Ch 12 Heron's Formula Class 9 Important Questions NCERT Maths

Q.1 **Find the Area of a Triangle whose two sides are 18 cm and 10 cm respectively and the perimeter is 42cm.**

Solution:

Let us consider the third side of the triangle to be "c".

Now, the three sides of the triangle are a = 18 cm, b = 10 cm, and "c" cm

It is given that the perimeter of the triangle = 42cm

So,

18 + 10 + c = 42

c = 42 – (18 + 10) cm = 14 cm

∴ The semi perimeter of triangle (s) = 42/2 = 21 cm

Using Heron's formula,

Area of the triangle A = $s(s-a)(s-b)(s-c)$$----------$$\sqrt{s(s-a)(s-b)(s-c)}$

= $21(21-18)(21-10)(21-14)$$----------$$\sqrt{21(21-18)(21-10)(21-14)}$

= $21\times3\times11\times7$$----------$$\sqrt{21\times3\times11\times7}$

= $21\times21\times11$$----------$$\sqrt{21\times21\times11}$

= 2111−−$\sqrt{2111}cm^2$

Q2: Sides of a Triangle are in the ratio of 14 : 20: 25 and its perimeter is 590cm. Find its area.

Solution:

The ratio of the sides of the triangle is given as 14: 20: 25

Let us consider the common ratio between the sides of the triangle be "a"

∴ The sides are 14a, 20a and 25a

It is also given that the perimeter of the triangle = 590 cm

12a + 17a + 25a = 590

=> 59a = 590

So, a = 10

Now, the sides of the triangle are 140 cm, 200 cm, 250 cm.

So, the semi perimeter of the triangle (s) = 590/2 = 295 cm

Using Heron's formula for Area of the triangle

$= s(s-a)(s-b)(s-c)----------\sqrt{s(s-a)(s-b)(s-c)}$

$= 295(295-140)(295-200)(295-250)----------$
$-\sqrt{295(295-140)(295-200)(295-250)}$

$= 295\times155\times95\times45----------\sqrt{295\times155\times95\times45}$

$= 195,474,375----------\sqrt{195,474,375}$

$= 13981.21cm^2$

Q3: A field is in the Shape of a Trapezium whose parallel sides are 22 m and 10 m. The non-parallel sides are given as 13 m and 14 m. Find the area of the field.

Solution:

Draw a line segment BE line AD. Then, draw a perpendicular on the line segment CD from point B

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Now, it can be seen that the quadrilateral ABED is a parallelogram. So,

AB = DE = 10 m

AD = BE = 13 m

EC = DC– ED

= 22 – 10 = 12 m

Now, consider the triangle BEC,

Its semi perimeter (s) = (13+ 14 + 12)/2

= 39/2 m

=19.5m

By using Heron's formula,

Area of ΔBEC =

= s(s−a)(s−b)(s−c)−−−−−−−−−−−−−−−−−√s(s−a)(s−b)(s−c)

= 19.5(19.5−13)(19.5−14)(19.5−15)−−−−−−−−−−−−−−−−−−−−−−−−−−
−√19.5(19.5−13)(19.5−14)(19.5−15)

= 19.5×6.5×5.5×4.5−−−−−−−−−−−−−−−−−√19.5×6.5×5.5×4.5

= 3137.06−−−−−−√3137.06

= 56m^2

We also know that the area of ΔBEC = (½) × EC × BF

56 cm^2 = (½) × 12 × BF

BF = 56 x 2 /12cm

= 9.3 cm

So, the total area of ABED will be BF × DE i.e. 9.3 × 10 = 93 m^2

∴ Area of the field = 93 + 56 = 149 m^2

Q4: Find the Area of the Triangular field of sides 55 m, 60 m, and 65 m. Find the cost of laying the grass in the triangular field at the rate of Rs 8 per m^2.

Solution:

Given that

Sides of the triangular field are 50 m, 60 m and 65 m.

Cost of laying grass in a triangular field = Rs 8 per m^2

Let a = 55, b = 60, c = 65

Semi- Perimeter s = (a + b + c)/2

⇒ s = (55 + 60 + 65)/2

= 180/2

= 90.

By using Heron's formula,

Area of ΔBEC =

= s(s−a)(s−b)(s−c)−−−−−−−−−−−−−−−−−√s(s−a)(s−b)(s−c)

= 90(90−55)(90−60)(90−65)−−−−−−−−−−−−−−−−−−−−−−−√90(90−55)(90−60)(90−65)

= 90×35×30×25−−−−−−−−−−−−−√90×35×30×25

=2362500−−−−−−−√2362500

= 1537m^2

Cost of laying grass = Area of triangle × Cost of laying grass per m^2

= 1537×8

= Rs.12296

Q5: The Perimeter of an Isosceles triangle is 42 cm. The ratio of the equal side to its base is 3: 4. Find the area of the triangle.

Solution:

Given that,

The perimeter of the isosceles triangle = 42 cm

It is also given that,

Ratio of equal side to base = 3 : 4

Let the equal side = 3x

So, base = 4x

Perimeter of the triangle = 42

⇒ 3x + 3x + 4x = 42

⇒ 10x = 42

⇒ x = 4.2

Equal side = 3x = 3×4.2 = 12.6

Base = 4x = 4×4.2 = 16.8

The sides of the triangle = 12.6cm, 12.6cm and 16.8cm.

Let a = 12.6, b = 12.6, c = 16.8

s = (a + b + c)/2

⇒ s = (12.6 + 12.6 + 16.8)/2

= 42/2

= 21.

By using Heron's formula,

Area of ΔBEC =

= s(s−a)(s−b)(s−c)−−−−−−−−−−−−−−−−−−√s(s−a)(s−b)(s−c)

= 21(21−12.6)(21−12.6)(21−16.8)−−−−−−−−−−−−−−−−−−−−−−−−−−√21(21−12.6)(21−12.6)(21−16.8)

=21×8.4×8.4×4.2−−−−−−−−−−−−−−−−−√21×8.4×8.4×4.2

= 6223.39−−−−−−√6223.39

= 78.88cm^2

Q.6 Find the area of a triangle whose two sides are 18 cm and 10 cm and the perimeter is 42cm.

Solution:

Assume that the third side of the triangle to be "x".

Now, the three sides of the triangle are 18 cm, 10 cm, and "x" cm

It is given that the perimeter of the triangle = 42cm

So, x = 42 – (18 + 10) cm = 14 cm

∴ The semi perimeter of triangle = 42/2 = 21 cm

Using Heron's formula,

Area of the triangle,

= √[s (s-a) (s-b) (s-c)]

= √[21(21 – 18) (21 – 10) (21 – 14)] cm^2

= √[21 × 3 × 11 × 7] m^2

= 21√11 cm^2

Q.7: The sides of a triangle are in the ratio of 12: 17: 25 and its perimeter is 540cm. Find its area.

Solution:

The ratio of the sides of the triangle is given as 12: 17: 25

Now, let the common ratio between the sides of the triangle be “x”

∴ The sides are 12x, 17x and 25x

It is also given that the perimeter of the triangle = 540 cm

12x + 17x + 25x = 540 cm

=> 54x = 540cm

So, x = 10

Now, the sides of the triangle are 120 cm, 170 cm, 250 cm.

So, the semi perimeter of the triangle (s) = 540/2 = 270 cm

Using Heron’s formula,

Area of the triangle

$$= \sqrt{s(s-a)(s-b)(s-c)}$$
$$= \left[\sqrt{270(270-120)(270-170)(270-250)}\right] \text{cm}^2$$
$$= \left[\sqrt{270 \times 150 \times 100 \times 20}\right] \text{cm}^2$$

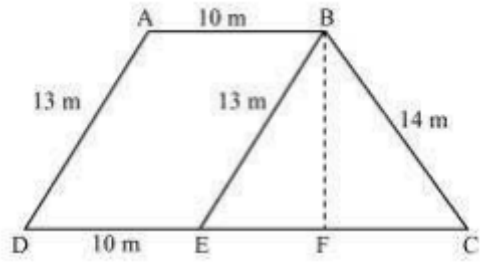
= 9000 cm²

Q.8: A field is in the shape of a trapezium whose parallel sides are 25 m and 10 m. The non-parallel sides are 14 m and 13 m. Find the area of the field.

Solution:

First, draw a line segment BE parallel to the line AD. Then, from B, draw a perpendicular on the line segment CD.

Now, it can be seen that the quadrilateral ABED is a parallelogram. So,



AB = ED = 10 m

AD = BE = 13 m

EC = 25 – ED = 25 – 10 = 15 m

Now, consider the triangle BEC,

Its semi perimeter (s) = (13+ 14 + 15)/2 = 21 m

By using Heron’s formula,

Area of ΔBEC =

$$\sqrt{s(s-a)(s-b)(s-c)}$$

$$\left(\sqrt{21 \times (21-13) \times (21-14) \times (21-15)}\right) m^2$$

$$\left(\sqrt{21 \times 8 \times 7 \times 6}\right) m^2$$

= 84 m^2

We also know that the area of ΔBEC = (½) × CE × BF

84 cm^2 = (½) × 15 × BF

=> BF = (168/15) cm = 11.2 cm

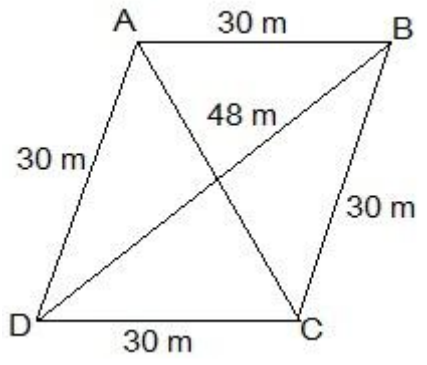
So, the total area of ABED will be BF × DE, i.e. 11.2 × 10 = 112 m^2

∴ Area of the field = 84 + 112 = 196 m^2

Q.9: A rhombus-shaped field has green grass for 18 cows to graze. If each side of the rhombus is 30 m and its longer diagonal is 48 m, how much area of grass field will each cow be getting?

Solution:

Draw a rhombus-shaped field first with the vertices as ABCD. The diagonal AC divides the rhombus into two congruent triangles which are having equal areas. The diagram is as follows.



Consider the triangle BCD,

Its semi-perimeter = (48 + 30 + 30)/2 m = 54 m

Using Heron's formula,

Area of the ΔBCD =

$$\sqrt{s(s-a)(s-b)(s-c)}$$

$$\left(\sqrt{54(54-48)(54-30)(54-30)}\right) m^2$$

$$\left(\sqrt{54 \times 6 \times 24 \times 24}\right) m^2$$

= 432 m^2

∴ Area of field = 2 × area of the ΔBCD = (2 × 432) m^2 = 864 m^2

Thus, the area of the grass field that each cow will be getting = (864/18) m^2 = 48 m^2

Q.10: Find the cost of laying grass in a triangular field of sides 50 m, 65 m and 65 m at the rate of Rs 7 per m^2.

Solution:

According to the question,

Sides of the triangular field are 50 m, 65 m and 65 m.

Cost of laying grass in a triangular field = Rs 7 per m2

Let a = 50, b = 65, c = 65

s = (a + b + c)/2

⇒ s = (50 + 65 + 65)/2

= 180/2

= 90.

Area of triangle = √(s(s-a)(s-b)(s-c))

= √(90(90-50)(90-65)(90-65))

= √(90×40×25×25)

= 1500m^2

Cost of laying grass = Area of triangle ×Cost per m^2

= 1500×7

= Rs.10500