Important Questions for Class 11 Maths Chapter 12 - Limits and Derivatives

Class 11 Chapter 12 – Limits and Derivatives Important Questions with Solutions

Practice the following important questions in class 11 Maths Limits and Derivatives that should help you to solve the problems faster with accuracy.

Question 1:

Find the derivative of the function $x^2\cos x$.

Solution:

Given function is $x^2\cos x$

Let $y = x^2\cos x$

Differentiate with respect to x on both sides.

Then, we get:

$dy/dx = (d/dx)x^2\cos x$

Now, using the formula, we can write the above form as:

$dy/dx = x^2 (d/dx) \cos x + \cos x (d/dx)x^2$

Now, differentiate the function:

$dy/dx = x^2 (-\sin x) + \cos x (2x)$

Now, rearrange the terms, we will get:

$dy/dx = 2x \cos x – x^2 \sin x$

Question 2:

Find the positive integer "n" so that $\lim_{x\to 3}[(x^n– 3^n)/(x – 3)] = 108$.

Solution:

Given limit: $\lim_{x\to 3}[(x^n– 3^n)/(x – 3)] = 108$

Now, we have:

$\lim_{x\to 3}[(x^n– 3^n)/(x-3)] = n(3)^{n-1}$

$n(3)^{n-1} = 108$

Now, this can be written as:

$n(3)^{n-1} = 4\ (27) = 4(3)^{4-1}$

Therefore, by comparing the exponents in the above equation, we get:

$n = 4$

Therefore, the value of positive integer "n" is 4.

Question 3:

Find the derivative of $f(x) = x^3$ using the first principle.

Solution:

By definition,

$f'(x) = \lim_{h\to 0} [f(x+h)-f(x)]/h$

Now, substitute $f(x)=x^3$ in the above equation:

$f'(x) = \lim_{h\to 0} [(x+h)^3-x^3]/h$

$f'(x) = \lim_{h\to 0} (x^3+h^3+3xh(x+h)-x^3)/h$

$f'(x) = \lim_{h\to 0} (h^2+3x(x+h))$

Substitute h = 0, we get:

$f'(x) = 3x^2$

Therefore, the derivative of the function $f'(x) = x^3$ is $3x^2$.

Question 4:

Determine the derivative of cosx/(1+sin x).

Solution:

Given function: cosx/(1+sin x)

Let y = cosx/(1+sin x)

Now, differentiate the function with respect to "x", we get

dy/dx = (d/dx) (cos x/(1+sin x))

Now, use the u/v formula in the above form, we get

$dy/dx = [(1+\sin x)(-\sin x) – (\cos x)(\cos x)]/(1+\sin x)^2$

$dy/dx = (-\sin x – \sin^2 x-\cos^2 x)/(1+\sin x)^2$

Now, take (-) outside from the numerator, we get:

$dy/dx = -(\sin x + \sin^2 x + \cos^2 x)/(1+\sin x)^2$

We know that $\sin^2 x + \cos^2 x = 1$

By substituting this, we can get:

$dy/dx = -(1+\sin x)/(1+\sin x)^2$

Cancel out (1+sin x) from both numerator and denominator, we get:

$dy/dx = -1/(1+\sin x)$

Therefore, the derivative of cosx/(1+sin x) is -1/(1+sin x).

Question 5:

$\lim_{x\to 0} |x|/x$ is equal to:

(a)1 (b)-1 (c)0 (d)does not exists

Solution:

A correct answer is an **option (d)**

Explanation:

The limit mentioned here is $x\to 0$

It has two possibilities:

Case 1: $x\to 0^+$

Now, substitute the limit in the given function:

$\lim_{x\to 0+} |x|/x = x/x = 1$

Case 2: $x\to 0^-$

Now, substitute the limit in the given function:

$\lim_{x\to 0-} |x|/x = -x/x = -1$

Hence, the result for both cases varies, the solution is an option (D)

Question 6:

Evaluate the derivative of $f(x) = \sin^2 x$ using Leibnitz product rule.

Solution:

Given function: $f(x) = \sin^2 x$

Let $y = \sin^2 x$

Now, by using Leibnitz product rule, we can write it as:

$dy/dx = (d/dx) \sin^2 x$

$\sin^2 x$ can be written as $(\sin x)(\sin x)$

Now, it becomes:

$dy/dx = (d/dx) (\sin x)(\sin x)$

$dy/dx = (\sin x)'(\sin x) + (\sin x)(\sin x)'$

$dy/dx = \cos x \sin x + \sin x \cos x$

$dy/dx = 2 \sin x \cos x$

$dy/dx = \sin 2x$

Therefore, the derivative of the function $\sin^2 x$ is $\sin 2x$.