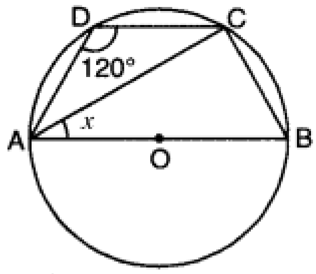
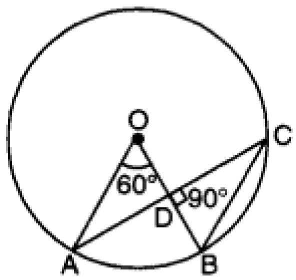
Chapter 10 Circles Class 9 Maths important questions

Question 1.
In the figure, O is the centre of a circle passing through points A, B, C and D and $\angle ADC = 120°$. Find the value of x.



Solution:
Since ABCD is a cyclic quadrilateral
$\angle ADC + \angle ABC = 180°$
[$\therefore$ opp. $\angle$s of a cyclic quad. are supplementary]
$120° + \angle ABC = 180°$
$\angle ABC = 180° - 120° = 60°$
Now, $\angle ACB = 90°$ [angle in a semicircle]
In rt. $\angle$ed ΔCB, $\angle ACB = 90°$
$\angle CAB + \angle ABC = 90°$
$x + 60° = 90°$
$x = 90° - 60°$
$x = 30°$

Question 2.
In the given figure, O is the centre of the circle, $\angle AOB = 60°$ and $CDB = 90°$. Find $\angle OBC$.

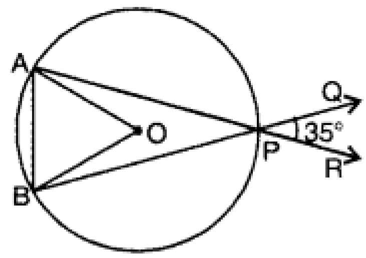


Solution:
Since angle subtended at the centre by an arc is double the angle subtended at the remaining part of the circle.
$\therefore$ $\angle ACB = 13 \angle AOB = 13 \times 60° = 30°$
Now, in ACBD, by using angle sum property, we have
$\angle CBD + \angle BDC + \angle DCB = 180°$
$\angle CBO + 90° + \angle ACB = 180°$
[$\because$ $\angle CBO = \angle CBD$ and $\angle ACB = \angle DCB$ are the same $\angle$s]
$\angle CBO + 90° + 30° = 180°$
$\angle CBO = 1800 - 90° - 30° = 60°$
or $\angle OBC = 60°$

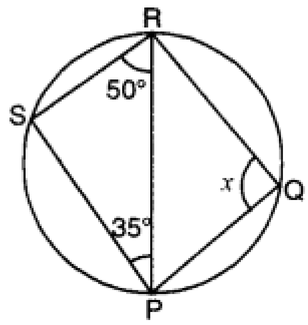
Question 3.
In the given figure, O is the centre of the circle with chords AP and BP being produced to R and Q respectively. If $\angle QPR = 35°$, find the measure of $\angle AOB$.

Solution:
$\angle APB = \angle RPQ = 35°$ [vert. opp. $\angle$s]

Now, ∠AOB and ∠APB are angles subtended by an arc AB at centre and at the remaining part of the circle.
∴ ∠AOB = 2∠APB = 2 × 35° = 70°

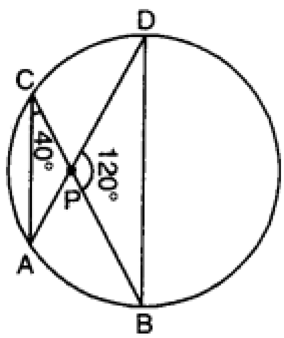


Question 4.
In the figure, PQRS is a cyclic quadrilateral. Find the value of x.



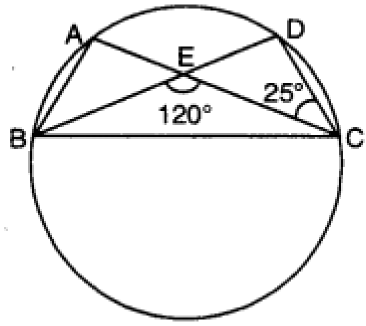
Solution:
In ΔPRS, by using angle sum property, we have
∠PSR + ∠SRP + ∠RPS = 180°
∠PSR + 50° + 350 = 180°
∠PSR = 180° – 850 = 95°
Since PQRS is a cyclic quadrilateral
∴ ∠PSR + ∠PQR = 180°
[∵ opp. ∠s of a cyclic quad. are supplementary]
95° + x = 180°
x = 180° – 95°
x = 85°

Question 5.
In the given figure, ∠ACP = 40° and BPD = 120°, then find ∠CBD.



Solution:
∠BDP = ∠ACP = 40° [angle in same segment]
Now, in ΔBPD, we have
∠PBD + ∠BPD + ∠BDP = 180°
⇒ ∠PBD + 120° + 40° = 180°
⇒ ∠PBD = 180° – 1600 = 20°
or ∠CBD = 20°

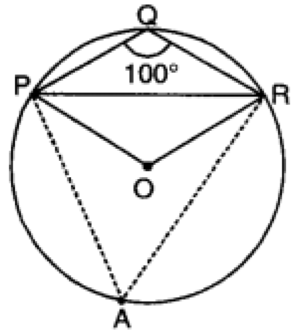
Question 6.
In the given figure, if ∠BEC = 120°, ∠DCE = 25°, then find ∠BAC.



Solution:
∠BEC is exterior angle of ΔCDE.
∴ ∠CDE + ∠DCE = ∠BEC
⇒ ∠CDE + 25° = 120°
⇒ ∠CDE = 95°
Now, ∠BAC = ∠CDE [∵ angle in same segment are equal]
⇒ ∠BAC = 95°

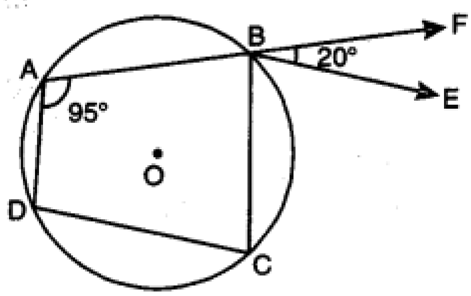
Question 7.
In the given figure, PQR = 100°, where P, Q and R are points on a circle with centre O. Find LOPR.

Solution:
Take any point A on the circumcircle of the circle.
Join AP and AR.



∵ APQR is a cyclic quadrilateral.
∴ ∠PAR + ∠PQR = 180° [sum of opposite angles of a cyclic quad. is 180°]
∠PAR + 100° = 180°
⇒ Since ∠POR and ∠PAR are the angles subtended by an arc PR at the centre of the circle and circumcircle of the circle.
∠POR = 2∠PAR = 2 x 80° = 160°
∴ In APOR, we have OP = OR [radii of same circle]
∠OPR = ∠ORP [angles opposite to equal sides]
Now, ∠POR + ∠OPR + ∠ORP = 180°
⇒ 160° + ∠OPR + ∠OPR = 180°
⇒ 2∠OPR = 20°
⇒ ∠OPR = 10°

Question 8.
In figure, ABCD is a cyclic quadrilateral in which AB is extended to F and BE || DC. If ∠FBE = 20° and DAB = 95°, then find ∠ADC.

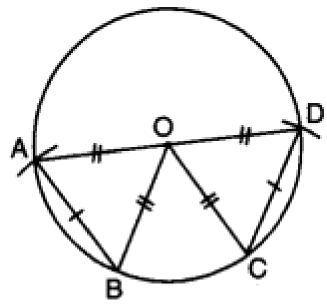


Solution:
Sum of opposite angles of a cyclic quadrilateral is 180°
∴ ∠DAB + ∠BCD = 180°
⇒ 95° + ∠BCD = 180°
⇒ ∠BCD = 180° – 95° = 85°
∵ BE || DC
∴ ∠CBE = ∠BCD = 85°[alternate interior angles]
∴ ∠CBF = CBE + ∠FBE = 85° + 20° = 105°
Now, ∠ABC + 2CBF = 180° [linear pair]
and ∠ABC + ∠ADC = 180° [opposite angles of cyclic quad.]
Thus, ∠ABC + ∠ADC = ∠ABC + 2CBF
⇒ ∠ADC = CBF
⇒ ∠ADC = 105° [∵ CBF = 105°]

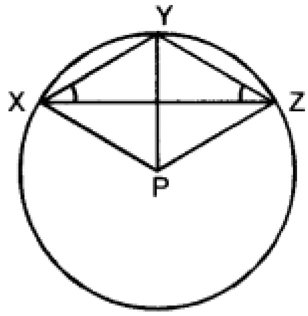
Question 9
Equal chords of a circle subtends equal angles at the centre.

Solution:
Given : In a circle C(O, r), chord AB = chord CD
To Prove : ∠AOB = ∠COD.
Proof : In ΔAOB and ΔCOD
AO = CO (radii of same circle]
BO = DO [radii of same circle]

Chord AB = Chord CD (given]
⇒ ΔAOB = ACOD [by SSS congruence axiom]
⇒ ∠AOB = COD (c.p.c.t.]



Question 10.
In the given figure, P is the centre of the circle. Prove that : ∠XPZ = 2(∠X∠Y + ∠YXZ).



Solution:
Arc XY subtends ∠XPY at the centre P and ∠XZY in the remaining part of the circle.
∴ ∠XPY = 2 (∠X∠Y)
Similarly, arc YZ subtends ∠YPZ at the centre P and ∠YXZ in the remaining part of the circle.
∴ ∠YPZ = 2(∠YXZ) ....(ii)
Adding (i) and (ii), we have
∠XPY + ∠YPZ = 2 (∠XZY + ∠YXZ)
∠XP2 = 2 (∠XZY + ∠YXZ)