Important Questions for Class 11 Maths Chapter 8 - Sequences and Series

Question 1:

The sums of n terms of two arithmetic progressions are in the ratio 5n+4: 9n+6. Find the ratio of their 18^{th} terms.

Solution:

Let a_1, a_2 and d_1, d_2 be the first term and the common difference of the first and second arithmetic progression respectively.

Then,

(Sum of n terms of the first A.P)/(Sum of n terms of the second A.P) = (5n+4)/(9n+6)

$\Rightarrow$ [(n/2)[$2a_1$+ (n-1)d_1]]/ [(n/2)[$2a_2$+ (n-1)d_2]]= (5n+4)/(9n+6)

Cancel out (n/2) both numerator and denominator on L.H.S

$\Rightarrow$[$2a_1$+ (n-1)d_1]/[$2a_2$+ (n-1)d_2]= (5n+4)/(9n+6) ...(1)

Now susbtitute n= 35 in equation (1), {Since (n-1)/2 = 17}

Then equation (1) becomes

$\Rightarrow$[$2a_1$+ $34d_1$]/[$2a_2$+ $34d_2$]= (5(35)+4)/(9(35+6)

$\Rightarrow$[a_1+ $17d_1$]/[a_2+ $17d_2$]= 179/321 ...(2)

Now, we can say that.

18th term of first AP/ 18th term of second AP = [a_1+ $17d_1$]/[a_2+ $17d_2$]....(3)

Now, from (2) and (3), we can say that,

18th term of first AP/ 18th term of second AP = 179/321

Hence, the ratio of the 18th terms of both the AP’s is 179:321.

Question 2:

Insert five numbers between 8 and 26 such that resulting sequence is an A.P.

Solution:

Assume that A_1, A_2, A_3, A_4, and A_5 are the five numbers between 8 and 26, such that the sequence of an A.P becomes 8, A_1, A_2, A_3, A_4, A_5, 26

Here, a= 8, l =26, n= 5

Therefore, 26= 8+(7-1)d

Hence it becomes,

26 = 8+6d

6d = 26-8 = 18

6d= 18

d = 3

A_1= a+d = 8+ 3 =11

A_2= a+2d = 8+ 2(3) =8+6 = 14

A_3= a+3d = 8+ 3(3) =8+9 = 17

A_4= a+4d = 8+ 4(3) =8+12 = 20

A_5= a+5d = 8+ 5(3) =8+15 = 23

Hence, the required five numbers between the number 8 and 26 are 11, 14, 17, 20, 23.

Question 3:

The 5th, 8th, and 11th terms of a GP are p, q and s respectively. Prove that $q^2 = ps$

Solution:

Given that:

5th term = P

8th term = q

11th term = s

To prove that: $q^2 = ps$

By using the above information, we can write the equation as:

$a_5 = ar^{5-1} = ar^4 = p$ ....(1)

$a_8 = ar^{8-1} = ar^7 = q$ ....(2)

$a_{11} = ar^{11-1} = ar^{10} = s$ ...(3)

Divide the equation (2) by (1), we get

$r^3 = q/p$ ...(4)

Divide the equation (3) by (2), we get

$r^3 = s/q$ ...(5)

Now, equate the equation (4) and (5), we get

$q/p = s/q$

It becomes, $q^2 = ps$

Hence proved.

Question 4:

Show that the sum of $(m + n)^{th}$ and $(m – n)^{th}$ terms of an A.P. is equal to twice the m^{th} term.

Solution:

Let a and d be the first term and the common difference of the A.P. respectively. It is known

that the k^{th} term of an A.P. is given by

$a_k = a + (k - 1)d$

Therefore, $a_{m+n} = a + (m+n - 1)d$

$a_{m-n} = a + (m-n - 1)d$

$a_m = a + (m-1)d$

Hence, the sum of $(m + n)^{th}$ and $(m – n)^{th}$ terms of an A.P is written as:

$a_{m+n} + a_{m-n} = a + (m+n - 1)d + a + (m-n - 1)d$

$= 2a + (m + n - 1 + m – n - 1)d$

$= 2a + (2m-2)d$

$= 2a + 2(m-1)d$

$= 2 [a + (m-1)d]$

$= 2 a_m$ [since $a_m = a + (m-1)d$]

Therefore, the sum of $(m + n)^{th}$ and $(m - n)^{th}$ terms of an A.P. is equal to twice the m^{th} term.

Question 5:

Find the sum of integers from 1 to 100 that are divisible by 2 or 5.

Solution:

The integers from 1 to 100, which are divisible by 2, are 2, 4, 6 ..... 100.

This forms an A.P. with both the first term and common difference equal to 2.

$\Rightarrow 100=2+(n-1)2$

$\Rightarrow n= 50$

Therefore, the sum of integers from 1 to 100 that are divisible by 2 is given as:

$2+4+6+...+100 = (50/2)[2(2)+(50-1)(2)]$

$= (50/2)(4+98)$

$= 25(102)$

$= 2550$

The integers from 1 to 100, which are divisible by 5, 10.... 100

This forms an A.P. with both the first term and common difference equal to 5.

Therefore, $100= 5+(n-1)5$

$\Rightarrow 5n = 100$

$\Rightarrow n= 100/5$

$\Rightarrow n= 20$

Therefore, the sum of integers from 1 to 100 that are divisible by 2 is given as:

$5+10+15+...+100= (20/2)[2(5)+(20-1)(5)]$

$= (20/2)(10+95)$

$= 10(105)$

$= 1050$

Hence, the integers from 1 to 100, which are divisible by both 2 and 5 are 10, 20, ... 100.

This also forms an A.P. with both the first term and common difference equal to 10.

Therefore, 100= 10+(n-1)10

⇒10n = 100

⇒ n= 100/10

⇒ n= 10

10+20+...+100= (10/2)[2(10)+(10-1)(10)]

= (10/2)(20+90)

= 5(110)

= 550

Therefore, the required sum is:

= 2550+ 1050 – 550

= 3050

Hence, the sum of the integers from 1 to 100, which are divisible by 2 or 5 is 3050.