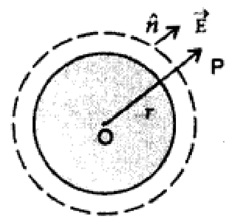
IMPORTANT QUESTIONS CLASS – 12 PHYSICS CHAPTER – 1 ELECTRIC CHARGES AND FIELDS

Question 1.
Using Gauss' law deduce the expression for the electric field due to a uniformly charged spherical conducting shell of radius R at a point
(i) outside and
(ii) inside the shell.
Plot a graph showing variation of electric field as a function of r > R and r < R (r being the distance from the centre of the shell)
Answer:
(i) Field Outside Shell :
Consider a thin spherical shell of radius R with centre O. Let charge +q be distributed uniformly over the surface of shell. To calculate electric field intensity at P where OP = r, imagine a sphere S, with centre at O and radius r. The surface of sphere is Gaussian surface over at every point. Electric field is same and directed radially outwards.



Applying Gauss' theorem

$$\oint_S \vec{E}.\vec{dS} = \frac{q}{\varepsilon_0} \Rightarrow E = \frac{q}{\varepsilon_0 4\pi r^2}$$

r→ is distance of point P from centre where E is calculated]

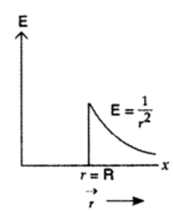
$$\Rightarrow E = \frac{q}{4\pi\varepsilon_0 r^2}$$

[S = 4pr^2

(ii) Inside Shell: As we know charge is located on its surface,

$$\therefore \ E.dS = \frac{q}{\varepsilon_0} \ (q = 0) \ \text{Hence,} \ \vec{E} = 0$$

(iii) at r < R E→ is zero and r = R, E is maximum at r > R, E is decreasing at E ∝ 1r2

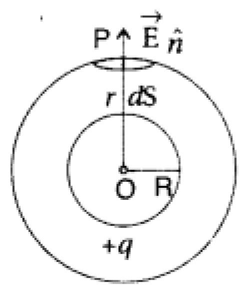


Question 2.
Using Gauss's law, derive the expression for the electric field at a point
(i) outside and
(ii) inside a uniformly charged thin spherical shell. Draw a graph showing electric field E as a function of distance from the centre.
Answer:
Electric field due to a uniformly charged spherical shell:
Suppose a thin spherical shell of radius R and centre O

Let the charge + q be distributed over the surface of sphere
Electric field intensity E→ is same at every point on the surface of sphere directed directly outwards
Let a point P be outside the shell with radius vector

$\vec{r}$ and small area element $\vec{dS} = \hat{n}\,dS$

According to the *Gauss's law*



$$\oint \vec{E}.\vec{dS} = \frac{q}{\varepsilon_0}$$

$$\Rightarrow \quad \oint E dS \frac{q}{\varepsilon_0}$$

Since $\vec{E}$ and $\hat{n}$ are in the same direction

$$\therefore \quad E\, 4\pi r^2 = \frac{q}{\varepsilon_0} \qquad \Rightarrow \qquad E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$$

where [Vectorially, $\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{r}$

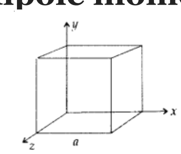
(*i*) If σ is the surface charge density on the shell, then $q = 4\pi R^2$

$$\therefore E = \frac{1}{4\pi\varepsilon_0}\frac{4\pi R^2\sigma}{R^2} = \frac{\sigma}{\varepsilon_0}$$

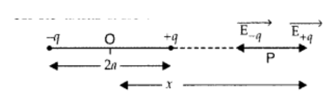
(*ii*) If the point P lies *inside* the spherical shell, then the *Gaussian surface* encloses no charge

$\therefore \quad q = 0$ Hence $E = 0$.

Question 3.
(a) An electric dipole of dipole moment p→ consists of point charges + q and – q separated by a distance 2a apart. Deduce the expression for the electric field E→ due to the dipole at a distance x from the centre of the dipole on its axial line in terms of the dipole moment p→?. Hence show that in the limit x >> a, E→ —>2p→ ($4\pi\varepsilon_0 x^3$).
(Delhi 2015)



Answer:
(a) Expression for magnetic field due to dipole on its axial lane:



Electric field intensity at point P due to charge $-q$,

$$\vec{E}_{-q} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{(x+a)^2}(\hat{x})$$

Due to charge $+q$,

$$\vec{E}_{+q} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{(x-a)^2}(\hat{x})$$

Net Electric field at point P, $\vec{E} = \vec{E}_{-q} + \vec{E}_{+q}$

$$= \frac{q}{4\pi\varepsilon_0}\times\left[\frac{1}{(x-a)^2} - \frac{1}{(x+a)^2}\right](\hat{x})$$

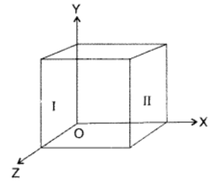
$$= \frac{1}{4\pi\varepsilon_0}\left[\frac{4aqx}{(x^2-a^2)^2}\right](\hat{x}) = \frac{1}{4\pi\varepsilon_0}\frac{(q\times 2a)2x}{(x^2-a^2)^2}(\hat{x})$$

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{2px}{(x^2-a^2)^2}\hat{x} \qquad \because p = (q\times 2a)$$

For $x >> a$

$$(x^2-a^2)^2 \simeq x^4 \qquad \vec{E} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{2p}{x^3}\hat{x}$$

(b)Only the faces perpendicular to the direction of x-axis, contribute to the Electric flux. The remaining faces of the cube given zero



Total flux $\phi = \phi_I + \phi_{II}$

$$= \oint_I \vec{E} \cdot \vec{ds} + \oint_{II} \vec{E} \cdot \vec{ds} = 0 + 2(a) \cdot a^2$$

$\therefore$ $\phi = 2a^3$

Charge enclosed $(q) = \phi\epsilon_0 = 2a^3\epsilon_0$ $\left[\because \phi = \frac{q}{\epsilon_0}\right.$

Question 4.
(a) Define electric flux. Write its S.I. unit. "Gauss's law in electrostatics is true for any closed surface, no matter what its shape or size is". Justify this statement with the help of a suitable example.
(b) Use Gauss's law to prove that the electric field inside a uniformly charged spherical shell is zero. (All India)
(a) Electric flux. The electric lines of force passing through that area, when held normally to the lines of force.
Answer:
Electric flux. The electric lines of force passing through that area, when held normally to the lines of force.

$$Mathematically,\ \phi_E = \oint_S \vec{E} \cdot \Delta \vec{S}$$

S.I. units: Vm, Nm^2C^{-1}
Gauss's Law states that the electric flux through a closed surface is given by

$$\phi = \frac{q}{\epsilon_0}$$

The law implies that the total electric flux through a closed surface depends on the quantity of total charge enclosed by the surface, and does not depend on its shape and size.

For example, net charge enclosed by the electric dipole (q, -q) is zero, hence the total elec¬tric flux enclosed by a surface containing electric dipole is zero.
(b) Electrical field inside a uniformly charged spherical shell. Let us consider a point 'P' inside the shell. The Gaussian surface is a sphere through P centred at O.

The flux through the Gaussian surface is E × $4\pi r^2$.

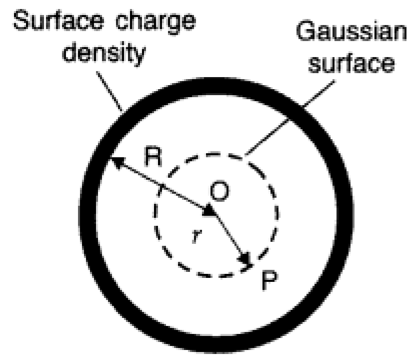
However, in this case, the Gaussian surface encloses no charge. Gauss's law then gives

$E \times 4\pi r^2 = 0$

or $E = 0$

$(r < R)$

that is, the field due to a uniformly charged thin shell is zero at all points inside the shell.



Question 5.
(a) Derive the expression for the energy stored in a parallel plate capacitor. Hence obtain the expression for the energy density of the electric field.
(b) A fully charged parallel plate capacitor is connected across an uncharged identical capacitor. Show that the energy stored in the combination is less than that stored initially in the single capacitor. (All India 2015)

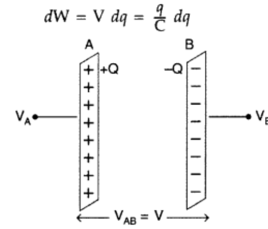
Answer:

(a) (i) Energy of a parallel plate capacitor. Consider a capacitor of capacitance C. Initial charge on plates is zero. Initial potential difference between the capacitor plates is zero. Let a charge Q be given to it in small steps. When charge is given to capacitor, the potential difference between its plates increases. Let at any instant when charge on capacitor be q, the potential difference between its plates be,

$$V = \frac{q}{C}$$

Now work done in giving an additional infinitesimal charge dq to capacitor

$$dW = V\,dq = \frac{q}{C}\,dq$$



The total work done in giving charge from 0 to Q will be equal to the sum of all such infinitesimal works, which may be obtained by integration. Therefore total work

$$W = \int_0^Q V\,dq = \int_0^Q \frac{q}{C}\,dq = \frac{1}{C}\left[\frac{q^2}{2}\right]_0^Q = \frac{1}{C}\left(\frac{Q^2}{2} - \frac{0}{2}\right) = \frac{Q^2}{2C}$$

If V is the final potential difference between capacitor plates, then Q = CV

$$\therefore \quad W = \frac{(CV)^2}{2C} = \frac{1}{2}CV^2 = \frac{1}{2}QV$$

This work is stored as electrostatic potential energy of capacitor i.e., Electriostatic potential energy,

$$U = \frac{Q^2}{2C} = \frac{1}{2}CV^2 = \frac{1}{2}QV$$

(ii) Expression for Energy Density.
Consider a parallel plate capacitor consisting of plates, each of area A, separated by a distance d. If space between the plates is filled with a medium of dielectric constant K, then
Capacitence of Capacitor,

$$C = \frac{K\varepsilon_0 A}{d}$$

If σ is the surface charge density of plates, then electric field strength between the plates.

$$E = \frac{\sigma}{K\varepsilon_0} \Rightarrow \sigma = K\varepsilon_0 E$$

Charge on each plate of capacitor

$$Q = \sigma A = K\varepsilon_0 EA$$

∴ Energy stored by capacitor,

$$U = \frac{Q^2}{2C} = \frac{(K\varepsilon_0 EA)^2}{2(K\varepsilon_0 A / d)} = \frac{1}{2} K\varepsilon_0 E^2 Ad$$

But $Ad = \tau$, volume of space between capacitor plates

∴ Energy stored,

$$U = \frac{1}{2}K\varepsilon_0 E^2 \tau$$

Electrostatic Energy stored per unit volume, $U_e = \frac{U}{\tau} = \frac{1}{2}\mathbf{K\varepsilon_0 E^2}$

This is the expression for electrostatic energy density in medium of dielectric constan K. In air of free space (K = 1), therefore energy density,

$$U_e = \frac{1}{2}\varepsilon_0 E^2$$

(b) The energy of the capacitor when fully charged is

$$E_i = \frac{1}{2}\frac{q^2}{C} \quad \ldots(i)$$

When this charged capacitor is connected to an identical capacitor C, then the charge will be distributed equally, q2 on each of the capacitors, then

$$\text{Similarly, } E_2 = \frac{1}{2}\frac{\left(\frac{q}{2}\right)^2}{C} \quad \ldots(iii)$$

The energy stored of the combination, Adding (*ii*) and (*iii*), we have

$$E_f = E_1 + E_2 = \frac{1}{2}\frac{q^2}{4C} + \frac{1}{2}\frac{q^2}{4C} = \frac{1}{2}\frac{q^2}{2C} \quad \ldots(iv)$$

On comparing (*i*) and (*iv*), we have

$$\frac{E_f}{E_i} = \frac{\frac{1}{2}\frac{q^2}{2C}}{\frac{1}{2}\frac{q^2}{C}} = \frac{1}{2}$$

Hence, the total energy stored is half of that stored initially in one capacitor which means the energy stored in combination is less than that stored initially in the single capacitor.

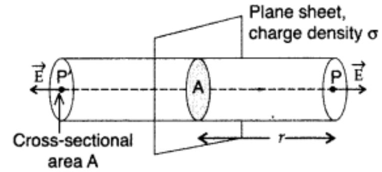
Question 6.
(i) Use Gauss's law to find the electric field due to a uniformly charged infinite plane sheet. What is the direction of field for positive and negative charge densities?
(ii) Find the ratio of the potential differences that must be applied across the parallel and series combination of two capacitors Cj and C2 with their capacitances in the ratio 1 : 2 so that the energy stored in the two cases becomes the same.
Answer:
(i)
(a) Electric flux: The electric flux through a given area held inside an electric field is the

measure of the total number of electric lines of force passing normally through that area.

S.I. units of electric flux = $NC^{-1}m^2$ = **Nm^2C^{-1} or Vm**

Mathematically, $\phi_E = \oint_s \vec{E} \Delta \vec{s}$

(b) Consider a thin, infinite plane sheet of charge with uniform surface charge density o. We wish to calculate its electric field at a point P at distance r from it.



By symmetry, electric field E points outwards normal to the sheet. Also, it must have same magnitude and opposite direction at two points P anel F equidistant from the sheet and on opposite sides. We choose cylindrical Gaussian surface of cross¬sectional area A and length 2r with its axis perpendicular to the sheet.

As the lines of force are parallel to the curved surface of the cylinder, the flux through the curved surface is zero. The flux through the plane-end faces of the cylinder is :

$\phi E = EA + EA = 2EA$

Charge enclosed by the *Gaussian surface*,

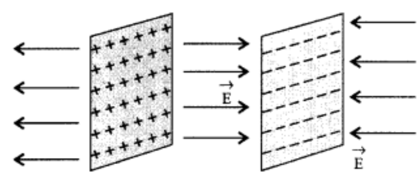
$q = \sigma A$

According to *Gauss's theorem*,

$\phi_E = \frac{q}{\varepsilon_0} \quad \therefore \; 2EA = \frac{\sigma A}{\varepsilon_0}$ or $E = \frac{\sigma}{2\varepsilon_0}$

Clearly, E is independent of *r*, the distance from the plane sheet.

(*c*)



(i) For positively charged sheet → away from the sheet

(ii)For negatively charged sheet →towards the sheet

(b) and

(*ii*) When two capacitors are connected in series

$$\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} \Rightarrow C_s = \frac{C_1C_2}{C_1 + C_2} \quad \ldots(i)$$

When connected in parallel, $C_p = C_1 + C_2$ ...(*ii*)

Given : $C_1 : C_2 :: 1 : 2$ or $C_2 = 2C_1$

$$U_s = \frac{1}{2}C_sV_s^2 \quad U_p = \frac{1}{2}C_pV_p^2$$

$\because \; U_s = U_p$

$$\Rightarrow \frac{V_{series}}{V_{parallel}} = \sqrt{\frac{C_p}{C_s}} = \sqrt{\frac{C_1 + C_2}{\frac{C_1C_2}{C_1 + C_2}}}$$

$$= \sqrt{\frac{(C_1 + C_2)(C_1 + C_2)}{C_1C_2}}$$

$$= \sqrt{\frac{(C_1 + C_2)^2}{C_1C_2}} = \sqrt{\frac{(C_1 + 2C_1)^2}{C_1 \times 2C_1}}$$

$$= \sqrt{\frac{(3C_1)^2}{2C_1^2}} = \frac{\mathbf{3}}{\sqrt{\mathbf{2}}}$$

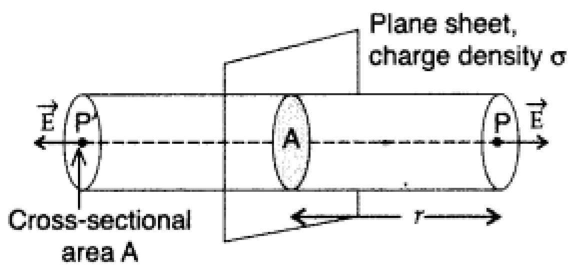
Question 7.
(a) Use Gauss's theorem to find
the electronic field due to a
uniformly charged infinitely large plane thin sheet with surface charge density a.
(b) An infinitely large thin plane sheet has a uniform surface charge density +a. Obtain the expression for the amount of work done in bringing a point charge q from infinity to a point, distant r, in front of the charged plane sheet.

Answer:
(a) Electric flux: The electric flux through a given area held inside an electric field is the measure of the total number of electric lines of force passing normally through that area.

S.I. units of electric flux = $NC^{-1}m^2$ = **Nm^2C^{-1} or Vm**

Mathematically, $\phi_E = \oint_s \vec{E} \Delta \vec{s}$

(b) Consider a thin, infinite plane sheet of charge with uniform surface charge density 0. We wish to calculate its electric field at a point P at distance r from it.



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As the lines of force are parallel to the curved surface of the cylinder, the flux through the curved surface is zero. The flux through the plane-end faces of the cylinder is :

$\phi E = EA + EA = 2EA$

Charge enclosed by the *Gaussian surface*,

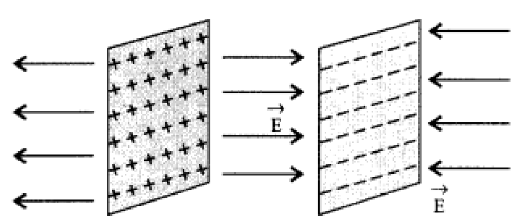
$q = \sigma A$

According to *Gauss's theorem*,

$\phi_E = \frac{q}{\varepsilon_0} \quad \therefore \; 2EA = \frac{\sigma A}{\varepsilon_0}$ or $E = \frac{\sigma}{2\varepsilon_0}$

Clearly, E is independent of *r*, the distance from the plane sheet.

(*c*)



(*i*) (*ii*)

(i) For positively charged sheet →
away from the sheet
(ii)For negatively charged sheet
→towards the sheet

$$(b)\ W = q\int_{\infty}^{r} \vec{E}.d\vec{r}$$

$$= q\int_{\infty}^{r}(-E\,dr) = q\int_{\infty}^{r}\left(\frac{\sigma}{2\varepsilon_0}\right)dr = \frac{q\sigma}{2\varepsilon}[\infty - r]$$

$$\Rightarrow (\infty)$$