	25	175	50	1	1	25	25
200-250	22	225	100	2	4	44	88
	$\Sigma f = 100$					$\Sigma fd = 19$	$\Sigma fd^2 = 193$

$$\bar{X} = A + \frac{\Sigma fd}{\Sigma f} \times i = 125 + \frac{19}{100} \times 50 = 134.5$$

$$\sigma = \sqrt{\frac{\Sigma fd^2}{\Sigma f} - \left(\frac{\Sigma fd}{\Sigma f}\right)^2} \times i$$

$$= \sqrt{\frac{193}{100} - \left(\frac{19}{100}\right)^2} \times 50 = \sqrt{\frac{193}{100} - \frac{361}{10000}} \times 50$$

$$= \sqrt{\frac{18939}{10000}} \times 50 = 68.5$$

$$CV = \frac{\sigma}{\bar{X}} \times 100 = \frac{68.5}{134.5} \times 100 = 50.93$$

For Brand B

Life (in hrs)	Number of Bulbs (f)	Mid Value m	A=125	$d = \frac{X-A}{c}$	d^2	fd	fd^2
0-50	2	25	-100	-2	4	-4	8
50-100	8	75	-50	-1	1	-8	8
100-150	60	125	0	0	0	0	0
150-200	25	175	50	1	1	25	25
200-250	5	225	100	2	4	10	20
	$\Sigma f = 100$					$\Sigma fd = 23$	$\Sigma fd^2 = 61$

$$\bar{X} = A + \frac{\Sigma fd}{\Sigma f} \times i$$

$$= 125 + \frac{23}{100} \times 50$$

$$= 136.5$$