Important Questions for Class 11 Maths Chapter 7 - Binomial Theorem

Question 1:

Expand the expression $(2x-3)^6$ using the binomial theorem.

Solution:

Given Expression: $(2x-3)^6$

By using the binomial theorem, the expression $(2x-3)^6$ can be expanded as follows:

$(2x-3)^6 = {}^6C_0(2x)^6 - {}^6C_1(2x)^5(3) + {}^6C_2(2x)^4(3)^2 - {}^6C_3(2x)^3(3)^3 + {}^6C_4(2x)^2(3)^4 - {}^6C_5(2x)(3)^5 + {}^6C_6(3)^6$

$(2x-3)^6 = 64x^6 - 6(32x^5)(3) + 15(16x^4)(9) - 20(8x^3)(27) + 15(4x^2)(81) - 6(2x)(243) + 729$

$(2x-3)^6 = 64x^6 - 576x^5 + 2160x^4 - 4320x^3 + 4860x^2 - 2916x + 729$

Thus, the binomial expansion for the given expression $(2x-3)^6$ is $64x^6 - 576x^5 + 2160x^4 - 4320x^3 + 4860x^2 - 2916x + 729$.

Question 2:

Evaluate $(101)^4$ using the binomial theorem

Solution:

Given: $(101)^4$.

Here, 101 can be written as the sum or the difference of two numbers, such that the binomial theorem can be applied.

Therefore, 101 = 100+1

Hence, $(101)^4 = (100+1)^4$

Now, by applying the binomial theorem, we get:

$(101)^4 = (100+1)^4 = {}^4C_0(100)^4 + {}^4C_1(100)^3(1) + {}^4C_2(100)^2(1)^2 + {}^4C_3(100)(1)^3 + {}^4C_4(1)^4$

$(101)^4 = (100)^4 + 4(100)^3 + 6(100)^2 + 4(100) + (1)^4$

$(101)^4 = 100000000 + 4000000 + 60000 + 400 + 1$

$(101)^4 = 104060401$

Hence, the value of $(101)^4$ is 104060401.

Question 3:

Using the binomial theorem, show that $6^n - 5n$ always leaves remainder 1 when divided by 25

Solution:

Assume that, for any two numbers, say x and y, we can find numbers q and r such that x = yq + r, then we say that b divides x with q as quotient and r as remainder. Thus, in order to show that $6^n - 5n$ leaves remainder 1 when divided by 25, we should prove that $6^n - 5n = 25k + 1$, where k is some natural number.

We know that,

$(1 + a)^n = {}^nC_0 + {}^nC_1\, a + {}^nC_2\, a^2 + \ldots + {}^nC_n\, a^n$

Now for a=5, we get:

$(1 + 5)^n = {}^nC_0 + {}^nC_1\, 5 + {}^nC_2\, (5)^2 + \ldots + {}^nC_n\, 5^n$

Now the above form can be weitten as:

$6^n = 1 + 5n + 5^2\, {}^nC_2 + 5^3\, {}^nC_3 + \ldots. + 5^n$

Now, bring 5n to the L.H.S, we get

$6^n - 5n = 1 + 5^2\, {}^nC_2 + 5^3\, {}^nC_3 + \ldots. + 5^n$

$6^n - 5n = 1 + 5^2\, ({}^nC_2 + 5\, {}^nC_3 + \ldots. + 5^{n-2})$

$6^n - 5n = 1 + 25\, ({}^nC_2 + 5\, {}^nC_3 + \ldots. + 5^{n-2})$

$6^n - 5n = 1 + 25\, k$ (where $k = {}^nC_2 + 5\, {}^nC_3 + \ldots. + 5^{n-2}$)

The above form proves that, when $6^n - 5n$ is divided by 25, it leaves the remainder 1.

Hence, the given statement is proved.

Question 4:

Find the value of r, If the coefficients of $(r - 5)^{th}$ and $(2r - 1)^{th}$ terms in the expansion of $(1 + x)^{34}$ are equal.

Solution:

For the given condition, the coefficients of $(r - 5)^{th}$ and $(2r - 1)^{th}$ terms of the expansion $(1 + x)^{34}$ are ${}^{34}C_{r-6}$ and ${}^{34}C_{2r-2}$ respectively.

Since the given terms in the expansion are equal,

$^{34}C_{r-6} = {}^{34}C_{2r-2}$

From this, we can write it as either

r-6=2r-2

(or)

r-6=34 -(2r-2) [We know that, if $^{n}C_{r} = {}^{n}C_{p}$, then either r = p or r = n – p]

So, we get either r = – 4 or r = 14.

We know that r being a natural number, the value of r = – 4 is not possible.

Hence, the value of r is14.