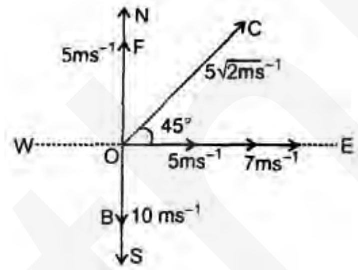
IMPORTANT QUESTIONS CLASS – 11 PHYSICS CHAPTER – 3 MOTION IN A PLANE



Question 1.
A body is acted upon by the following, velocities:
(i) 7 ms^{-1} due to E,
(ii) 10 ms^{-1} due S,
(iii) 52–√ ms^{-1} due N.E.
Find the magnitude and direction of the resultant velocity.

Answer:
Let OA, OB and OC represent the velocities given in the statement i.e.
OA = 7 ms^{-1}
OB = 10 ms^{-1}
and OC = 52–√ ms^{-1}
To find their resultant velocity, resolve OC into two rectangular components along east and north.

$\therefore$ Component of OC along East = $5\sqrt{2}\cos 45°$

$= 5\sqrt{2} \times \frac{1}{\sqrt{2}} = 5\ ms^{-1}$ represented by OD

Component of OC along north = $5\sqrt{2}\sin 45°$

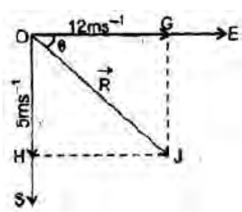
$= 5\sqrt{2} \times \frac{1}{\sqrt{2}} = 5\ ms^{-1}$ represented by OF.

Hence resultant velocity along east = 7 + 5 = 12 ms^{-1} and resultant velocity along south = OB – OF = 10 – 5 = 5 ms^{-1}.

If R be the resultant velocity, then the magnitude of R is obtained by applying the parallelogram law of vector addition as

$$R = \sqrt{OG^2 + OH^2} = \sqrt{(12)^2 + 5^2} = \sqrt{144 + 25} = 13ms^{-1}.$$

When OG = $12ms^{-1}$ and OH $5ms^{-1}$.



The direction of R: Let θ be the angle made by R with the east.
$\therefore$ in rt. ∠d ΔOGI,

$$\tan\theta = \frac{GI}{OG} = \frac{5}{12} = 0.4167 = \tan 22°37'$$

$$\therefore\ \theta = 22°37'.$$

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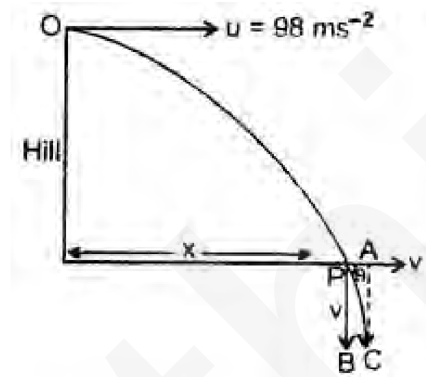
Question 2.

A projectile is fired horizontally with a velocity of 98 ms^{-1} from the top of a hill 490 m high. Find:
(i) the velocity with which it strikes the ground.
(ii) the time is taken to reach the ground.
(iii) the distance of the target from the hill.



Answer:

(i) h = 490 m, a = g = 9.8 ms^2
U_y = initial velocity along the y-axis at the top of the tower = 0

$$\therefore \text{using} \quad S = ut + \frac{1}{2}at^2, \text{ we get}$$
$$490 = 0 + \frac{1}{2}gt^2 = \frac{9.8}{2}t^2$$
$$\therefore \quad t = \sqrt{\frac{490}{4.9}} = \sqrt{100} = 10\text{s.}$$

(ii) Let v be the velocity along the y-axis with which the projectile hits the ground.

$$\therefore \quad v = u + at \text{ gives}$$
$$v = 0 + gt = 9.8 \times 10$$
$$= 98 \text{ ms}^{-1}$$

If V be the resultant velocity of hitting the ground

$$\text{Then} \quad V = \sqrt{u^2 + v^2} = \sqrt{(98)^2 + (98)^2}$$
$$= \sqrt{2(98)^2} = 98\sqrt{2}$$

Let θ be the angle made by V with the horizontal

$$\therefore \quad \tan\theta = \frac{AC}{PA} = \frac{v}{u} = \frac{gt}{u}\ \frac{98}{98} = 1$$
$$\therefore \quad \theta = 45^\circ$$

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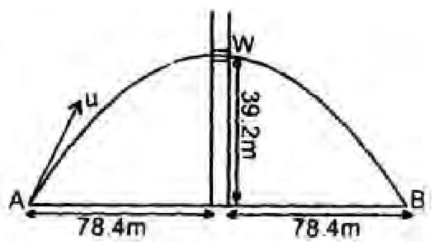
(iii) Let x, be the distance of the target from the hill.
∴ x = horizontal distance covered with u in a time t.
ut = 98 × 10 = 980 m.

Question 3.
A boy stands at 78.4 m from a building and throws a ball which just enters a window 39.2 m above the ground. Calculate the velocity of the projection of the ball.

Answer:
Let the boy standing at A throw a ball with initial velocity u.
θ = angle of the projection made with the horizontal.

As the boy is at 78.4 m from the building and the ball just enters above the ground.



∴ $h_m = 39.2$ m
R = horizontal range
= 2 × 78.4 m
= 156.8 m

Now we know that

$$R = \frac{u^2 \sin 2\theta}{g} = 2 \times 78.4 \quad .... (1)$$

$$h_m = \frac{u^2 \sin^2 \theta}{2g} = 39.2 \quad .... (2)$$

$\frac{(1)}{(2)}$ gives $\frac{R}{h_m} = 2 \times \frac{78.4}{39.2}$

$$= \frac{u^2 \sin 2\theta}{g} \times \frac{2g}{u^2 \sin^2 \theta}$$

or $4 = 2 \times \frac{2 \sin\theta \cos\theta}{\sin^2 \theta}$

or $4 = 4 \cot\theta = \frac{4}{\tan\theta}$

or $\tan\theta = \frac{4}{4} = 1 = \tan 45$

∴ $\theta = 45^{\circ}$

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$\therefore$ From (1) $\quad \frac{u^2 \sin 90}{9.8} = 2 \times 78.4$

or $\quad u = \sqrt{2 \times 9.8 \times 78.4} = \sqrt{39.2 \times 2 \times 19.6}$

$= \sqrt{(39.2)^2}$

$u = 39.2\ ms^{-1}$.

Question 4.
Two particles located at a point begin to move with velocities 4 ms^{-1} and 1 ms^{-1} horizontally in opposite directions. Determine the time when their velocity vectors become perpendicular. Assuming that the motion takes place in a uniform gravitational field of strength g.

Answer:
Let v_1 and v_2 be the velocities of first and 2nd particles respectively after a time t.
$\therefore v_1 = 4\hat{i} - gt\,\hat{j}$
$v_2 = -\hat{i} - gt\,\hat{j}$
For v_1 and v_2 to be ⊥ to each other, then their dot product must be zero.

i.e. $\quad v_1 \cdot v_2 = 0$

or $\quad (4\hat{i} - gt\hat{j}) \cdot (-\hat{i} - gt\hat{j}) = 0$

or $\quad 4(-1) + (-gt)(-gt) = 0$

or $\quad -4 + g^2t^2 = 0$

or $\quad g^2t^2 = 4$

or $\quad t^2 = \frac{4}{g^2} = \left(\frac{2}{g}\right)^2$

$\therefore \quad t = \frac{2}{g}$.

Question 5.

A body is projected with a velocity of 40 ms^{-1}. After two seconds, it crosses a verticle pole of 20.4 m. Find the angle of projection and the horizontal range.

Answer:
Here, u = 40 ms^{-1}
height of vertical pole, h = 20.4 m
t = 2 seconds
Let us take vertical motion

$$y = u_y t + \frac{1}{2} a_y t^2 \quad \text{.....(1)}$$

$\therefore \quad y = h = 20.4 \quad$ .....(2)

$t = 2s,\ u_y = u \sin\theta = 40 \sin\theta$

$a_y = -g = -9.8\ ms^{-2}$

$\therefore$ From (1) and (2), we get

$$20.4 = 40 \sin\theta \times 2 + \frac{1}{2}(-9.8) \times 2^2$$

$$= 80 \sin\theta - 19.6$$

or $\quad 80 \sin\theta = 20.4 + 19.6 = 40$

or $\quad \sin\theta = \frac{40}{80} = \frac{1}{2} = \sin 30^\circ$

or $\quad \theta = 30^\circ$

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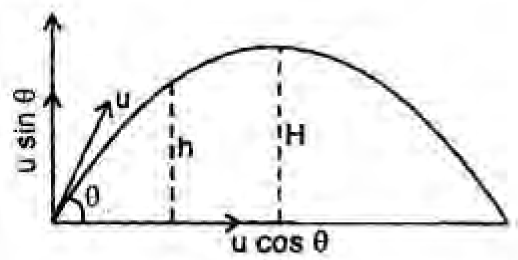
∴ The horizontal range is given by the relation,



$$R = \frac{u^2 \sin 2\theta}{g} = \frac{(40)^2 \sin 60^\circ}{9.8}$$

$$= \frac{1600}{9.8} \times \frac{\sqrt{3}}{2} = \frac{400\sqrt{3}}{4.9}$$

$$= 141.39 \text{ m.}$$

Question 6.
The greatest and the least resultant of two forces acting at a point are 29 N and 5 N respectively. If each force is increased by 3 N, find the resultant of two new forces when acting at a point at an angle of 90° with each other.

Answer:
Let A and B be the two forces.
∴ Greatest Resultant = A + B = 29 N ….(1)
least Resultant = A – B = 5 N ….(2)

Adding (1) and (2), we get

$$2\mathbf{A} = 34 \text{ N}$$

or $\mathbf{A} = 17 \text{ N}$

∴ from (1), $17 + \mathbf{B} = 29$

or $\mathbf{B} = 29 - 17 = 12 \text{ N}$

Let A and B be the new forces such that
A' = A + 3 = 17 + 3 = 20N and
B' = B + 3 = 12 + 3 = 15 N

Here, θ = angle between A' and B' = 90°
Let R be the resultant of A' and B'.
∴ according to parallelogram law of vector addition

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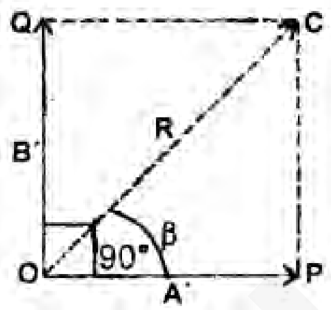
$$R = \sqrt{A'^2 + B'^2 + 2A'B'\cos 90^\circ}$$
$$= \sqrt{A'^2 + B'^2} = \sqrt{20^2 + 15^2}$$
$$= \sqrt{400 + 225} = \sqrt{625}$$
$$= \sqrt{(25)^2} = 25$$
$$\therefore \quad R = 25 \text{ N}$$

The direction of R:
Let β be the angle made by R with A'

$$\therefore \quad \tan\beta = \frac{B'}{A'} = \frac{15}{20} = 0.7500 = \tan 36^\circ.52'$$
$$\therefore \quad \beta = 36^\circ.52'.$$



Question 7.
An aircraft is trying to fly due north at a speed of 100 ms^{-1} but is subjected to a crosswind blowing from west to east at 50 ms^{-1}. What is the actual velocity of the aircraft relative to the surface of the earth?

Answer:
Let V_a and V_w be the velocities of aircraft and wind respectively.
∴ V_a = 100 ms^{-1} along N direction
V_w = 50 ms^{-1} along E direction

If V be the resultant velocity of the aircraft, then these may be represented as in the figure given below. So the magnitude of V is given by,

$$V = \sqrt{V_a^2 + V_w^2} = \sqrt{(100)^2 + (50)^2}$$
$$= \sqrt{10000 + 2500} = \sqrt{12500}$$
$$= 111.8 \text{ ms}^{-1}.$$

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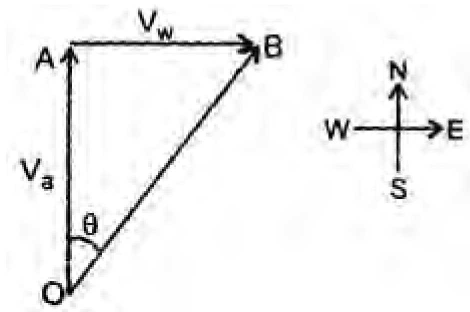


Let ∠AOB = θ be the angle which the resultant makes with the north direction.

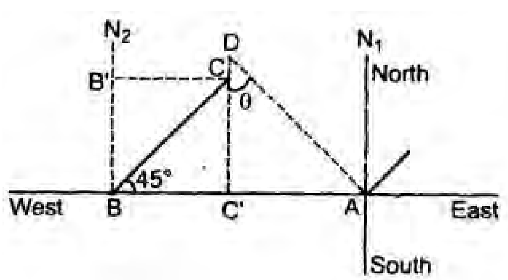
$$\therefore \quad \tan\theta = \frac{V_w}{V_a} = \frac{50}{100} = \frac{1}{2} = 0.5$$
$$= \tan 26°34'$$
$$\therefore \quad \theta = 26°34'.$$

Question 8.
Calculate the total linear acceleration of a particle moving in a circle of radius 0.4 m at the instant when its angular velocity is 2 rad s^{-1} and angular acceleration is 5 rad s^{-2}.

Answer:
Since the particle possesses angular acceleration, so its total linear acceleration (a) is the vector sum of the tangential acceleration (a,) and the centripetal acceleration (ac). a_1 and a_c, are at right angles to each other.
$a = \sqrt{a_{t}^{2}+a_{c}^{2}}$ .... (1)

Question 9.
An airplane flies 400 km west from city A to city B then 300 km north-east to city C and finally 100 km north to city D. How far is it from city A to city D? In what direction must the airplane go to return directly to the city A from city D?

Answer:
Given, AB = 400 km
BC = 300 km
CD = 100 km
AD =?



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Let N_1, N_2 represent north directions.
$\angle ABC = 45°$
Draw CC' ⊥ AB, And CB' ⊥ BN_2
Now in ΔBC' C

$$\frac{CC'}{BC} = \sin 45° = \frac{1}{\sqrt{2}}$$

$$\therefore \quad CC' = \frac{BC}{\sqrt{2}} = \frac{300}{\sqrt{2}} = 150\sqrt{2} = 212 \text{ km.}$$

∴ Distance C′D is given by

$$C'D = C'C + CD = 212 + 100 = 312 \text{ km.}$$

Also in ΔBC′C,

$$\frac{BC'}{BC} = \cos 45° = \frac{1}{\sqrt{2}}$$

$$\text{or} \quad BC' = \frac{BC}{\sqrt{2}} = \frac{300}{\sqrt{2}} = 150\sqrt{2} = 212 \text{ km.}$$

$$\therefore \quad AC' = AB - BC' = 400 - 212 = 188 \text{ km.}$$

From AAC'D, AD is given by

$$AD = \sqrt{(AC')^2 + (C'D)^2} = \sqrt{(188)^2 + (312)^2}$$

$$\text{or} \quad AD = 364 \text{ km}$$

Let $\angle C'DA = \theta = ?$

$$\therefore \quad \tan\theta = \frac{AC'}{DC'} = \frac{188}{312} = 0.6026 = \tan 31°$$

$$\therefore \quad \theta = 31° \text{ East of South.}$$

Question 10.
Which of the following quantities are independent of the choice of the orientation of the coordinates axes:
a + b, $3a_x + 2b_y$, [a + b – c], angle between b and c, a?

Answer:
a + b, |a + b – c|, angle between b and c, a are the quantities that are independent of the choice of the orientation of the coordinate axes.

But the value of $3a_x + 2b_y$ depends on the orientation of the axes.