

IMPORTANT QUESTIONS CLASS – 12 D < MG 7 G

CHAPTER – 2 'ELECTROSTATIC POTENTIAL AND CAPACITANCE |

1. Explain what would happen if in the capacitor , a 3 mm thick mica sheet (of dielectric constant = 6) were inserted between the plates,

1. While the voltage supply remained connected.

2. After the supply was disconnected.

Ans. Dielectric constant of the mica sheet, $k = 6$

1. Initial capacitance,

New capacitance, $= 106 \text{ pF}$

$$C = 1.771 \times 10^{-11}$$

Supply voltage, $V = 100 \text{ V}$

New charge s

$$C' = kC = 6 \times 1.771 \times 10^{-11}$$

Potential across the plates remains 100 V .

2. Dielectric constant, $k = 6$

$$q^1 = C^1 V = 6 \times 1.7717 \times 10^{-9} = 1.06 \times 10^{-8} \text{ C}$$

Initial capacitance

New capacitance

If supply voltage is removed, then there will be no effect on the amount of charge in the plates. $C = 1.771 \times 10^{-11} \text{ F}$

Charge =

$$C^1 = kC = 6 \times 1.771 \times 10^{-11} = 106 \text{ pF}$$

Potential across the plates is given by,

$$\begin{aligned} \therefore V' &= \frac{q}{C^1} \\ &= \frac{1.771 \times 10^{-9}}{106 \times 10^{-12}} \\ &= 16.7 \text{ V} \end{aligned} \qquad = 1.771 \times 10^{-9} \text{ C}$$

2. A 12 pF capacitor is connected to a 50V battery. How much electrostatic energy is stored in the capacitor?

Ans. Capacitor of the capacitance,

Potential difference, $V = 50 \text{ V}$

$$C = 12 \text{ pF} = 12 \times 10^{-12} \text{ F}$$

Electrostatic energy stored in the capacitor is given by the relation,

$$E_B = \frac{q_2}{4\pi\epsilon_0 (BZ)^2}$$

$$\begin{aligned} \therefore 2\theta &= 112.5^\circ \\ \cos 2\theta &= -038 \end{aligned}$$

$$\cos \theta = \frac{0.10}{0.18} = \frac{5}{9} = 0.5556$$

$$\theta = \cos^{-1} 0.5556 = 56.25$$

$$\vec{E}_2 - \vec{E}_1 = \frac{\sigma}{2\epsilon_0} \hat{n} + \frac{\sigma}{2\epsilon_0} \hat{n} = \frac{\sigma}{\epsilon_0} \hat{n}$$

$$E(2\pi dL) = \frac{\lambda L}{\epsilon_0}$$

$$= \frac{q_1 q_2}{4\pi \epsilon_0 d_1} = 27.2 \text{ eV}$$

$$= \frac{9 \times 10^9 \times (1.6 \times 10^{-19})^2}{1.06 \times 10^{-10}} = 27.2 \text{ eV}$$

$$= 21.73 \times 10^{-10} \text{ J} - 27.2 \text{ eV}$$

$$= 13.58 \text{ eV} - 27.2 \text{ eV}$$

$$= 13.6 \text{ eV}$$

$$E = \frac{1}{2} CV^2$$

$$= \frac{1}{2} \times 12 \times 10^{-12} \times (50)^2$$

$$= 1.5 \times 10^{-8} \text{ J}$$

Therefore, the el

ectrostatic energy stored in the capacitor is $1.5 \times 10^{-8} \text{ J}$.

3. A 600 pF capacitor is charged by a 200 V supply. It is then disconnected from the supply and is connected to another uncharged 600 pF capacitor. How much electrostatic energy is lost in the process?

Ans. Capacitance of the capacitor, $C = 600 \text{ pF}$ Potential difference, $V = 200 \text{ V}$

Electrostatic energy stored in the capacitor is given by,

$$E = \frac{1}{2} CV^2$$

$$= \frac{1}{2} \times (600 \times 10^{-12}) \times (200)^2$$

$$= 1.2 \times 10^{-5} \text{ J}$$

If supply is disconnected from the capacitor and another capacitor of capacitance $C = 600 \text{ pF}$ is connected to it, then equivalent capacitance (C') of the combination is given by,

$$\frac{1}{C'} = \frac{1}{C} + \frac{1}{C}$$

$$= \frac{1}{600} + \frac{1}{600} = \frac{2}{600} = \frac{1}{300}$$

$$\therefore C' = 300 \mu F$$

New electrostatic energy can be calculated as

$$E^1 = \frac{1}{2} \times C^1 \times V^2$$

$$= 0.6 \times 10^{-5} J \qquad = \frac{1}{2} \times 300 \times (200)^2$$

Loss in electrostatic energy = $E - E^1$

$$= 1.2 \times 10^{-5} - 0.6 \times 10^{-5}$$

$$= 0.6 \times 10^{-5}$$

$$= 6 \times 10^{-6} J$$

Therefore, the electrostatic energy lost in the process is $6 \times 10^{-6} J$.

4. A spherical conducting shell of inner radius and outer radius has a charge

1. A charge q is placed at the centre of the shell. What is the surface charge density on the inner and outer surfaces of the shell?

2. Is the electric field inside a cavity (with no charge) zero, even if the shell is not spherical, but has any irregular shape? Explain.

Ans. (a) Charge placed at the centre of a shell is +q. Hence, a charge of magnitude -q will be induced to the inner surface of the shell. Therefore, total charge on the inner surface of the shell is -q.

Surface charge density at the inner surface of the shell is given by the relation,

$$\sigma_1 = \frac{\text{Total charge}}{\text{Outer surface area}} = \frac{-q}{4\pi r_1^2} \dots (i)$$

A charge of +q is induced on the outer surface of the shell. A charge of magnitude Q is placed on the outer surface of the shell. Therefore, total charge on the outer surface of the shell is Q + q. Surface charge density at the outer surface of the shell,

$$\sigma_2 = \frac{\text{Total charge}}{\text{Outer surface area}} = \frac{Q+q}{4\pi r_2^2} \dots (ii)$$

1. Yes

The electric field intensity inside a cavity is zero, even if the shell is not spherical and has any irregular shape. Take a closed loop such that a part of it is inside the cavity along a field line while the rest is inside the conductor. Net work done by the field in carrying a test charge over a closed loop is zero because the field inside the conductor is zero. Hence, electric field is zero, whatever is the shape.

5. If one of the two electrons of a molecule is removed, we get a hydrogen molecular ion. In the ground state of an, the two protons are separated by roughly 1.5 , and the electron is roughly 1 from each proton. Determine the

potential energy of the system. Specify your choice of the zero of potential energy.

Ans.



The system of two protons and one electron is represented in the given figure.

Charge on proton 1,

$$q_1 = 1.6 \times 10^{-19} \text{ C}$$

Charge on proton 2,

$$q_2 = 1.6 \times 10^{-19} \text{ C}$$

Charge on electron,

$$q_3 = -1.6 \times 10^{-19} \text{ C}$$

Distance between protons 1 and 2, $d_1 = 1.5 \times 10^{-10} \text{ m}$

Distance between proton 1 and electron, $d_2 = 1 \times 10^{-10} \text{ m}$

Distance between proton 2 and electron, $d_3 = 1 \times 10^{-10} \text{ m}$

The potential energy at infinity is zero.

Potential energy of the system,

$$V = \frac{q_1 q_2}{4\pi \epsilon_0 d_1} + \frac{q_2 q_3}{4\pi \epsilon_0 d_3} + \frac{q_3 q_1}{4\pi \epsilon_0 d_2}$$

$$\frac{1}{4\pi \epsilon_0 d} = 9 \times 10^9 \text{ Nm}^2 \text{C}^{-2}$$

Substituting

$$V = \frac{9 \times 10^9 \times 10^{-19} \times 10^{-19}}{10^{-10}} \left[-(16)^2 + \frac{(1.6)^2}{1.5} + -(1.6)^2 \right]$$

$$= -30.7 \times 10^{-19} \text{ J}$$

$$= -19.2 \text{ eV}$$

Therefore, the potential energy of the system is -19.2 eV .

6. Two charged conducting spheres of radii a and b are connected to each other by a wire. What is the ratio of electric fields at the surfaces of the two spheres? Use the result obtained to explain why charge density on the sharp and pointed ends of a conductor is higher than on its flatter portions.

Ans .Let a be the radius of a sphere A, QA be the charge on the sphere, and CA be the capacitance of the sphere. Let b be the radius of a sphere B, QB be the charge on the sphere, and CB be the capacitance of the sphere. Since the two spheres are connected with a wire, their potential (V) will become equal.

Let EA be the electric field of sphere A and EB be the electric field of sphere B. Therefore,

their ratio,

$$\frac{E_A}{E_B} = \frac{Q_A}{4\pi\epsilon_0 \times a_1} \times \frac{b^2 4\pi\epsilon_0}{Q_B}$$

$$\frac{E_A}{E_B} = \frac{Q_A}{Q_B} \times \frac{b^2}{a^2} \quad \dots(1)$$

However $\frac{Q_A}{Q_B} = \frac{C_A V}{C_B V}$

$$\therefore \frac{Q_A}{Q_B} = \frac{a}{b} \quad \dots(2)$$

And $\frac{C_A}{C_B} = \frac{a}{b}$

Putting the value of (2) in (1), we obtain $\therefore \frac{E_A}{E_B} = \frac{a}{b} \times \frac{b^2}{a^2} = \frac{b}{a}$

Therefore, the ratio of electric fields at the surface is $\frac{b}{a}$.

7. What is the area of the plates of a 2 F parallel plate capacitor, given that the separation between the plates is 0.5 cm? [You will realize from your answer why ordinary capacitors are in the range of or less. However, electrolytic capacitors do have a much larger capacitance (0.1 F) because of very minute separation between the conductors.]

Ans. Capacitance of a parallel capacitor, $C = 2 \text{ F}$

Distance between the two plates, $d = 0.5 \text{ cm} = 0.5 \times 10^{-2} \text{ m}$ Capacitance of a parallel plate capacitor is given by the relation,

Where,

$\epsilon_0 =$ Permittivity of free space =

$$= 8.85 \times 10^{-12} \text{ C}^2 \text{ N}^{-1} \text{ m}^{-2}$$

Hence, the area of the plates is too large. To avoid this situation, the capacitance is taken in the range of μF .

$$\therefore A = \frac{2 \times 0.5 \times 10^{-2}}{8.85 \times 10^{-12}} = 1130 \text{ km}^2$$

8. A spherical capacitor consists of two concentric spherical conductors, held in position by suitable insulating supports

Show that the capacitance of a spherical capacitor is given by where and are the radii of outer and inner spheres, respectively.

Ans. Radius of the outer shell = r_1

Radius of the inner shell =

The inner surface of the outer shell has charge $+Q$.

The outer surface of the inner shell has induced charge $-Q$.

Potential difference between the two shells is given by,

$$V = \frac{Q}{4\pi\epsilon_0 r_2} - \frac{Q}{4\pi\epsilon_0 r_1}$$

Where,

ϵ_0 = Permittivity of free space

$$= \frac{Q(r_1 - r_2)}{4\pi\epsilon_0 r_1 r_2}$$

$$V = \frac{Q}{4\pi\epsilon_0} \left[\frac{1}{r_2} - \frac{1}{r_1} \right]$$

Capacitance of the given system is given by,

$$= \frac{4\pi\epsilon_0 r_1 r_2}{r_1 - r_2}$$

$$C = \frac{\text{charge}(Q)}{\text{potential difference}(V)}$$

Hence, proved.

9. A cylindrical capacitor has two co-axial cylinders of length 15 cm and radii 1.5 cm and 1.4 cm. The outer cylinder is earthed and the inner cylinder is given a charge of 3.5. Determine the capacitance of the system and the potential of the inner cylinder. Neglect end effects (i.e., bending of field lines at the ends).

Ans. Length of a co-axial cylinder, $l = 15 \text{ cm} = 0.15 \text{ m}$

Radius of outer cylinder, $r_1 = 1.5 \text{ cm} = 0.015 \text{ m}$

Radius of inner cylinder, $r_2 = 1.4 \text{ cm} = 0.014 \text{ m}$

Charge on the inner cylinder, $q = 3.5$

Capacitance of a co-axial cylinder of radii r_1 and r_2 is given by the relation,

$$\mu C = 3.5 \times 10^{-6} \text{ C}$$

Where,

ϵ_0 = Permittivity of free space =

$$C = \frac{2\pi\epsilon_0 l}{\log_2 \frac{r_1}{r_2}}$$

Potential difference of the inner cylinder is given by,

$$8.85 \times 10^{-12} \text{ N}^{-1} \text{ m}^{-2} \text{ C}^2$$

$$V = \frac{q}{C}$$

$$\therefore C = \frac{2\pi \times 8.85 \times 10^{-12} \times 0.15}{2.3026 \log_{10} \left(\frac{0.15}{0.14} \right)}$$

$$= \frac{2\pi \times 8.85 \times 10^{-12} \times 0.15}{2.3026 \times 0.0299} = 1.2 \times 10^{-10} \text{ F}$$

$$= \frac{3.5 \times 10^{-6}}{1.2 \times 10^{-10}} = 2.92 \times 10^4 \text{ V}$$

10. A parallel plate capacitor is to be designed with a voltage rating 1 kV, using a material of dielectric constant 3 and dielectric strength about. (Dielectric strength is the maximum electric field a material can tolerate without breakdown, i.e., without starting to conduct electricity through partial ionisation.) For safety, we should like the field never to exceed, say 10% of the dielectric strength. What minimum area of the plates is required to have a capacitance of 50 pF?

Ans. Potential rating of a parallel plate capacitor, $V = 1 \text{ kV} = 1000 \text{ V}$

Dielectric constant of a material, $\epsilon_r = 3$ Dielectric strength = 10^7 V/m

For safety, the field intensity never exceeds 10% of the dielectric strength. Hence, electric field intensity, $E = 10\% \text{ of } 10^7 = 10^6 \text{ V/m}$

Capacitance of the parallel plate capacitor, $C = 50 \text{ pF} = 50 \times 10^{-12} \text{ F}$

Distance between the plates is given by,

$$d = \frac{V}{E}$$

Capacitance is given by the relation,

$$C = \frac{\epsilon_0 \epsilon_r A}{d}$$

$$= \frac{1000}{10^6} = 10^{-3} \text{ m}$$

Where,

A = Area of each plate

ϵ_0 = Permittivity of free space =

$$\therefore A = \frac{CD}{\epsilon_0 \epsilon_r}$$

$$8.85 \times 10^{-12} \text{ N}^{-1} \text{ C}^2 \text{ m}^{-2}$$

Hence, the area of each plate is about 19 cm^2 .Search

$$= \frac{50 \times 10^{-12} \times 10^{-3}}{8.85 \times 10^{-12} \times 3} \approx 19 \text{ cm}^2$$