Important Questions For Class 11 Maths Chapter 3 - Trigonometric Functions

Q. No.1: In any triangle ABC, prove that a sin (B – C) + b sin (C – A) + c sin (A – B) = 0.

Solution:

In any triangle ABC,

a/sin A = b/sin B = c/sin C = k

a = k sin A, b = k sin B, c = k sin C

LHS

= a sin (B – C) + b sin (C – A) + c sin (A – B)

= k sin A [sin B cos C – cos B sin C] + k sin B [sin C cos A – cos C sin A] + k sin C [sin A cos B – cos A sin B]

= k sin A sin B cos C – k sin A cos B sin C + k sin B sin C cos A – k sin B cos C sin A + k sin C sin A cos B – k sin C cos A sin B

= 0

= RHS

Hence proved that a sin (B – C) + b sin (C – A) + c sin (A – B) = 0.

Q.No.2: Find the radius of the circle in which a central angle of 60° intercepts an arc of length 37.4 cm (use π = 22/7).

Solution:

Given,

Length of the arc = l = 37.4 cm

Central angle = θ = 60° = 60π/180 radian = π/3 radians

We know that,

r = l/θ

= (37.4) * (π / 3)

= (37.4) / [22 / 7 * 3]

= 35.7 cm

Hence, the radius of the circle is 35.7 cm.

Q. No.3: A wheel makes 360 revolutions in one minute. Through how many radians does it turn in one second?

Solution:

Given,

Number of revolutions made by the wheel in 1 minute = 360

1 minute = 60 seconds

Number of revolutions in 1 second = 360/60 = 6

Angle made in 1 revolution = 360°

Angles made in 6 revolutions = 6 × 360°

Radian measure of the angle in 6 revolutions = 6 × 360 × π/180

= 6 × 2 × π

= 12π

Hence, the wheel turns 12π radians in one second.

Q. No. 4: Find the value of √3 cosec 20° – sec 20°.

Solution:

√3 cosec 20° – sec 20°

$$= \frac{\sqrt{3}}{\sin 20^\circ} - \frac{1}{\cos 20^\circ}$$

$$= \frac{\sqrt{3}\cos 20^\circ - \sin 20^\circ}{\sin 20^\circ \cos 20^\circ} = 4\left(\frac{\frac{\sqrt{3}}{2}\cos 20^\circ - \frac{1}{2}\sin 20^\circ}{2\sin 20^\circ \cos 20^\circ}\right)$$

$$= 4\left(\frac{\sin 60^\circ \cos 20^\circ - \cos 60^\circ \sin 20^\circ}{\sin 40^\circ}\right)$$

$$= 4\left(\frac{\sin (60^\circ - 20^\circ)}{\sin 40^\circ}\right) = 4$$

Q. No. 5: Show that tan 3x tan 2x tan x = tan 3x – tan 2x – tan x.

Solution:

Let 3x = 2x + x

Taking "tan" on both sides,

tan 3x = tan (2x + x)

We know that,

tan 3x = (tan 2x + tan x)/ (1- tan 2x tan x)

$$\tan(x+y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$$

tan 3x(1 – tan 2x tan x) = tan 2x + tan x

tan 3x – tan 3x tan 2x tan x = tan 2x + tan x

tan 3x – (tan 2x + tan x) = tan 3x tan 2x tan x

Therefore, tan 3x – tan 2x – tan x = tan 3x tan 2x tan x.

Q. No. 6: Prove that:

$$\cos^2 x + \cos^2\left(x + \frac{\pi}{3}\right) + \cos^2\left(x - \frac{\pi}{3}\right) = \frac{3}{2}$$

Solution:

LHS

$$= \cos^2 x + \cos^2\left(x + \frac{\pi}{3}\right) + \cos^2\left(x - \frac{\pi}{3}\right)$$

$$= \cos^2 x + [\cos(x + \tfrac{\pi}{3})]^2 + [\cos(x - \tfrac{\pi}{3})]^2$$

$$= \cos^2 x + (\cos x \cos\tfrac{\pi}{3} - \sin x \sin\tfrac{\pi}{3})^2 + (\cos x \cos\tfrac{\pi}{3} + \sin x \sin\tfrac{\pi}{3})^2$$

$$= \cos^2 x + [\cos x \left(\tfrac{1}{2}\right) - \sin x (\tfrac{\sqrt{3}}{2})]^2 + [\cos x \left(\tfrac{1}{2}\right) + \sin x (\tfrac{\sqrt{3}}{2})]^2$$

$$= \cos^2 x + \tfrac{1}{4}(\cos x - \sqrt{3}\sin x)^2 + \tfrac{1}{4}(\cos x + \sqrt{3}\sin x)^2$$

$$= \cos^2 x + \tfrac{1}{4}(\cos^2 x + 3\sin^2 x - 2\sqrt{3}\cos x \sin x) + \tfrac{1}{4}(\cos^2 x + 3\sin^2 x + 2\sqrt{3}\cos x \sin x)$$

$$= \cos^2 x + \tfrac{1}{4}(\cos^2 x + 3\sin^2 x - 2\sqrt{3}\cos x \sin x + \cos^2 x + 3\sin^2 x + 2\sqrt{3}\cos x \sin x)$$

$$= \cos^2 x + \tfrac{1}{4}(2\cos^2 x + 6\sin^2 x)$$

$$= \cos^2 x + \tfrac{1}{2}\cos^2 x + \tfrac{3}{2}\sin^2 x$$

$$= \tfrac{3}{2}\cos^2 x + \tfrac{3}{2}\sin^2 x$$

$$= \tfrac{3}{2}(\cos^2 x + \sin^2 x)$$

$$= \tfrac{3}{2}(1)$$

$$= \tfrac{3}{2}$$

= RHS

Hence proved.

Q. No. 7: Find the value of cos 570° sin 510° + sin (-330°) cos (-390°).

Solution:

LHS =cos (570)sin (510) + sin (- 330)cos (- 390)

= cos (570) sin (510) + [– sin (330)]cos (390) [because sin(– x) = – sin x and cos(– x) = cos x]

= cos (570)sin(510) – sin (330)

= cos (90 * 6 + 30) sin (90 * 5 + 60) – sin (90 * 3 + 60) cos (90 * 4 + 30)

= – cos (30) cos (60) – [– cos (60)] cos (30)

= – cos (30) cos (60) + cos (30) sin (60)

= 0

Q. No. 8: Find the general solution of the following equation.

$$\cot^2\theta + \frac{3}{\sin\theta} + 3 = 0$$

Solution:

Given,

$$\cot^2\theta + \frac{3}{\sin\theta} + 3 = 0$$

$$\frac{\cos^2\theta}{\sin^2\theta} + \frac{3}{\sin\theta} + 3 = 0$$

$\cos^2\theta + 3\sin\theta + 3\sin^2\theta = 0$

$1 - \sin^2\theta + 3\sin\theta + 3\sin^2\theta = 0$

$2\sin^2\theta + 3\sin\theta + 1 = 0$

$2\sin^2\theta + 2\sin\theta + \sin\theta + 1 = 0$

$2\sin\theta(\sin\theta + 1) + 1(\sin\theta + 1) = 0$

$(2\sin\theta + 1)(\sin\theta + 1) = 0$

$2\sin\theta + 1 = 0, \sin\theta + 1 = 0$

$\sin\theta = -½, \sin\theta = -1$

$\theta = n\pi - (-1)^n \pi/6, \theta = n\pi - (-1)^n \pi/2; n \in Z$

Q. No. 9: Show that 2 sin²β + 4 cos (α + β) sin α sin β + cos 2 (α + β) = cos 2α.

Solution:

LHS = $2\sin^2\beta + 4\cos(\alpha + \beta)\sin\alpha\sin\beta + \cos 2(\alpha + \beta)$

$= 2\sin^2\beta + 4(\cos\alpha\cos\beta - \sin\alpha\sin\beta)\sin\alpha\sin\beta + (\cos 2\alpha\cos 2\beta - \sin 2\alpha\sin 2\beta)$

$= 2\sin^2\beta + 4\sin\alpha\cos\alpha\sin\beta\cos\beta - 4\sin^2\alpha\sin^2\beta + \cos 2\alpha\cos 2\beta - \sin 2\alpha\sin 2\beta$

$= 2 \sin^2\beta + \sin 2\alpha \sin 2\beta - 4 \sin^2\alpha \sin^2\beta + \cos 2\alpha \cos 2\beta - \sin 2\alpha \sin 2\beta$

$= (1 - \cos 2\beta) - (2 \sin^2\alpha)(2 \sin^2\beta) + \cos 2\alpha \cos 2\beta$

$= (1 - \cos 2\beta) - (1 - \cos 2\alpha)(1 - \cos 2\beta) + \cos 2\alpha \cos 2\beta$

$= \cos 2\alpha$

= RHS

Therefore, $2 \sin^2\beta + 4 \cos(\alpha + \beta) \sin\alpha \sin\beta + \cos 2(\alpha + \beta) = \cos 2\alpha$

Q. No. 10: Prove that:

$$\frac{\sec 8\theta - 1}{\sec 4\theta - 1} = \frac{\tan 8\theta}{\tan 2\theta}$$

Solution:

LHS=$(\sec 8\theta - 1)/(\sec 4\theta - 1)$

$=(1/(\cos 8\theta) - 1)/(1/(\cos 4\theta) - 1)$

$=((1-\cos 8\theta)\cos 4\theta)/((1-\cos 4\theta)\cos 8\theta)$

$=(2\sin^2 4\theta \cos 4\theta)/((2\sin^2 2\theta)\cos 8\theta)$

$=(2\sin 4\theta \cos 4\theta \sin 4\theta)/((2\sin^2 2\theta)\cos 8\theta)$

$=(\sin 8\theta * 2\sin 2\theta \cos 2\theta)/((2\sin^2 2\theta)\cos 8\theta)$

$=(\tan 8\theta * \cos 2\theta)/(\sin 2\theta)$

$= (\tan 8\theta)/(\tan 2\theta)$

= RHS

Hence proved.