Important Questions for Class 11 Maths Chapter 4 - Complex Numbers and Quadratic Equations

Question 1:

Write the given complex number (1 – i) – (–1 + i6) in the form a + ib

Solution:

Given Complex number: (1 – i) – (–1 + i6)

Multiply (-) by the term inside the second bracket (–1 + i6)

= 1 – i +1 – i6

= 2 – 7i, which is of the form a + ib.

Question 2:

Express the given complex number (-3) in the polar form.

Solution:

Given, complex number is -3.

Let r cos θ = -3 ...(1)

and r sin θ = 0 ...(2)

Squaring and adding (1) and (2), we get

$r^2\cos^2\theta + r^2\sin^2\theta = (-3)^2$

Take r^2 outside from L.H.S, we get

$r^2(\cos^2\theta + \sin^2\theta) = 9$

We know that, $\cos^2\theta + \sin^2\theta = 1$, then the above equation becomes,

$r^2 = 9$

r = 3 (Conventionally, r > 0)

Now, subsbtitute the value of r in (1) and (2)

3 cos θ = -3 and 3 sin θ = 0

cos θ = -1 and sin θ = 0

Therefore, $\theta = \pi$

Hence, the polar representation is,

$-3 = r \cos \theta + i\, r \sin \theta$

$3 \cos \pi + 3 \sin \pi = 3(\cos \pi + i \sin \pi)$

Thus, the required polar form is $3 \cos \pi + 3i \sin \pi = 3(\cos \pi + i \sin \pi)$

Question 3:

Solve the given quadratic equation $2x^2 + x + 1 = 0$.

Solution:

Given quadratic equation: $2x^2 + x + 1 = 0$

Now, compare the given quadratic equation with the general form $ax^2 + bx + c = 0$

On comparing, we get

$a = 2$, $b = 1$ and $c = 1$

Therefore, the discriminant of the equation is:

$D = b^2 - 4ac$

Now, substitute the values in the above formula

$D = (1)^2 - 4(2)(1)$

$D = 1 - 8$

$D = -7$

Therefore, the required solution for the given quadratic equation is

$x = [-b \pm \sqrt{D}]/2a$

$x = [-1 \pm \sqrt{-7}]/2(2)$

We know that, $\sqrt{-1} = i$

$x = [-1 \pm \sqrt{7}i] / 4$

Hence, the solution for the given quadratic equation is $(-1 \pm \sqrt{7}i) / 4$.

Question 4:

For any two complex numbers z_1 and z_2, show that $\mathrm{Re}(z_1z_2) = \mathrm{Re}z_1\, \mathrm{Re}z_2 - \mathrm{Im}z_1 \mathrm{Im}z_2$

Solution:

Given: z_1 and z_2 are the two complex numbers

To prove: $Re(z_1z_2) = Rez_1\ Rez_2 - Imz_1Imz_2$

Let $z_1 = x_1+iy_1$ and $z_2 = x_2+iy_2$

Now, $z_1\ z_2 =(x_1+iy_1)(x_2+iy_2)$

Now, split the real part and the imaginary part from the above equation:

$\Rightarrow x_1(x_2+iy_2) +iy_1(x_2+iy_2)$

Now, multiply the terms:

$= x_1x_2+ix_1y_2+ix_2y_1+i^2y_1y_2$

We know that, $i^2 = -1$, then we get

$= x_1x_2+ix_1y_2+ix_2y_1-y_1y_2$

Now, again seperate the real and the imaginary part:

$= (x_1x_2 -y_1y_2) +i\ (x_1y_2+x_2y_1)$

From the above equation, take only the real part:

$\Rightarrow Re\ (z_1z_2) =(x_1x_2 -y_1y_2)$

It means that,

$\Rightarrow Re(z_1z_2) = Rez_1\ Rez_2 - Imz_1Imz_2$

Hence, the given statement is proved.

Question 5:

Find the modulus of [(1+i)/(1-i)] – [(1-i)/(1+i)]

Solution:

Given: [(1+i)/(1-i)] – [(1-i)/(1+i)]

Simplify the given expression, we get:

$[(1+i)/(1-i)] - [(1-i)/(1+i)] = [(1+i)^2 - (1-i)^2]/ [(1+i)(1-i)]$
$= (1+i^2+2i-1-i^2+2i)) / (1^2+1^2)$

Now, cancel out the terms,

$= 4i/2$

$= 2i$

Now, take the modulus,

$| [(1+i)/(1-i)] – [(1-i)/(1+i)]| = |2i| = \sqrt{2^2} = 2$

Therefore, the modulus of [(1+i)/(1-i)] – [(1-i)/(1+i)] is 2.