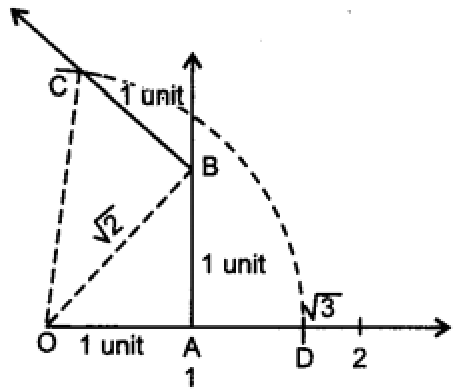
Chapter 1 Number Systems Class 9 maths Important Questions NCERT

Question 1.
Represent √3 on the number line.
Solution:

Here, $\sqrt{3} = \sqrt{1+2} = \sqrt{(1)^2 + (\sqrt{2})^2}$

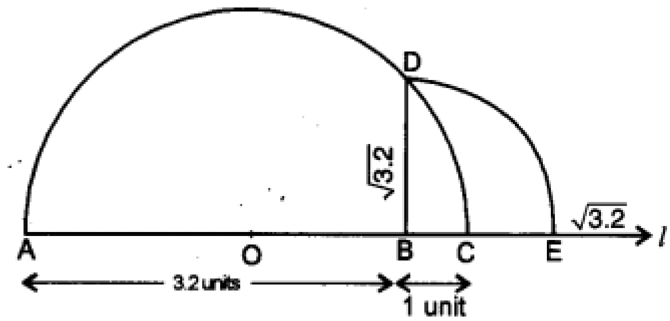
And, $\sqrt{2} = \sqrt{1+1} = \sqrt{(1)^2 + (1)^2}$

On the number line, take OA = 1 unit. Draw AB = 1 unit perpendicular to OA. Join OB.
Again, on OB, draw BC = 1 unit perpendicular to OB. Join OC.
By Pythagoras Theorem, we obtain OC = √3. Using compasses, with centre O and radius OC, draw an arc, which intersects the number line at point D. Thus, OD = √3 and D corresponds to √3.



Question 2.
Represent √3.2 on the number line.
Solution:
First of all draw a line of length 3.2 units such that AB = 3.2 units. Now, from point B, mark a distance of 1 unit. Let this point be 'C'. Let 'O' be the mid-point of the distance AC. Now, draw a semicircle with centre 'O' and radius OC. Let us draw a line perpendicular to AC passing through the point 'B' and intersecting the semicircle at point 'D'.
∴ The distance BD = √3.2

Now, to represent √3.2 on the number line. Let us take the line BC as number line and point 'B' as zero, point 'C' as '1' and so on. Draw an arc with centre B and radius BD, which intersects the number line at point 'E'.
Then, the point 'E' represents √3.2.



Question 3.
Express 1.32 + 0.35 as a fraction in the simplest form.
Solution:
Let . x = 1.32 = 1.3222.....(i)

Multiplying eq. (i) by 10, we have
10x = 13.222...
Again, multiplying eq. (i) by 100, we have

$100x = 132.222\ldots \ldots$(iii)
Subtracting eq. (ii) from (iii), we have
$100x - 10x = (132.222\ldots) - (13.222\ldots)$
$90x = 119$
$\Rightarrow x = 11990$
Again, $y = 0.35 = 0.353535\ldots\ldots$
Multiply (iv) by 100, we have ...(iv)
$100y = 35.353535\ldots$ (v)
Subtracting (iv) from (u), we have
$100y - y = (35.353535\ldots) - (0.353535\ldots)$
$99y = 35$
$y = 3599$

$$\text{Now, } 1.3\overline{2} + 0.\overline{35} = x + y = \frac{119}{90} + \frac{35}{99} = \frac{1309+350}{990} = \frac{1659}{990} = \frac{553}{330}$$

Question 4.
Find the square root of 10 + √24 + √60 + √40.
Solution:

$$\text{Let } \quad x = 10 + \sqrt{24} + \sqrt{60} + \sqrt{40}$$
$$\text{Now, } \quad \sqrt{x} = \sqrt{10+\sqrt{24}+\sqrt{60}+\sqrt{40}}$$
$$= \sqrt{2+3+5+2\sqrt{6}+2\sqrt{15}+2\sqrt{10}}$$
$$= \sqrt{(\sqrt{2})^2+(\sqrt{3})^2+(\sqrt{5})^2+2\sqrt{2}\sqrt{3}+2\sqrt{3}\sqrt{5}+2\sqrt{5}\sqrt{2}}$$
Using $a^2 + b^2 + c^2 + 2ab + 2bc + 2ca = (a + b + c)^2$, we obtain
$$= \sqrt{(\sqrt{2}+\sqrt{3}+\sqrt{5})^2} = \sqrt{2}+\sqrt{3}+\sqrt{5}.$$

Question 5.
If x = 9 + 4√5, find the value of √x – 1x√.
Solution:
Here,
$x = 9 + 4\sqrt{5}$
$x = 5 + 4 + 2 \text{ x } 2\sqrt{5}$
$x = (\sqrt{5}^2 + (2^2 + 2 \text{ x } 2\text{x } \sqrt{5})$.
$x = (\sqrt{5} + 2)^2$
$\sqrt{x} = \sqrt{5} + 2$
Now, 1x√ = 15√+2

$$\frac{1}{\sqrt{x}} = \frac{1}{\sqrt{5}+2} \times \frac{\sqrt{5}-2}{\sqrt{5}-2} = \frac{\sqrt{5}-2}{5-4} = \sqrt{5}-2$$
$$\text{Hence, } \sqrt{x} - \frac{1}{\sqrt{x}} = \sqrt{5}+2-\sqrt{5}+2 = 4.$$

Question 6.
If x = 15√−2 , find the value of
$x^3 - 3^2 - 5x + 3$
Solution:

$\therefore x - 2 = \sqrt{5}$
Squaring both sides, we have

$x^2 - 4x + 4 = 5$
$x^2 - 4x - 1 = 0$ ...(i)
Now, $x^3 - 3^2 - 5x + 3 = (x^2 - 4x - 1)(x + 1) + 4$
$= 0(x + 1) + 4 = 4$ [using (i)]

$$x = \frac{1}{\sqrt{5}-2}$$

$$x = \frac{1}{\sqrt{5}-2} \times \frac{\sqrt{5}+2}{\sqrt{5}+2} = \frac{\sqrt{5}+2}{5-4} = \sqrt{5}+2$$

Question 7.
Find 'x', if $2^{x-7} \times 5^{x-4} = 1250$.
Solution:
We have $2^{x-7} \times 5^{x-4} = 1250$
$\Rightarrow 2^{x-7} \times 5^{x-4} = 2\,5 \times 5 \times 5 \times 5$
$\Rightarrow 2^{x-7} \times 5^{x-4} = 21 \times 54$
Equating the powers of 2 and 5 from both sides, we have
$\Rightarrow x - 7 = 1$ and $x - 4 = 4$
$\Rightarrow x = 8$ and $x = 8$
Hence, $x = 8$ is the required value.

Question 8.
Evaluate:

$$(27)^{-\frac{1}{3}}.(27)^{-\frac{1}{3}}\left[27^{\frac{1}{3}} - 27^{\frac{2}{3}}\right].$$

Solution:

$$(27)^{-\frac{1}{3}}.(27)^{-\frac{1}{3}}\left[27^{\frac{1}{3}} - 27^{\frac{2}{3}}\right] = 27^{-\frac{2}{3}}\left[27^{\frac{1}{3}} - 27^{\frac{2}{3}}\right]$$
$$= (27)^{-\frac{2}{3}+\frac{1}{3}} - (27)^{-\frac{2}{3}+\frac{2}{3}}$$
$$= (27)^{-\frac{1}{3}} - (27)^0 = (3)^{-\frac{1}{3}} - 1$$
$$= 3^{-1} - 1 = \frac{1}{3} - 1 = -\frac{2}{3}.$$

`**Question 9.**
If x = p+q√+p−q√p+q√−p−q√,
then prove that $q^2 - 2px + 9 = 0$.
Solution:

$$x = \frac{\sqrt{p+q}+\sqrt{p-q}}{\sqrt{p+q}-\sqrt{p-q}} = \frac{\sqrt{p+q}+\sqrt{p-q}}{\sqrt{p+q}-\sqrt{p-q}} \times \frac{\sqrt{p+q}+\sqrt{p-q}}{\sqrt{p+q}+\sqrt{p-q}}$$
$$= \frac{\left(\sqrt{p+q}+\sqrt{p-q}\right)^2}{\left(\sqrt{p+q}\right)^2 - \left(\sqrt{p-q}\right)^2} = \frac{p+q+p-q+2\times\sqrt{p+q}\times\sqrt{p-q}}{(p+q)-(p-q)}$$
$$= \frac{2p+2\sqrt{p^2-q^2}}{2q} = \frac{p+\sqrt{p^2-q^2}}{q}$$

$$\Rightarrow \quad qx = p+\sqrt{p^2-q^2}$$
$$\Rightarrow \quad qx - p = \sqrt{p^2-q^2}$$

Squaring both sides, we have
$\Rightarrow q^2x^2 + p^2 - 2pqx = p^2 - q^2$
$\Rightarrow q^2x^2 - 2pqx + q^2 = 0$
$\Rightarrow q(q^2 - 2px + q) = 0$
$\Rightarrow qx^2 - 2px + q = 0$ ($\because q \neq 0$)

Question 10.

If a = 13−11√ and b = 1a, then find a² – b²

Solution:

Here, $a = \frac{1}{3-\sqrt{11}} \times \frac{3+\sqrt{11}}{3+\sqrt{11}} = \frac{3+\sqrt{11}}{9-11} = \frac{3+\sqrt{11}}{-2}$

$b = \frac{1}{a} = 3-\sqrt{11}$

Now, $a^2 - b^2 = (a+b)(a-b)$

$$= \left(\frac{3+\sqrt{11}}{-2} + 3 - \sqrt{11}\right)\left(\frac{3+\sqrt{11}}{-2} - 3 + \sqrt{11}\right)$$

$$= \left(\frac{-3-\sqrt{11}+6-2\sqrt{11}}{2}\right)\left(\frac{-3-\sqrt{11}-6+2\sqrt{11}}{2}\right)$$

$$= \left(\frac{3-3\sqrt{11}}{2}\right)\left(\frac{-9+\sqrt{11}}{2}\right) = \frac{-27+3\sqrt{11}+27\sqrt{11}-33}{4}$$

$$= \frac{-60+30\sqrt{11}}{4} = \frac{-30+15\sqrt{11}}{2} = \frac{1}{2}(15\sqrt{11}-30)$$