Important Questions Class 12 Maths Chapter 1 Relations & Functions

Very Short Answer Questions [1 Mark Question]

Ques. If R = { (a, a^3): a is a prime number less than 5 } be a relation. Find the range of R.[Foreign 2014]

Ans. Given, R = {(a, a^3): a is a prime number less than 5} .

We know that 2 and 3 are the prime numbers less than 5.

Therefore, a can take values of 2 and 3.

R= {(2,2^3), (3,3^3)} = {(2,8), (3,27)}

Hence, the range of R is {8,27}.

Ques. If f: {1,3,4} {1,2,5} and g: {1,2,5} {1,3} given by f = {(1,2), (3,5), (4,1)} and g = {(1,3), (2,3), (5,1)). Write down gof. [All India 2014]

Ans. The functions f: {1,3,4} {1,2,5} and g: {1,2,5} {1,3} are defined as,

f = {(1,2), (3,5), (4,1)}

and, g = {(1,3), (2,3), (5,1)}

gof (1) = g (f(1)} = g(2) = 3 [since, f(1) = 2 and g(2) = 3]

gof(3) = g(f(3)) = g(5) = 1 [since, f(3) = 5 and g(5) = 1]

gof (4) = g(f(4)) = g(1) = 3 [since, f(4) = 1 and g(1) = 3]

Therefore, gof={(1,3), (3,1), (4,3)}.

Ques. Let R be the equivalence relation in the set A = {0,1,2,3,4,5} given by

R = {(a,b): 2 divides (a - b)}. Write the equivalence class [0]. [Delhi 2014]

Ans. Given, R = {(a, b):2 divides (a-b)}. Here, all even integers are related to zero, i.e. (0, 2),

(0, 4).

Hence, the equivalence class of [0] = {2,4} .

Ques. If A={1,2,3}, B = {4, 5, 6, 7} and f = {(1, 4), (2, 5), (3, 6)) is a function from A to B. State whether f is one-one or not. [All India 2011]

Ans. Given, A = {1, 2, 3} and B = {4, 5, 6, 7}

Now, f: A→ B is defined as f = {(1, 4), (2, 5), (3, 6)}

Therefore, f(1) = 4, f(2)= 5 and f(3) = 6.

It is seen that the images of distinct elements of A under f are distinct.So, f is one-one.

Ques. If: R R is defined by f(x) = 3x + 2, then define f[f(x)]. [Foreign 2011; Delhi 2010]

Ans. Given, f(x) = 3x + 2

Now, f[f(x)] = f(3x + 2) = 3(3x + 2) + 2 = 9x + 6 + 2 = 9x+8.

Ques. Write fog, if f: R→ R and g: R→ R are given by f(x) = |x| and g(x) = |5x-2|. [Foreign 2011].

Ans. Given, f(x)= |x|, g(x) = 5x - 21 Now, fog (x) = f[g(x)] = f{|5x - 21}

Short Answer Questions [2 Marks Questions]

Ques. If R= {(x, y): x + 2y = 8} is a relation on N, then write the range of R. [All India 2014]

Ans. Given, the relation R is defined on the set of natural numbers, i.e.,

N as R = {(x, y) : x + 2y = 8}

To find the range of R, x + 2y = 8 can be rewritten as y= 8-x2

On putting x = 2, we get y = 8-22= 3

On putting x = 4, we get y = 2

On putting x = 6, we get y = 1

As, x, y N, then R= {(2, 3) (4, 2) (6, 1)}. Hence, the range of relation is {3, 2, 1}.

Ques. If f is an invertible function, defined as f(x) = 3x - 4 / 5 then write $f^{-1}(x)$. [Foreign 2010]

Ans. We are given f(x)=3x-4/5 which is invertible.

Let,

y= 3x - 4/ 5

5y = 3x - 4

X= 5y + 4/ 3

f^{-1} (y) = 5y + 3/ 3 and f(x)= 5x +4/ 3

Ques.Show that the Signum Function f: R → R, given by

f(x)={ 1 for x greater than 0

0 for x=0 is neither one-one nor onto

-1 for x less than 0

Ans. In the given function f(x)={ 1 for x greater than 0

0 for x=0 is neither one-one nor onto

-1 for x less than 0

the value of f(x) is defined only for when x= -1, 0, 1.

For any other real number, say y=2, there is no corresponding element x. Therefore, the function is not an **onto function**.

Also, for any value of x, say f_1 or f_2, the value will be the same image, that is, 1, 0, or -1. Therefore, it is not a one to one function.

Hence, the given function is neither a one-one function nor an onto function.

Ques. State whether the function f: N→ N given by f(x) = 5x is injective, surjective or both. [All India 2008C, HOTS]

Ans. f: N→ N is given by f (x) = 5x

Let x_1, x_2 ∈ N such that f (x_1) = f (x_2)

∴ 5 x_1 = 5 x_2

⇒ x_1 x_2

∴ f is a one-one, or, an injective function.

Now, let $y=f(x)$

$\Rightarrow y=5x$

For $y=1$, x does not belong to N.

Therefore, the function is not an onto or surjective function.

Ques. If f: R→ R is defined by f(x) = (3-x³)1/3, then find fof(x). [All India 2010]

Ans. Given, function is $f:R\rightarrow R$ such that,

$f(x) = (3-x^3)^{1/3}$

Now, $fof(x) = f[f(x)] = f[(3 - x^3)^{1/3}]$

$= [3-\{(3-x^3)^{1/3}\}^3]^{1/3}$

$= [3-(3-x^3)]^{1/3} = (x^3)^{1/3}$

$= x$

Long Answer Questions [3 Marks Questions]

Ques. If f: R→ R is defined by f(x)=x²-3x + 2, find f(f(x)). [NCERT, MISC]

Ans. Given, $f(x)=x^2-3x+2$.

$\Rightarrow f(f(x)) = f(x)^2-3f(x) + 2.$

$=(x^2-3x + 2)^2-3(x^2-3x+2)+2$

Using $(a-b+c)^2=a^2 + b^2+c^2-2ab + 2ac-2ab$

$= (x^2)^2 + (3x)^2 +2^2- 2x2(3x) + 2x^2(2) - 2x^2(3x) - 3(x^2-3x + 2) + 2$

$=x^4 + 9x^2 + 4- 6x^3 - 12x + 4x^2 - 3x^2 +9x -6 + 2$

$=x^4 - 6x^3 + 9x^2 + 4x^2 - 3x^2 - 12x + 9x- 6 + 2 + 4$

$=x^4 - 6x^3 + 10x^2 - 3x.$

Ques. Given a non empty set X, consider P(X) which is the set of all subsets of X. Define the relation R in P(X) as follows: For subsets A, B in P(X), A R B if and only if A⊂B. Is R an equivalence relation on P(X)? Justify you answer.

Ans. A R B means $A \subset B$

Here, relation is

R= {(A, B): A & B are sets, $A \subset B$}

Reflexive

Since every set is a subset of itself,

This implies, $A \subset A$

$\Rightarrow (A, A) \in R$.

Therefore, R is reflexive.

Symmetric

To check whether symmetric or not,

If $(A, B) \in R$, then $(B,A) \in R$

If $(A, B) \in R$,

Then, $A \subset B$.

But, $B \subset A$ is not true

As all elements of A are in B, therefore, $A \subset B$.

But all elements of B are not in A.

So $B \subset A$ is not true.

Therefore, R is not symmetric.

Transitive

Since $(A, B) \in R$ & $(B, C) \in R$

If, $A \subset B$ and $B \subset C$, then $A \subset C$.

$\Rightarrow (A,C) \in R$

So, If $(A, B) \in R$ & $(B, C) \in R$, then $(A, C) \in R$

Therefore, R is transitive.

Hence, R is reflexive and transitive but not symmetric.

Therefore, R is not an equivalence relation since it is not symmetric.

Ques. Show that the relation R in the set {1, 2, 3} given by R= {(1, 1), (2, 2), (3, 3), (1, 2), (2, 3)} is reflexive but neither symmetric nor transitive.

Ans. R={(1, 1), (2, 2), (3, 3), (1, 2), (2, 3)}

Reflexive

If the relation is reflexive,then $(a, a)\in R$ for every $a\in(1, 2, 3)$.

Since $(1, 1)\in R$, $(2, 2)\in R$ & $(3, 3)\in R$.

Therefore, R is reflexive

Symmetric

To check whether symmetric or not, If $(a, b)\in R$, then $(b, a)\in R$.

Here $(1, 2)\in R$, but $(2, 1)\notin R$.

Therefore, R is not symmetric.

Transitive

To check whether transitive or not,

If $(a,b)\in R$ & $(b,c)\in R$, then $(a,c)\in R$.

Here, $(1, 2)\in R$ and $(2, 3)\in R$ but $(1, 3)\notin R$.

Therefore, R is not transitive.

Hence, R is reflexive but neither symmetric nor transitive.

Very Long Answer Questions [5 Marks Questions]

Ques. If f,g: $R\rightarrow R$ are two functions defined as $f(x) = |x|+x$ and $g(x) = |x|-x$, $\forall x\in R$, then, find fog and gof. [India 2014]

Ans. Here,

$f(x) = |x|+x$ and $g(x) = |x|-x$

$f(x)= \{ x+x$, if $x \geq 0$

$-x+x$, if $x < 0$

$f(x) = \{2x$, if $x \geq 0$

0, if $x<0$

$g(x) = \{ x-x$,if $x \geq 0$

$-x-x$, if $x<0$

or

$g(x)= \{0$, if $x \geq 0$

$-2x$, if $x < 0$

Thus for $x \geq 0$, $(gof)(x) = g(2x) = 0$

and for $x < 0$, $(gof)(x) = g(0) = 0$

or

$(gof)(x) =0$ for all $x \in R$

Thus for $x \geq 0$, $(fog)(x) = f(0) = 2(0) = 0$

and for $x < 0$,

$(fog)(x) = f(-2x) = - 4x$

or

$(fog)(x) = \{ 0$, if $x \geq 0$

$-4x$, if $x < 0$

Ques. If A = R-{3} and B = R-{1}. Consider the function f: A→B defined by f(x) x-2/ x-3 for

all x∈A. Then show that f is bijective. Find $f^{-1}(x)$. [Delhi 2012,2014]

Ans. Given, function is f: A→B, where A= R-{3}

and B=R-{1}, such that f(x)= x-2/x-3.

For One-one

Let $f(x_1) = f(x_2)$, for all $x_1, x_2 \in A$

$\Rightarrow x_2-2/ x_1-3= x_2-2/ x_2-3$

$\Rightarrow (x_1 - 2) (x_2-3) = (x_1-2)(x_? - 3)$

$\Rightarrow x_1x_2 - 3x_1 -2x_2 + 6 = x_1x_2 - 3x_1 - 2x_2 +6$

$\Rightarrow - 3x_12x_2 = - 3x_1 - 2x_2$

$- 3 (x_1 - x_2)+2 (x_1 - x_2) = 0$

$- (x_1 - x_2) = 0$

Or, $x_1 - x_2= 0$

This implies, $x_1 = x_2$.

Since, $f(x_1) = f(x_2)$

$\Rightarrow x_1 = x_2$, for all $x_1, x_2 \in A$.

So, f(x) is a one-one function.

Onto

To show f(x) is onto, we show that range of f(x) and its codomain are same.

Now,

let. y =x-2 /x - 3

or, xy-3y=x-2

$\Rightarrow$xy - x = 3y - 2

$\Rightarrow$x(y-1) = 3y - 2

$\Rightarrow$x=3y-2/y-1 ……Eqn (1)

Since, x ∈R-{3}, for all y ∈ R- {1}, the range of f(x)=R-{1}.

Also, the given codomain of f(x) = R-{1}

Therefore, Range = Codomain.

Hence, f(x) is an onto function.

Therefore, f(x) is a **bijective function**.

From Eq. (i), we get,

f-1(y)= 3y - 2/ y - 1

⇒$f^{-1}(x)$ = 3x - 2/ x-1

which is the inverse function of f(x).

Ques. If A={1,2,3,...,9} and R be the relation in AxA defined by (a, b) R (c,d). If a +d=b+c for (a,b), (c,d) in Ax A. Prove that R is an equivalence relation, Also, obtain the equivalence class [(2, 5)]. [Delhi 2014]

Ans. Given, relation R defined by (a, b) R (c, d), if a+d=b+c for (a, b), (c, d) in A x A.

Here, A = {1, 2, 3, ..., 9}

We observe the following properties on R:

Reflexive

Let (1, 2) be an element of A X A.

Then, (1, 2) belongs to AxA

This implies, 1,2 belongs to A

Or, 1+2=2+1 [since, addition is commutative]

Or, (1, 2) R (1, 2)

Thus, (1, 2) R (1, 2), for all(1, 2) belonging to

AxA.

So, R is reflexive on A x A.

Symmetric

Let (1, 2), (3, 4)∈A × A such that (1, 2) R (3, 4).

Then, 1+ 4 = 2 + 3

⇒3+2 =4+1 [since, addition is commutative]

⇒ (3, 4) R (1, 2)

Thus, (1, 2) R (3, 4)

⇒ (3, 4) R (1, 2), for all (1, 2), (3, 4) belonging to AXA

So, R is symmetric on A x A.

Transitive

Let (1, 2), (3, 4), (5, 6)∈ A x A such that

(1, 2)R (3, 4) and (3, 4) R (5, 6).

Then,

(1, 2) R (3, 4)

Or, 1+4=2+3

Or, (3, 4) R (5, 6)

Or, 3+6=4+5

This implies, (1+4) + 3+ 6 = (2+3)+(4+5)

or, 1+6=2+5 = (1, 2) R (5, 6)

Thus, (1, 2) R (3, 4) and (3, 4) R (5, 6)

⇒ (1, 2) R (5, 6), (1, 2), (3, 4), (5, 6) = A × A

So, R is transitive on A x A.

Hence, it is an equivalence relation on A X A.

Ques. If Z is the set of all integers and R is the relation on Z defined as R={(a,b): a,b∈Z and a - b is divisible by 5}. Prove that R is an equivalence relation. [Delhi 2010, HOTS]

Ans. The given relation is R = {(a, b): a,b∈Z and a - b is divisible by 5}. We shall prove that R is reflexive, symmetric and transitive.

(i) Reflexive

As for any x ∈Z, we have x - x = 0 and 0 is divisible by 5.

⇒(x-x) is divisible by 5.

⇒(x,x)∈R for all X∈Z.

Therefore, R is reflexive.

(ii) Symmetric

As $(x, y) \in R$, where $(x, y) \in Z$.

$\Rightarrow (x - y)$ is divisible by 5. [by definition of R]

$\Rightarrow x - y = 5A$ for some $A \in Z$

$\Rightarrow y - x = 5(-A)$

$\Rightarrow (y - x)$ is also divisible by 5

Therefore, $(y,x) \in R$

Therefore, R is symmetric.

iii) Transitive

As $(x, y) \in R$, where $x, y \in Z$

$\Rightarrow (x - y)$ is divisible by 5.

$\Rightarrow x - y = 5A$ for some $A \in Z$

Again, for $(y, z) \in R$, where $y, z \in Z$

$\Rightarrow (y-z)$ is divisible by 5

$\Rightarrow y-z = 5B$ for some $B \in Z$

$(y-z)$ is divisible by 5

Now, $(x - y) + (y - z) = 5A + 5B$.

$\Rightarrow x-z = 5(A + B)$

$\Rightarrow (x-z)$ is divisible by 5 for some $A+B \in Z$

Therefore, R is transitive.

Thus, R is reflexive, symmetric and transitive. Hence, it is an equivalence relation.

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