

Important Questions Class 11 Maths Chapter 2 Relations and Functions

Important Questions & Answers For Class 11 Maths Chapter 2 Relations and Functions

Q.1: Write the range of a Signum function.

Solution:

The real function $f: \mathbb{R} \rightarrow \mathbb{R}$ defined by

is called the signum function. Domain of $f = \mathbb{R}$, Range of $f = \{1, 0, -1\}$

Q.2: The Cartesian product $A \times A$ has 9 elements among which are found $(-1, 0)$ and $(0, 1)$. Find the set A and the remaining elements of $A \times A$.

$$f(x) = \begin{cases} 1, & \text{if } x > 0 \\ 0, & \text{if } x = 0 \\ -1, & \text{if } x < 0 \end{cases}$$

Solution:

We know that,

If $n(A) = p$ and $n(B) = q$, then $n(A \times B) = pq$

From the given,

$$n(A \times A) = 9$$

$$n(A) \times n(A) = 9,$$

$$n(A) = 3 \dots\dots(i)$$

The ordered pairs $(-1, 0)$ and $(0, 1)$ are two of the nine elements of $A \times A$.

Therefore, $A \times A = \{(a, a) : a \in A\}$

Hence, $-1, 0, 1$ are the elements of A(ii)

From (i) and (ii),

$$A = \{-1, 0, 1\}$$

The remaining elements of set $A \times A$ are $(-1, -1)$, $(-1, 1)$, $(0, -1)$, $(0, 0)$, $(1, -1)$, $(1, 0)$ and $(1, 1)$.

Q.3: Express the function $f: A \rightarrow \mathbb{R}$, $f(x) = x^2 - 1$, where $A = \{-4, 0, 1, 4\}$ as a set of ordered pairs.

Solution:

Given,

$$A = \{-4, 0, 1, 4\}$$

$$f(x) = x^2 - 1$$

$$f(-4) = (-4)^2 - 1 = 16 - 1 = 15$$

$$f(0) = (0)^2 - 1 = -1$$

$$f(1) = (1)^2 - 1 = 0$$

$$f(4) = (4)^2 - 1 = 16 - 1 = 15$$

Therefore, the set of ordered pairs = $\{(-4, 15), (0, -1), (1, 0), (4, 15)\}$

Q.4: Assume that $A = \{1, 2, 3, \dots, 14\}$. Define a relation R from A to A by $R = \{(x, y) : 3x - y = 0, \text{ such that } x, y \in A\}$. Determine and write down its range, domain, and codomain.

Solution:

It is given that the relation R from A to A is given by $R = \{(x, y) : 3x - y = 0, \text{ where } x, y \in A\}$.

It means that $R = \{(x, y) : 3x = y, \text{ where } x, y \in A\}$

$$\text{Hence, } R = \{(1, 3), (2, 6), (3, 9), (4, 12)\}$$

We know that the domain of R is defined as the set of all first elements of the ordered pairs in the given relation.

$$\text{Hence, the domain of } R = \{1, 2, 3, 4\}$$

To determine the codomain, we know that the entire set A is the codomain of the relation R .

$$\text{Therefore, the codomain of } R = A = \{1, 2, 3, \dots, 14\}$$

As it is known that, the range of R is defined as the set of all second elements in the relation ordered pair.

$$\text{Hence, the Range of } R \text{ is given by } = \{3, 6, 9, 12\}$$

Q.5: Let $f(x) = x^2$ and $g(x) = 2x + 1$ be two real functions. Find

$(f + g)(x)$, $(f - g)(x)$, $(fg)(x)$, $(f/g)(x)$

Solution:

Given,

$$f(x) = x^2 \text{ and } g(x) = 2x + 1$$

$$(f + g)(x) = x^2 + 2x + 1$$

$$(f - g)(x) = x^2 - (2x + 1) = x^2 - 2x - 1$$

$$(fg)(x) = x^2(2x + 1) = 2x^3 + x^2$$

$$(f/g)(x) = x^2/(2x + 1), x \neq -1/2$$

Q.6: Redefine the function: $f(x) = |x - 1| - |x + 6|$. Write its domain also.

Solution:

Given function is $f(x) = |x - 1| - |x + 6|$

Redefine of the function is:

The domain of this function is \mathbb{R} .

Q.7: The function f is defined by

$$f(x) = \begin{cases} -x + 1 + x + 6, & x \leq -6 \\ -x + 1 - x - 6, & -6 \leq x < 1 \\ x - 1 - x - 6, & x \geq 1 \end{cases}$$

Draw the graph of $f(x)$.

Solution:

$$= \begin{cases} 7, & x \leq -6 \\ -2x - 5, & -6 \leq x < 1 \\ -7, & x \geq 1 \end{cases}$$

$f(x) = 1 - x$, $x < 0$, this gives

$$f(-4) = 1 - (-4) = 5;$$

$$f(-3) = 1 - (-3) = 4,$$

$$f(-2) = 1 - (-2) = 3$$

$$f(-1) = 1 - (-1) = 2; \text{ etc,}$$

Also, $f(1) = 2$, $f(2) = 3$, $f(3) = 4$, $f(4) = 5$ and so on for $f(x) = x + 1$, $x > 0$.

$$f(x) = \begin{cases} 1 - x, & x < 0 \\ 1, & x = 0 \\ x + 1, & x > 0 \end{cases}$$

Thus, the graph of f is as shown in the below figure.

Q.8: Find the domain and range of the real function $f(x) = x/1+x^2$.

Solution:

Given real function is $f(x) = x/1+x^2$.

$$1 + x^2 \neq 0$$

$$x^2 \neq -1$$

Domain : $x \in \mathbb{R}$

Let $f(x) = y$

$$y = x/1+x^2$$

$$\Rightarrow x = y(1 + x^2)$$

$$\Rightarrow yx^2 - x + y = 0$$

This is quadratic equation with real roots.

$$(-1)^2 - 4(y)(y) \geq 0$$

$$1 - 4y^2 \geq 0$$

$$\Rightarrow 4y^2 \leq 1$$

$$\Rightarrow y^2 \leq 1/4$$

$$\Rightarrow -1/2 \leq y \leq 1/2$$

$$\Rightarrow -1/2 \leq f(x) \leq 1/2$$

$$\text{Range} = [-1/2, 1/2]$$

