Important Questions for Class 11 Maths Chapter 10 - Conic Sections

Question 1:

Determine the equation of the circle with radius 4 and Centre (-2, 3).

Solution:

Given that:

Radius, r = 4, and center (h, k) = (-2, 3).

We know that the equation of a circle with centre (h, k) and radius r is given as

$(x - h)^2 + (y - k)^2 = r^2$ ....(1)

Now, substitute the radius and center values in (1), we get

Therefore, the equation of the circle is

$(x + 2)^2 + (y - 3)^2 = (4)^2$

$x^2 + 4x + 4 + y^2 - 6y + 9 = 16$

Now, simplify the above equation, we get:

$x^2 + y^2 + 4x - 6y - 3 = 0$

Thus, the equation of a circle with center (-2, 3) and radius 4 is $x^2 + y^2 + 4x - 6y - 3 = 0$

Question 2:

Compute the centre and radius of the circle $2x^2 + 2y^2 - x = 0$

Solution:

Given that, the circle equation is $2x^2 + 2y^2 - x = 0$

This can be written as:

$\Rightarrow (2x^2 - x) + y^2 = 0$

$\Rightarrow 2\{[x^2 - (x/2)] + y^2\} = 0$

$\Rightarrow \{x^2 - 2x(1/4) + (1/4)^2\} + y^2 - (1/4)^2 = 0$

Now, simplify the above form, we get

$\Rightarrow (x - (1/4))^2 + (y-0)^2 = (1/4)^2$

The above equation is of the form $(x - h)^2 + (y - k)^2 = r^2$

Therefore, by comparing the general form and the equation obtained, we can say

$h = 1/4$, $k = 0$, and $r = 1/4$.

Question 3:

Determine the focus coordinates, the axis of the parabola, the equation of the directrix and the latus rectum length for $y^2 = -8x$

Solution:

Given that, the parabola equation is $y^2 = -8x$.

It is noted that the coefficient of x is negative.

Therefore, the parabola opens towards the left.

Now, compare the equation with $y^2 = -4ax$, we obtain

$-4a = -8$

$\Rightarrow a = 2$

Thus, the value of a is 2.

Therefore, the coordinates of the focus = $(-a, 0) = (-2, 0)$

Since the given equation involves y^2, the axis of the parabola is the x-axis.

Equation of directrix, $x = a$ i.e., $x = 2$

We know the formula to find the length of a latus rectum

Latus rectum length $= 4a$

Now, substitute $a = 2$, we get

Length of latus rectum = 8

Question 4:

Determine the foci coordinates, the vertices, the length of the major axis, the minor axis, the eccentricity and the length of the latus rectum of the ellipse $(x^2/49) + (y^2/36) = 1$

Solution:

The given equation is $(x^2/49) + (y^2/36) = 1$

It can be written as $(x^2/7^2) + (y^2/6^2) = 1$

It is noticed that the denominator of $x^2/49$ is greater than the denominator of the $y^2/36$

On comparing the equation with $(x^2/a^2) + (y^2/b^2) = 1$, we will get

$a = 7$ and $b = 6$

Therefore, $c = \sqrt{(a^2 - b^2)}$

Now, substitute the value of a and b

$\Rightarrow \sqrt{(a^2 - b^2)} = \sqrt{(7^2 - 6^2)} = \sqrt{(49-36)}$

$\Rightarrow \sqrt{13}$

Hence, the foci coordinates are $(\pm \sqrt{13}, 0)$

Eccentricity, $e = c/a = \sqrt{13}/7$

Length of the major axis $= 2a = 2(7) = 14$

Length of the minor axis $= 2b = 2(6) = 12$

The coordinates of the vertices are $(\pm 7, 0)$

Latus rectum Length $= 2b^2/a = 2(6)^2/7 = 2(36)/7 = 72/7$

Question 5:

Determine the equation for the ellipse that satisfies the given conditions: Centre at (0, 0), the major axis on the y-axis and passes through the points (3, 2) and (1, 6).

Solution:

Centre = (0, 0), and major axis that passes through the points (3, 2) and (1, 6).

We know that the equation of the ellipse will be of the form when the centre is at (0, 0) and the major axis is on the y-axis,

$(x^2/b^2) + (y^2/a^2) = 1$ …. (1)

Here, a is the semi-major axis.

It is given that, the ellipse passes through the points (3, 2) and (1, 6).

Hence, equation (1) becomes

$(9/b^2) + (4/a^2) = 1$ …(2)

$(1/b^2) + (36/a^2) = 1$ …(3)

Solving equation (2) and (3), we get

$b^2 = 10$ and $a^2 = 40$

Therefore, the equation of the ellipse becomes: $(x^2/10) + (y^2/40) = 1$

Question 6:

Determine the equation of the hyperbola which satisfies the given conditions: Foci (0, ±13), the conjugate axis is of length 24.

Solution:

Given that: Foci (0, ±13), Conjugate axis length = 24

It is noted that the foci are on the y-axis.

Therefore, the equation of the hyperbola is of the form:

$(y^2/a^2)-(x^2/b^2) = 1$ ...(1)

Since the foci are (0, ±13), we can get

C = 13

It is given that, the length of the conjugate axis is 24,

It becomes 2b = 24

b= 24/2

b= 12

And, we know that $a^2 + b^2 = c^2$

To find a, substitute the value of b and c in the above equation:

$a^2 + 12^2 = 13^2$

$a^2 = 169-144$

$a^2 = 25$

Now, substitute the value of a and b in equation (1), we get

$(y^2/25)-(x^2/144) = 1$, which is the required equation of the hyperbola.