Q. 1: Write the following sets in the roster form.

(i) A = {x | x is a positive integer less than 10 and $2^{x} - 1$ is an odd number}

(ii) C = {x : $x^2 + 7x - 8 = 0, x \in R$ }

Solution:

(i) $2^x - 1$ is always an odd number for all positive integral values of x since 2^x is an even number.

In particular, $2^{x} - 1$ is an odd number for x = 1, 2, ..., 9.

Therefore, $A = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

(ii) $x^2 + 7x - 8 = 0$

(x+8)(x-1) = 0

x = -8 or x = 1

Therefore, $C = \{-8, 1\}$

Q. 2: Write the following sets in roster form:

(i) A = {x : x is an integer and $-3 \le x < 7$ }

(ii) B = {x : x is a natural number less than 6}

Solution:

(i) A = {x : x is an integer and $-3 \le x < 7$ }

Integers are ...-5, -4, -3, -2, -2, 0, 1, 2, 3, 4, 5, 6, 7, 8,....

 $A = \{-3, -2, -1, 0, 1, 2, 3, 4, 5, 6\}$

(ii) $B = {x : x is a natural number less than 6}$

Natural numbers are 1, 2, 3, 4, 5, 6, 7,

B = {1, 2, 3, 4, 5}

Q. 3: Given that N = {1, 2, 3, ..., 100}, then

(i) Write the subset A of N, whose elements are odd numbers.

(ii) Write the subset B of N, whose elements are represented by x + 2, where x ∈ N.

Solution:

(i) $A = \{x \mid x \in N \text{ and } x \text{ is odd}\}$

A = {1, 3, 5, 7, ..., 99}

(ii) B = {y | y = x + 2, x \in N}

 $1 \in N, y = 1 + 2 = 3$

 $2 \in N,$ y = 2 + 2 = 4, and so on.

Therefore, $B = \{3, 4, 5, 6, \dots, 100\}$

Q. 4: Let $X = \{1, 2, 3, 4, 5, 6\}$. If n represent any member of X, express the following as sets:

(i) $n \in X$ but $2n \notin X$

(ii) n + 5 = 8

(iii) n is greater than 4

Solution:

(i) For X = $\{1, 2, 3, 4, 5, 6\}$, it is given that $n \in X$, but $2n \notin X$.

Let, $A = \{x \mid x \in X \text{ and } 2x \notin X\}$

Now, $1 \notin A$ as $2.1 = 2 \in X$

 $2 \notin A$ as $2.2 = 4 \in X$

 $3 \notin A$ as $2.3 = 6 \in X$

But $4 \in A$ as $2.4 = 8 \notin X$

 $5 \in A as 2.5 = 10 \notin X$

6 ∈ A as 2.6 = 12 ∉ X

Therefore, $A = \{4, 5, 6\}$

(ii) Let $B = \{x \mid x \in X \text{ and } x + 5 = 8\}$

Here, $B = \{3\}$ as $x = 3 \in X$ and 3 + 5 = 8 and there is no other element belonging to X such that x + 5 = 8.

(iii) Let $C = \{x \mid x \in X, x > 4\}$

Therefore, $C = \{5, 6\}$

Q. 5: Let U = {1, 2, 3, 4, 5, 6}, A = {2, 3} and B = {3, 4, 5}.

Find A', B', A' \cap B', A \cup B and hence show that (A \cup B)' = A' \cap B'.

Solution:

Given,

 $U = \{1, 2, 3, 4, 5, 6\}, A = \{2, 3\} and B = \{3, 4, 5\}$

 $A' = \{1, 4, 5, 6\}$

 $B' = \{ 1, 2, 6 \}.$

Hence, $A' \cap B' = \{ 1, 6 \}$

Also, $A \cup B = \{2, 3, 4, 5\}$

 $(A \cup B)' = \{\,1, 6\,\}$

Therefore, $(A \cup B)' = \{1, 6\} = A' \cap B'$

Q. 6: Use the properties of sets to prove that for all the sets A and B, A – (A \cap B) = A – B

Solution:

 $A - (A \cap B) = A \cap (A \cap B)'$ (since $A - B = A \cap B'$)

 $= A \cap (A' \cup B')$ [by De Morgan's law)

= $(A \cap A') \cup (A \cap B')$ [by distributive law]

 $= \phi \cup (A \cap B')$

 $= \mathbf{A} \cap \mathbf{B'} = \mathbf{A} - \mathbf{B}$

Hence, proved that $A - (A \cap B) = A - B$.

Q. 7: Let U = {1, 2, 3, 4, 5, 6, 7}, A = {2, 4, 6}, B = {3, 5} and C = {1, 2, 4, 7}, find

(i) A' ∪ (B ∩ C')

(ii) $(B - A) \cup (A - C)$

Solution:

Given,

 $U = \{1, 2, 3, 4, 5, 6, 7\}, A = \{2, 4, 6\}, B = \{3, 5\} \text{ and } C = \{1, 2, 4, 7\}$ (i) A' = $\{1, 3, 5, 7\}$ C' = $\{3, 5, 6\}$ B \cap C' = $\{3, 5\}$ A' \cup (B \cap C') = $\{1, 3, 5, 7\}$ (ii) B - A = $\{3, 5\}$ A - C = $\{6\}$ (B - A) \cup (A - C) = $\{3, 5, 6\}$

Q. 8: In a class of 60 students, 23 play hockey, 15 play basketball,20 play cricket and 7 play hockey and basketball, 5 play cricket and basketball, 4 play hockey and cricket, 15 do not play any of the three games. Find

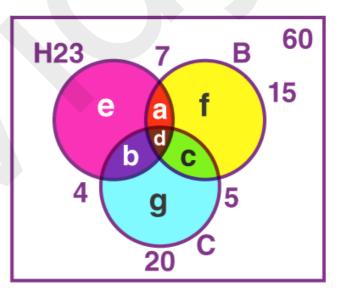
(i) How many play hockey, basketball and cricket

(ii) How many play hockey but not cricket

(iii) How many play hockey and cricket but not basketball

Solution:

Venn diagram of the given data is:



15 students do not play any of three games.

 $n(H \cup B \cup C) = 60 - 15 = 45$

 $n(H \cup B \cup C) = n(H) + n(B) + n(C) - n(H \cap B) - n(B \cap C) - n(C \cap H) + n(H \cap B \cap C)$

45 = 23 + 15 + 20 - 7 - 5 - 4 + d

45 = 42 + d

d = 45- 42 = 3

Number of students who play all the three games = 3

Therefore, the number of students who play hockey, basketball and cricket = 3

a + d = 7 a = 7 - 3 = 4 b + d = 4 b = 4 - 3 = 1 a + b + d + e = 23 4 + 1 + 3 + e = 23 e = 15Similarly, c = 2, g = 14, f = 6

Number of students who play hockey but not cricket = a + e

= 4 + 15

= 19

Number of students who play hockey and cricket but not basketball = b = 1

Q. 9: Let U = $\{x : x \in N, x \le 9\}$; A = $\{x : x \text{ is an even number, } 0 < x < 10\}$; B = $\{2, 3, 5, 7\}$. Write the set (A U B)'.

Solution:

Let U = {x : x \in N, x \leq 9}; A = {x : x is an even number, 0 < x < 10}; B = {2, 3, 5, 7}

Q. 10: In a survey of 600 students in a school, 150 students were found to be drinking Tea and 225 drinking Coffee, 100 were drinking both Tea and Coffee. Find how many students were drinking neither Tea nor Coffee.

Solution:

Given,

Total number of students = 600

Number of students who were drinking Tea = n(T) = 150

Number of students who were drinking Coffee = n(C) = 225

Number of students who were drinking both Tea and Coffee = $n(T \cap C) = 100$

 $n(T \cup C) = n(T) + n(C) - n(T \cap C)$

= 150 + 225 -100

= 375 - 100

= 275

Hence, the number of students who are drinking neither Tea nor Coffee = 600 - 275 = 325