IMPORTANT QUESTIONS CLASS – 11 PHYSICS CHAPTER – 4 LAWS OF MOTION

Question 1.
(a) State and prove impulse-momentum Theorem.

Answer:
It states that the impulse of force on a body is equal to the change in momentum of the body.
i.e. $J = F_t = P_2 - P_1$

Proof: From Newton's Second law of motion, we know that

$$\mathbf{F} = \frac{d\mathbf{p}}{dt} \text{ or } \mathbf{F}dt = d\mathbf{p} \quad \ldots (i)$$

Let P_1 and P_2 be the linear momenta of the body at time t = 0 and t respectively.
∴ integrating equation (i) within these limits, we get

$$\int_0^t \mathbf{F}\,dt = \int_{p_1}^{p_2} d\mathbf{p}$$

$$\text{or} \quad \mathbf{F}\int_0^t dt = \int_{p_1}^{p_2} \mathbf{p}^\circ\,d\mathbf{p}$$

$$\text{or} \quad \mathbf{F}[t]_0^t = [\mathbf{p}]_{p_1}^{p_2}$$

$$\mathbf{F}t = \mathbf{p}_2 - \mathbf{p}_1$$

$$\text{or} \quad \mathbf{J} = \mathbf{p}_2 - \mathbf{p}_1.$$

Hence proved.

(b) Prove that Newton's Second law is the real law of motion.

Answer:
Proof: If we can show that Newton's first and third laws are contained in the second law, then we can say that it is the real law of motion.
1. First law is contained in second law: According to Newton's second law of motion,
F = ma ...(i)
where m = mass of the body on which an external force F is applied and a = acceleration produced in it.

If F = 0, then from equation (1), we get
ma = 0, but as m ≠ 0
∴ a = 0

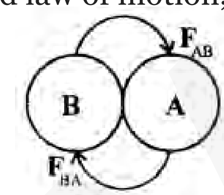
which means that there will be no acceleration in the body if no external force is applied. This shows that a body at rest will remain at rest and a body in uniform motion will continue to move along the same straight line in the absence of an external force. This is the statement of Newton's first law of motion. Hence, the First law of motion is contained in the Second law of motion.

2. Third law is contained in second law: Consider an isolated system of two bodies A and B. Let them act and react internally.
Let FAB = force applied on body A by body B
and FBA = force applied on body B by body A

It $\frac{\mathrm{d} \mathbf{p}_{\boldsymbol{A}}}{\mathrm{dt}}$ = rate of change of momentum of body A
and
$\frac{\mathrm{d} \mathbf{p}_{\boldsymbol{B}}}{\mathrm{dt}}$ = rate of change of momentum of body B
Then according to Newton's second law of motion,

$$\mathbf{F}_{AB} = \frac{d\mathbf{p}_A}{dt} \quad \text{.... (2)}$$

$$\mathbf{F}_{BA} = \frac{d\mathbf{p}_B}{dt} \quad \text{.... (3)}$$



(2) and (3) gives

$$\mathbf{F}_{AB} + \mathbf{F}_{BA} = \frac{d}{dt}(\mathbf{p}_A) + \frac{d}{dt}(\mathbf{p}_B)$$

$$= \frac{d}{dt}(\mathbf{p}_A + \mathbf{p}_B)$$

As no external force acts on the system (∵ it is isolated), therefore according to Newton's second law of motion,

$$\frac{d}{dt}(\mathbf{p}_1 + \mathbf{p}_2) = 0 \text{ or}$$

$$\mathbf{F}_{AB} + \mathbf{F}_{BA} = 0$$

or

$$\text{or,} \quad \mathbf{F}_{AB} = -\mathbf{F}_{BA}$$

Action = – Reaction,
which means that action and reaction are equal and opposite. It is the statement of Newton's 3rd law of motion. Thus 3rd law is contained in the second law of motion.

As both First and Third Law is contained in Second law, so Second law is the real law of motion.

Question 2.
Derive the general expression for the velocity of a rocket in flight and obtain the expression for the thrust acting on it.

Answer:
The working of a rocket is based upon the principle of conservation of momentum. Consider the flight of the rocket in outer space where no external forces act on it.

Let m_o = initial mass of rocket with fuel.
V_u = initial velocity of the rocket,
m = mass of the rocket at any instant t.
v = velocity of the rocket at that instant.
d_m = mass of the gases ejected by the rocket, in a small-time it.
u =H velocity of exhaust gases,
DV = increase in the velocity of the rocket in a time dt.

∴ Change in the momentum of exhaust gases = dm. u
Change in momentum of rocket = – (m – dm) dv.

A negative sign shows that the rocket is moving in a direction opposite to the motion of exhaust gases.

Applying the law of conservation of linear momentum,
dm.u = – (m – dm) dv ...(1)

As dm being very small as compared to m, so it can be neglected, Thus, eqn. (1) reduces to
dm.u = – m dv
or
$dv = -u \frac{dm}{m}$ ...(2)

Instantaneous velocity of the rocket:
At t = 0, mass of rocket = m0, velocity of rocket = v_o.
At t = t, mass of rocket = m, velocity of rocket = v.

∴ Integrating Eqn. (1) within these limits, we get

$$\int_{v_0}^{v} dv = -\int_{m_0}^{m} u \frac{dm}{m}$$

In actual practice, the velocity of exhaust gases nearly remains constant.

$$\therefore \quad \int_{v_0}^{v} dv = -u \int_{m_0}^{m} \frac{dm}{m}$$

$$\text{or} \quad [u]_{v_0}^{v} = -[\log_e m]_{m_0}^{m}$$

$$\text{or} \quad v - v_0 = -u(\log_e m - \log_e m_0)$$

$$= u(\log_e m_0 - \log m)$$

$$= u \log_e \left(\frac{m_0}{m}\right)$$

$$\text{or} \quad v = v_0 + u \log_e \left(\frac{m_0}{m}\right) \quad \text{....(3)}$$

equation (3) gives the instantaneous velocity of the rocket. In general $v_0 = 0$ at $t = 0$,

∴ Eqn. (3) reduces to

$$v = u \log_e \frac{m_0}{m} \quad \text{.... (4)}$$

From Eqn. (4), we conclude that the velocity of the rocket at any instant depends upon:

1. speed (u) of the exhaust gases.
2. Log of the ratio of initial mass (m0) of the rocket to its mass (m) at that instant of time.

Upthrust on the rocket (F): It is the upward force exerted on the rocket by the expulsion of exhaust gases. It is obtained as follows:
Dividing Eqn. (2) by dt, we get

$$\frac{dv}{dt} = -\frac{u}{m}\frac{dm}{dt}$$

or $$m\frac{dv}{dt} = -u\frac{dm}{dt}$$

But $$\frac{dv}{dt} = a, \text{ is the instantaneous acceleration}$$

∴ $$ma = -u\frac{dm}{dt}$$

or $$F = -u\frac{dm}{dt} \quad \text{.... (5)}$$

where F = ma is the instantaneous force (thrust).

From Eqn. (5), we conclude that the thrust (F) on the rocket at any instant is the product of the velocity of exhaust gases and the rate of combustion of fuel at that instant. Here negative sign shows that the thrust and velocity of exhaust gases are in opposite direction.

Question 3.
(a) Define inertia. What are its different types? Give examples.

Answer:
The tendency of bodies to remain in their state of rest or uniform motion along a straight line in the absence of an external force is called inertia.

Inertia is of the following three types:
1. The inertia of rest: When a body continues to lie at the same position with respect to its surrounding, it is said to possess inertia of rest. This situation may be changed only by the application of external force. For example, if a cot or sofa is lying in a particular place in the house, it will remain there even after days or years unless someone removes (by applying force) the same from its position. This is an example of the inertia of rest.

2. The inertia of motion: When a body is moved on a frictionless surface or a body is thrown in a vacuum, it will continue to move along its original path unless acted upon by an external force. In actual situations, air or floor etc. exert friction on the moving bodies so we are unable to visualize a force-free motion. This type of inertia when a body continues to move is called the inertia of direction.

3. In the above examples it is found that the direction of motion of the body or particle also does not change unless an external force acts on it. This tendency to preserve the direction of motion is called the inertia of direction.

(b) Explain Newton's First law of motion. Why do we call it the law of inertia?

Answer:

According to the First law of motion, "Everybody continues to be in the state of rest or of uniform motion along a straight line until it is acted upon by an external force."

It means that if a book lying on a table,-it will remain there for days or years together unless force is applied on it from outside to pick it.

Similarly, if a body is moving along a straight line with some speed, it will continue to do so until some external force is applied to it to change its direction of motion.

Thus First law tells us the following:

1. It tells us about the tendency of bodies to remain in the state of rest or of motion and the bodies by themselves can neither change the state of rest nor of uniform motion. This tendency is called inertia. To break the inertia of rest or motion or direction, we need an external force. Thus the definition of the first law matches with the definition of inertia and hence first law is called the law of inertia.
2. The first law of motion also provides the definition of another important physical quantity called force. Thus force is that agency which changes or tends to change the state of rest or of uniform motion of a body along a straight line.

(c) State Newton's Second law of motion. How does it help to measure force? Also, state the units of force.

Answer:

It states that the time rate of change of momentum of a body is directly proportional to the force applied to it.

$$
\begin{aligned}
\text{i.e. mathematically,} \quad F &\propto \frac{d}{dt}(\vec{p}) \\
&\propto \frac{d}{dt}(mv) \\
&\propto m\frac{dv}{dt} \\
&\propto ma \\
\text{or} \quad F &= kma \qquad \ldots (1)
\end{aligned}
$$

where $a = \frac{dv}{dt}$ = acceleration produced in the body of mass m.
k = proportionality constant which depends on the system of units chosen to measure F, m, and a.

In the S.I. system, k = l,
∴ F = ma

The magnitude of the force is given by
F = ma …. (2)

Note: We have assumed that the magnitude of velocity is smaller and much less than the speed of light. Only under this condition Eqns. (1) and (2) hold good.

The definition of the Second law and its mathematical form is given in Eqn. (2) provide us a mean of measuring force.
One can easily find the change in velocity of a body in a certain interval of time. Both velocity and time can be easily measured. Thus by knowing the mass of the body one can determine both change in momentum as well as the acceleration of the body produced by an external force. If the force is increased, the rate of change of momentum is also found to increase. So also is the acceleration. Now with known values of m and we can find F.

Units of force: Force in S.I. units is measured in newton or N. From Eqn. (1) or (2) we can see that a newton of force is that fore? which produces 1 ms^{-2} acceleration in the body of mass 1 kg.
1 newton = 1 kilogram × 1 metre/$(\text{second})^2$
or
1 N = 1 kg × 1 ms^{-2} = 1 kg ms^{-2}

In CGS system force is measured in dyne
1 dyne = 1 gram × 1 cm/s2 = 1 g cm s^{-2}
Since 1 N = 1 kgm s^{-2}= 1000 g × 100 cm s^{-2}
= 10^5 g cm s^{-2} = 105 dyne
1 N = 10^5 dyne
or
1 dyne = 10^5 N

Gravitational Unit: If a falling mass of 1 kg is accelerated towards the Earth with 9.8 ms^{-2}, then the force generated is called 1 kg wt (1-kilogram weight) force. It is the S.I. gravitational unit of force.

We know that the earth accelerates the mass with g = 9.8 ms^{-2}
1 Kg wt = 9.8 N [1 kg × 9.8 ms 2 = 9.8 N]

C.G.S. gravitational unit is gf or g wt.
1 gf = 1g × 980 cms^{-2}
= 980 dyne

Question 4.

A hunter has a machine gun that can fire 50 g bullets with a velocity of 150 ms^{-1}. A 60 kg tiger springs at him with a velocity of 10 ms^{-1}. How many bullets must the hunter fire into the tiger so as to stop him in his track?

Answer:
m = mass of bullet = 50 gm = 0.050 kg
M = mass of tiger = 60 kg
v = velocity of bullet = 150 ms^{-1}
V = velocity of tiger = – 10 ms^{-1}
(∵ it is coming from opposite direction) .

Let n = no. of bullets fired per second at the tiger so as to stop it.
∴ p_i = 0 before firing .... (i)
p_f = n(mv) + MV .... (ii)

∴ According to the law of conservation of momentum,

$$p_i = p_f$$

or $$0 = n(mv) + MV$$

or $$n = -\frac{MV}{mv} = \frac{-60 \times (-10)}{0.050 \times 150} = 80$$

Question 5.
A mass of 200 kg rests on a rough inclined plane of angle 300. If the coefficient of limiting friction is \frac{1}{\sqrt{3}} find the greatest and the least forces in newton, acting parallel to the plane to keep the mass in equilibrium.

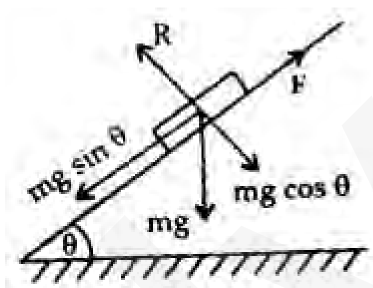
Answer:
Here, m = mass of body = 200 kg
Let angle of inclination = θ
μ_s = coefficient of limiting friction = \frac{1}{\sqrt{3}}

We know that $\mu_s = \tan\theta$

or $\tan\theta = \frac{1}{\sqrt{3}} = \tan 30^\circ$

or $\theta = 30^\circ$

The rectangular components of mg are as shown in fig.



Here mg sin θ acts along the plane in the downward direction and is given by

$$mg\sin\theta = 200 \times 9.8 \times \sin 30^\circ$$
$$= 200 \times 9.8 \times \frac{1}{2} = 980\text{ N}$$

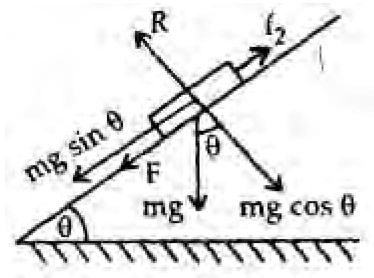
Also $R = mg\cos\theta$

and $F = \mu_s R = \mu_s\, mg\cos\theta$

$$= \frac{1}{\sqrt{3}} \times 200 \times 9.8 \times \frac{\sqrt{3}}{2}$$
$$= 980\text{ N}$$

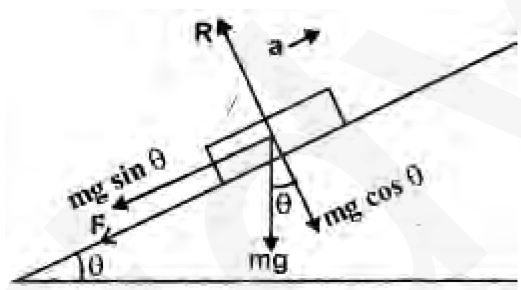
(i) the least forces in newton, acting parallel to the plane to keep the mass in equilibrium. given by
f_{21} =mg sinθ – F = 980 – 9800 = 0

(ii) The greatest force to be applied to keep the mass in equilibrium is given by
f_2 = mg sin θ + F = 980 + 980 = 1960 N.



Question 6.
Find the force required to move a train of 2000 quintals up an incline of 1 in 50, with an acceleration of 2 ms^{-2}, the force of friction being 0.5 newtons per quintal.

Answer:
Here, m = 2000 quintals
= 2000 × 100 kg (v 1 quintal = 100 kg)
$\sin\theta = \frac{1}{50}$, acceleration, a = 2 ms^{-2}
F = force of friction
= 0.5 N per quintal
= 0.5 × 2000 = 1000 N



In moving up an inclined plane, force required against gravity
= mg sin 0
$= 2000 \times 100 \times 9.8 \times \frac{1}{50}$
= 39200N.
Also if f = force required to produce acce. = 2 ms^{-2}.
Then f = ma = 200000 × 2 = 400000 N

∴Total force required = F + mg sin θ + f
= 1000 + 39200 + 400000 = 440200 N.

Question 7.
A bullet of mass 0.01 kg is fired horizontally into a 4 kg wooden block at rest on a horizontal surface. The coefficient of kinetic friction between the block and the surface is 0.25. The bullet remains embedded in the block and the combination moves 20 m before coming to rest. With what speed did the bullet strike the block?

Answer:
Here, m_1 = mass of the bullet = 0.01 kg
m_2 = mass of the wooden block = 4 kg
μ_2 = coefficient of kinetic friction = 0.25
initial velocity ofblock, u_2 = 0, s = distance moved by combination=20 m

Let u_1 = initial velocity of the bullet
If v = velocity of the combination, then according to the principle of conservation of linear momentum,

$$(m_1 + m_2)v = m_1u_1 + m_2u_2$$

or $$(0.01 + 4)v = 0.01 \times u_1 + 4 \times 0$$

$\therefore$ $$v = \frac{0.01}{4.01} u_1 = \frac{1}{401} u_1 \qquad \text{.... (1)}$$

If F = kinetic force of friction,
Then

$$\begin{aligned} F &= \mu R = \mu(m_1 + m_2)g \\ &= 0.25(0.01 + 4.00) \times 9.8 \\ &= 0.25 \times 4.01 \times 9.8 \end{aligned}$$

Then retardation 'a' produced is given by

$$\begin{aligned} a &= \frac{F}{m_1 + m_2} \\ &= 0.25 \times 4.01 \times \frac{9.8}{4.01} = 2.45 \text{ ms}^{-2} \end{aligned}$$

using the relation,

$$v^2 - u^2 = 2as, \text{ we get}$$
$$0 - v^2 = 2(-2.45) \times 20$$

or $$v^2 = 4.9 \times 20 = 98$$

$\therefore$ $$v = \sqrt{98} \text{ ms}^{-1}$$

From (1),

$$u_1 = 401v = 401 \times \sqrt{98} = 3969.7 \text{ ms}^{-1}.$$

Question 8.
A force of 100 N gives a mass m_1 an acceleration of 10 ms^{-2}, and of 20 ms^{-2} to a mass m_2. What acceleration would it give if both the masses are tied together?

Answer:
Let a = acceleration produced if m_1 and m_2 are tied together.
F = 100 N, Let a_1 and a_2 be the acceleration produced in m_1 and m_2 respectively.
$\therefore a_1 = 10\ ms^{-2}, a_2 = 20\ ms^{-2}$ (given)

Now $\qquad m_1 = \frac{F}{a_1}$ and $m_2 = \frac{F}{a_2}$

or $\qquad m_1 = \frac{100}{10} = 10\ ms^{-2}$

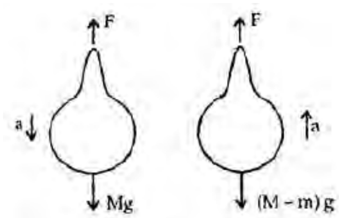
and $\qquad m_2 = \frac{100}{20} = 5\ ms^{-2}$

$\therefore \qquad m_1 + m_2 = 10 + 5 = 15$

Thus $\qquad a = \frac{F}{m_1 + m_2} = \frac{100}{15} = \frac{20}{3}$

$= 6.67\ ms^{-2}$.

Question 9.
A balloon with mass M is descending down with an acceleration 'a' < g. What mass m of its contents must be removed so that it starts moving up with an acceleration 'a'?
Answer:
Let F = retarding force acting on the balloon in the vertically upward direction.



When the balloon is descending down with an acceleration 'a', then the net force acting on the balloon in the downward direction is given by
Ma = Mg – F
or
F = Mg – Ma ....(i)
When the mass m is taken out of the balloon, then its weight is
= (M – m)g

Now as the balloon is moving upward with acceleration 'a', so the net force acting on the balloon in the upward direction is given by:

$$F - (M - m)g = (M - m)a$$

or $$Mg - Ma - (M - m)g = (M - m)a \text{ by using } (i)$$

or $$Mg - Ma - Mg + mg = Ma - ma$$

or $$m(g + a) = 2Ma$$

$$\therefore \quad m = \frac{2Ma}{a + g}$$

Question 10.
Three blocks are connected as shown below and are on a horizontal frictionless table. They are pulled to right with a force F = 50 N. If m_1 = 5 kg, m_2 = 10 kg and m_3 = 15 kg, find tensions T_3 and T_2.

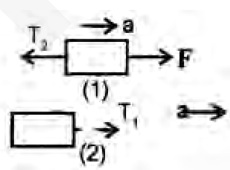
Answer:
Here, F = 50 N, m_1 = 5 kg, m_2 = 10 kg m_3 = 15 kg.

As the three blocks move with an acceleration 'a'

$$\therefore \quad a = \frac{F}{m_1 + m_2 + m_3}$$

$$= \frac{50}{5 + 10 + 15} = \frac{50}{30} = \frac{5}{3} \text{ ms}^{-2}$$

To determine T_2: Consider the free body diagram (1). Here F and T2 act towards the right and left respectively.



As the motion is towards the right side, so according to Newton's Second law of motion:

$$F - T_2 = m_3 a$$

or $$50 - T_2 = 15 \times \frac{5}{3} = 25$$

or $$T_2 = 50 - 25 = 25 \text{ N}$$

To determine T_3: Consider the free body diagram (2)

Here, $$m_1 a = T_1 \text{ or } T_1 = m_1 a$$

$$= 5 \times \frac{5}{3} = \frac{25}{3} = 8.33 \text{ N.}$$