Important Questions Class 12 Maths Chapter 10 Vector Algebra

1 Mark Questions

1. Is the measure of 5 seconds is scalar or vector?
Ans: Scalar

2. Find the sum of the vectors.

$$\vec{a}=\vec{i}-2\vec{j}+\vec{k}, \quad \vec{b}=-2\vec{i}+4\vec{j}+5\vec{k} \quad \vec{c}=\hat{i}-6\hat{j}-7\hat{k}$$

Ans: $\vec{d}=\vec{a}+\vec{b}+\vec{c}$
$=0\hat{i}-4\hat{j}-\hat{k}$

3. Find the direction ratios and the direction cosines of the vector

$$\vec{r}=2\hat{i}-7\hat{j}-3\hat{k}$$

Ans: D.R of $\vec{r}$ $are\ 2,-7,-3$

$$\left|\vec{r}\right|=\sqrt{4+49+9}=\sqrt{62}$$

D.C of

$$\vec{r}\ are\ \frac{2}{\sqrt{62}},\frac{-7}{\sqrt{62}},\frac{-3}{\sqrt{62}}$$

4. Find the angle between vectors

$$\vec{a}\ and\ \vec{b}\ if\left|\vec{a}\right|=\sqrt{3},\ \left|\vec{b}\right|=2 \quad \vec{a}.\vec{b}=\sqrt{6}$$

Ans: $\cos\theta = \frac{\vec{a}.\vec{b}}{|\vec{a}||\vec{b}|}$

$$= \frac{\sqrt{6}}{(\sqrt{3}).(2)} = \frac{\sqrt{2}\times\sqrt{3}}{\sqrt{3}.2} = \frac{\sqrt{2}}{\sqrt{2}.\sqrt{2}}$$

$$\cos\theta = \frac{1}{\sqrt{2}} \quad \theta = \pi/4$$

---

5. Vectors $\vec{a}$ *and* $\vec{b}$ **be such that**

$$|\vec{a}| = 3, \text{ and } |\vec{b}| = \frac{\sqrt{2}}{3},$$

then $\vec{a}\times\vec{b}$ **is a unit vector. Find angle between** $\vec{a}$ *and* $\vec{b}$ **.**

Ans:

$$|\vec{a}\times\vec{b}| = |\vec{a}||\vec{b}|\sin\theta$$

$$1 = 3\times\frac{\sqrt{2}}{3}\times\sin\theta$$

$\frac{1}{\sqrt{2}} = \sin\theta$

$\theta = \pi/4$

---

6. Is the measure of 10 Newton is scalar or vector.

Ans: Vector

---

7. Write two different vectors having same magnitude.

Ans: $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$

$$|\vec{a}| = \sqrt{1+4+9} = \sqrt{14}$$

$\vec{b} = 3\hat{i} + 2\hat{j} + 1\hat{k}$

$$\left|\vec{b}\right| = \sqrt{9+4+1} = \sqrt{14}$$

---

8. Find the direction ratios and the direction cosines of the vector $\vec{r} = \hat{i} + \hat{j} + \hat{k}$

Ans: D.R of $\vec{r}$ $are\ 1, 1, 1$

$$\vec{r} = \sqrt{1+1+1} = \sqrt{3}$$

$$D.C\ of\ are \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$$

---

9. Find

$$\left|\vec{a} - \vec{b}\right|\ if\ \left|\vec{a}\right| = 2,\ \left|\vec{b}\right| = 3\ and\ \vec{a}.\vec{b} = 4$$

Ans:

$$\left|\vec{a} - \vec{b}\right|^2 = \left(\vec{a} - \vec{b}\right).\left(\vec{a} - \vec{b}\right)$$

$$= \vec{a}.\vec{a} - \vec{a}.\vec{b} - \vec{b}.\vec{a} + \vec{b}.\vec{b}$$
$$= \left|\vec{a}\right| - 2\vec{a}.\vec{b} + \left|\vec{b}\right|^2$$

$= 4 - 2 \times 4 + 9$
$= 5$
$\left|\vec{a} - \vec{b}\right| = \sqrt{5}$

---

10. If

$$\vec{a} = 4\hat{i} + 3\hat{j} + 2\hat{k}\ \vec{b} = 3\hat{i} + 2\hat{k}\ find\ \left|\vec{b} \times 2\vec{a}\right|$$

Ans: $\vec{b} = 3\hat{i} + 2\hat{k}$

$$2\vec{a} = 8\hat{i} + 6\hat{j} + 4\hat{k}$$

$$\vec{b} \times 2\vec{a} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 0 & 2 \\ 8 & 6 & 4 \end{vmatrix}$$

$$= \hat{i}(0-12) - \hat{j}(12-16) + \hat{k}(18-0)$$

$$-12\hat{i} + 4\hat{j} + 18\hat{k}$$
$$\left|\vec{b} \times 2\vec{a}\right| = \sqrt{(-12)^2 + (4)^2 + (18)^2}$$

$= \sqrt{484}$
$= 22$

---

11. Is the measure of 20 m/s towards north is scalar or vector.
Ans: Vector
12.

$$\vec{a} = \hat{i} + 2\hat{j} \ \vec{b} = 2\hat{i} + \hat{j} \ Is \left|\vec{a}\right| = \left|\vec{b}\right|$$

Ans:

$$\left|\vec{a}\right| = \sqrt{(1)^2 + (2)^2} = \sqrt{5}$$

$$\left|\vec{b}\right| = \sqrt{(2)^2 + (1)^2} = \sqrt{5}$$

---

13. Find the direction ratios and the direction cosines of the vector $\vec{r} = \hat{i} + 2\hat{j} + 3\hat{k}$

Ans: D.R of $\vec{r}$ $are\ 1, 2, 3$

$$\left|\vec{r}\right| = \sqrt{1+4+9} = \sqrt{4}$$

D.C of

$$\vec{r}\ are\ \frac{1}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}}$$

---

14. Evaluate the product

$$\left(3\vec{a} - 5\vec{b}\right).\left(2\vec{a} + 7\vec{b}\right)$$

Ans:

$$\left(3\vec{a} - 5\vec{b}\right).\left(2\vec{a} + 7\vec{b}\right)$$

$$= 6\left|\vec{a}\right|^2 - 11\vec{a}.\vec{b} - 35\left|\vec{b}\right|^2$$

$$\left[\because \vec{a}.\vec{a} = \left|\vec{a}\right|^2\ \vec{b}.\vec{b} = \left|\vec{b}\right|^2\ \vec{a}.\vec{b} = \vec{b}.\vec{a}\right.$$

---

15. Find $\vec{a} \times \vec{b}$ **if**

$$\vec{a} = 2\hat{i} + \hat{j} + 3\hat{k},\ \vec{b} = 3\hat{i} + 5\hat{j} - 2\hat{k}$$

Ans:

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & 3 \\ 3 & 5 & -2 \end{vmatrix}$$

$$= \hat{i}(-2-15) - \hat{j}(-4-9) + \hat{k}(10-3)$$

$$= -17\hat{i} + 13\hat{j} + 7\hat{k}$$

---

16. Is the measure of 30 m/s towards north is scalar or vector.
Ans: Scalar

---

17. Compute the magnitude of

$$\vec{b} = 2\hat{i} - 7\hat{j} - 3\hat{k}$$

Ans:

$$\left|\vec{b}\right| = \sqrt{(2)^2 + (-7)^2 + (-3)^2}$$

$= \sqrt{4+49+9}$
$= \sqrt{62}$

---

18. Find the direction ratios and the direction cosines of the vector $\vec{r} = \hat{i} + 2\hat{j} - \hat{k}$
Ans: D.R of $\vec{r}$ *are* $1, 2, -1$

$$\vec{r} = \sqrt{1+4+1} = \sqrt{6}$$

D.C of

$$\vec{r} \text{ are } \frac{1}{\sqrt{6}}, \frac{2}{\sqrt{6}}, \frac{-1}{\sqrt{6}}$$

---

19. $\vec{a}$ **Is unit vector and**

$$(\vec{x} - \vec{a})(\vec{x} + \vec{a}) = 8,$$

Then find $\left|\vec{x}\right|$
Ans: $\left|\vec{a}\right| = 1$

$$\left(\vec{x}-\vec{a}\right).\left(\vec{x}+\vec{a}\right)=8$$
$$\left|\vec{x}\right|^2-1=8$$

$$\left|\vec{x}\right|^2=9$$
$$\left|\vec{x}\right|=3$$

---

20. Show that

$$\left(\vec{a}-\vec{b}\right)\times\left(\vec{a}+\vec{b}\right)=2\left(\vec{a}\times\vec{b}\right)$$

$\vec{a}\ and\ \vec{b}.$

Ans:

$$L.H.S=\left(\vec{a}-\vec{b}\right)\times\left(\vec{a}+\vec{b}\right)$$

$$=\vec{a}\times\vec{a}+\vec{a}\times\vec{b}-\vec{b}\times\vec{a}-\vec{b}\times\vec{b}$$
$$=0+\vec{a}\times\vec{b}+\vec{a}\times\vec{b}-0$$

$$=2\left(\vec{a}\times\vec{b}\right)\left[\begin{array}{l}\because \vec{a}\times\vec{b}=0\\ \vec{a}\times\vec{b}=-\vec{b}\times\vec{a}\end{array}\right.$$

Back to Top ↑Questions

4 Mark Questions

1. Find the unit vector in the direction of the sum of the vectors

$$\vec{a}=2\hat{i}+2\hat{j}-5\hat{k},\ \ \vec{b}=2\hat{i}+\hat{j}+3\hat{k}$$

Ans: Let $\vec{c}=\vec{a}+\vec{b}$

$$=\left(2\hat{i}+2\hat{j}-5\hat{k}\right)+\left(2\hat{i}+\hat{j}+3\hat{k}\right)$$

$$=4\hat{i}+3\hat{j}-2\hat{k}$$

$$\left|\vec{c}\right|=\sqrt{16+9+4}$$

$$=\sqrt{29}$$

The required unit vector is

$$\hat{c}=\frac{\vec{c}}{\left|\vec{c}\right|}$$

$$=\frac{4\hat{i}+3\hat{j}-2\hat{k}}{\sqrt{29}}$$

$$=\frac{4}{\sqrt{29}}\hat{i}+\frac{3}{\sqrt{29}}\hat{j}-\frac{2}{\sqrt{29}}\hat{k}$$

---

2. Show that the points

$$A\left(2\hat{i}-\hat{j}+\hat{k}\right),B\left(\hat{i}-3\hat{j}-5\hat{k}\right),C\left(3\hat{i}-4\hat{j}-4\hat{k}\right)$$

are the vertices of right angled triangle.
Ans:

$$\overrightarrow{AB}=-\hat{i}-2\hat{j}-6\hat{k}$$

$$\overrightarrow{BC}=2\hat{i}-\hat{j}+\hat{k}$$

$$\overrightarrow{CA}=-\hat{i}+3\hat{j}+5\hat{k}$$

$$\left|\overrightarrow{AB}\right|^2=41$$

$$\left|\overrightarrow{BC}\right|=6$$

$$\left|\overrightarrow{CA}\right|=35$$

$$\left|\overrightarrow{AB}\right|^2 = \left|\overrightarrow{BC}\right|^2 + \left|\overrightarrow{CA}\right|^2$$

Hence, the Δ is a right angled triangle.

---

3. Show that the points

$$A\left(-2\hat{i}+3\hat{j}+5\hat{k}\right), B\left(\hat{i}+2\hat{j}+3\hat{k}\right) and\ C\left(7\hat{i}-\hat{k}\right)$$

are collinear.
Ans:

$$\overrightarrow{AB} = 3\hat{i} - \hat{j} - 2\hat{k}$$

$$\overrightarrow{BC} = 6\hat{i} - 2\hat{j} - 4\hat{k}$$

$$\overrightarrow{CA} = 9\hat{i} - 3\hat{j} - 6\hat{k}$$

$$\left|\overrightarrow{AB}\right| = \sqrt{14}, \overrightarrow{BC} = 2\sqrt{14}$$

$$and \left|\overrightarrow{AC}\right| = 3\sqrt{14}$$

$$\left|\overrightarrow{AC}\right| = \left|\overrightarrow{AB}\right| + \left|\overrightarrow{BC}\right|$$

Hence points A, B, C are collinear.

---

4. If $\vec{a}, \vec{b}, \vec{c}$ are unit vector such that $\vec{a}+\vec{b}+\vec{c}=0$ find the value of $\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}$
Ans:

$$\left|\vec{a}\right| = 1, \left|\vec{b}\right| = 1, \left|\vec{c}\right| = 1,$$

$$\vec{a}+\vec{b}+\vec{c}=0 \quad (Given)$$

$$\vec{a}.\left(\vec{a}+\vec{b}+\vec{c}\right)$$
$$\vec{a}.\vec{a}+\vec{a}.\vec{b}+\vec{a}.\vec{c}=0$$

$$\left(\vec{a}\right)^2+\vec{a}.\vec{b}+\vec{a}.\vec{c}=0$$
$$1+\vec{a}.\vec{b}+\vec{a}.\vec{c}=0$$

$$\vec{a}.\vec{b}+\vec{a}.\vec{c}=-1--------(i)$$
similiorly

$$\vec{b}.\vec{a}+\vec{b}.\vec{c}=-1--------(ii)$$
again

$$\vec{c}.\vec{a}+\vec{c}.\vec{b}=-1--------(iii)$$
adding(i),(ii)and(iii)

$$2\left(\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}\right)=-3 \qquad \left[\vec{a}.\vec{b}=\vec{b}.\vec{a}\right.$$

$$\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}=-3/2$$

---

5. If

$$\vec{a}=2\hat{i}+2\hat{j}+3\hat{k}\ ,\ \vec{b}=-\hat{i}+2\hat{j}+\hat{k}\ ,\ \vec{c}=3\hat{i}+\hat{j}$$

are such that

$$\vec{a}+\lambda\vec{b} \text{ is } \perp \text{ to } \vec{c}$$

is then find the value of λ **.**

Ans:

$$\vec{a}+\lambda\vec{b}=\left(2\hat{i}+2\hat{j}+3\hat{k}\right)+\lambda\left(-\hat{i}+2\hat{j}+\hat{k}\right)$$

$$=(2-\lambda)\hat{i}+(2+2\lambda)\hat{j}+(3+\lambda)\hat{k}$$

$$\left(\vec{a}+\lambda\vec{b}\right).\vec{c}=0\left[\because \vec{a}+\lambda\vec{b}\perp\vec{c}\right.$$

$$\left[(2-\lambda)\hat{i}+(2+2\lambda)\hat{j}+(3+\lambda)\hat{k}\right].\left(3\hat{i}+\hat{j}\right)=0$$

$$3(2-\lambda)+(2+2\lambda)=0$$

$-\lambda=-8$

$\lambda=8$

---

6. Consider two point P and Q with position vectors

$$O\overline{P}=3\vec{a}-2\vec{b} \text{ and } O\overline{Q}=\vec{a}+\vec{b}$$

. Find the positions vector of a point R which divides the line joining P and Q in the ratio 2:1 (i) internally (ii) externally.

Ans: (i)

$$O\overline{R}=\frac{2\left(\vec{a}+\vec{b}\right)+1\left(3\vec{a}-2\vec{b}\right)}{2+1}$$

$=\frac{5\vec{a}}{3}$

(ii)

$$\overrightarrow{OR}=\frac{2\left(\vec{a}+\vec{b}\right)-\left(3\vec{a}-2\vec{b}\right)}{2-1}$$

$$=\frac{2\vec{a}+2\vec{b}-3\vec{a}+2\vec{b}}{1}$$

$=4\vec{b}-\vec{a}$

---

7. Show that the points A, B, C with position vectors

$$-2\vec{a}+3\vec{b}+5\vec{c},\ \vec{a}+2\vec{b}+3\vec{c}\ and\ 7\vec{a}-\vec{c}$$

respectively are collinear.
Ans:

$$\overrightarrow{OA}=-2\vec{a}+3\vec{b}+5\vec{c}$$

$$\overrightarrow{OB}=\vec{a}+2\vec{b}+3\vec{c}$$

$\overrightarrow{OC}=7\vec{a}-\vec{c}$

$$\overline{AB}=\overrightarrow{OB}-\overrightarrow{OA}=3\vec{a}-\vec{b}-2\vec{c}$$

$$\overline{BC}=\overline{OC}-\overline{OB}=6\vec{a}-2\vec{b}-4\vec{c}$$

$$=2\left(3\vec{a}-\vec{b}-2\vec{c}\right)$$

$\overrightarrow{BC}=2\overrightarrow{AB}$
Thus $\overline{AB}\parallel\overline{BC}$ but one point B is common to both vectors hence A, B, C are collinear.

---

8. Find a unit vector $\perp$ to each of the vectors

$$\left(\vec{a}+\vec{b}\right) and \left(\vec{a}-\vec{b}\right) where\ \vec{a}=\hat{i}+\hat{j}+\hat{k},\ \vec{b}=\hat{i}+2\hat{j}+3\hat{k}$$

Ans: A vector which is $\perp$ to both

$$\left(\vec{a}+\vec{b}\right) are \left(\vec{a}-\vec{b}\right)$$

is giving by

$$\left(\vec{a}+\vec{b}\right)\times\left(\vec{a}-\vec{b}\right)=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & 3 & 4\\ 0 & -1 & -2\end{vmatrix}$$

$$=-2\hat{i}+4\hat{j}-2\hat{k}$$
$$Let\ \vec{c}=-2\hat{i}+4\hat{j}-2\hat{k}$$

$$\left|\vec{c}\right|=\sqrt{4+16+4}$$

$=\sqrt{24}$
$=2\sqrt{6}$
Req. unit vector is

$$\frac{\vec{c}}{\left|\vec{c}\right|}=-\frac{1}{\sqrt{6}}\hat{i}+\frac{2}{\sqrt{6}}\hat{j}-\frac{1}{\sqrt{6}}\hat{k}$$

---

9. The scalar product of the vector $\hat{i}+\hat{j}+\hat{k}$ with a unit vector along the sum of vectors

$$2\hat{i}+4\hat{j}-5\hat{k},\ \text{and}\ \lambda\hat{i}+2\hat{j}+3\hat{k}$$

is equal the one. Find the value of λ
Ans:

$$\vec{a}=2\hat{i}+4\hat{j}-5\hat{k}$$

$\vec{b} = \lambda\hat{i} + \hat{j} + 3\hat{k}$

$$\vec{a} + \vec{b} = (2+\lambda)\hat{i} + 6\hat{j} - 2\hat{k}$$

Unit vector along

$\vec{a} + \vec{b} = \frac{\vec{a} + \vec{b}}{|\vec{a} + \vec{b}|}$

$$= \frac{(2+\lambda)\hat{i} + 6\hat{j} - 2\hat{k}}{\sqrt{(2+\lambda)^2 + (6)^2 + (-2)^2}}$$

$$= \frac{(2+\lambda)\hat{i} + 6\hat{j} - 2\hat{k}}{\sqrt{(2+\lambda)^2 + 40}}$$

$$ATQ \ \vec{c}.\left(\vec{a} + \vec{b}\right) = 1$$

$$\left(\hat{i} + \hat{j} + \hat{k}\right).\left(\frac{(2+\lambda)\hat{i} + 6\hat{j} - 2\hat{k}}{(2+\lambda)^2 + 40}\right) = 1$$

$$\frac{(2+\lambda) + 6 - 2}{\sqrt{(2+\lambda)^2 + 40}} = 1$$

$$2 + \lambda + 4 = \sqrt{(2+\lambda)^2 + 40}$$

sq.both site

$$\lambda^2 + 36 + 12\lambda = (2+\lambda)^2 + 40$$

$\lambda = 1$

10. Find the area of the Δ with vertices A (1, 1, 2) B (2, 3, 4) and C (1, 5, 5).

Ans: A (1, 1, 2) B(2, 3, 4) C (1, 5, 5)

$$\overline{OA} = \hat{i} + \hat{j} + 2\hat{k}$$

OB=2i^+3j^+4k^OB¯=2i^+3j^+4k^

OC=i^+5j^+5k^OC¯=i^+5j^+5k^

$$\overrightarrow{AB} = \overline{OB} - \overline{OA} = \hat{i} + 2\hat{j} + 2\hat{k}$$
$$\overrightarrow{AC} = \overline{OC} - \overline{OA} = 4\hat{j} + 3\hat{k}$$

$$\overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 2 \\ 0 & 4 & 3 \end{vmatrix}$$

$$= -2\hat{i} - 3\hat{j} + 4\hat{k}$$

$$Arae\ of\ \Delta ABC = \frac{1}{2}\left|\overrightarrow{AB} \times \overrightarrow{AC}\right|$$

$$= \frac{1}{2}\left|\sqrt{(-2)^2 + (-3)^2 + (4)^2}\right|$$

$$= \frac{1}{2}\sqrt{29}\ sq.unit$$

11. Show that the points A (1, -2, -8) B (5, 0, -2) and C (11, 3, 7) are collinear, and find the ratio in which B divides AC.

Ans:A (1, -2, -8), B (5, 0, -2), C (11, 3, 7)

$$\overline{OA} = 1\hat{i} - 2\hat{j} - 8\hat{k}$$

$$\overline{OB} = 5\hat{i} - 0\hat{j} - 2\hat{k}$$

$$\overline{OC} = 11\hat{i} + 3\hat{j} + 7\hat{k}$$

$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$
$= 4\hat{i} + 2\hat{j} + 6\hat{k}$

$$\overrightarrow{BC} = \overrightarrow{OC} - \overrightarrow{OB}$$
$$= 3\left(2\hat{i} + \hat{j} + 3\hat{k}\right)$$

$$= \frac{3}{2}\left(4\hat{i} + 2\hat{j} + 6\hat{k}\right)$$

$\overrightarrow{BC} = \frac{3}{2}\overline{AB}$

Thus $\overrightarrow{BC} \parallel \overrightarrow{AB}$ and one point B is common there fore A, B, C are collinear and B divides AC in 2:3.

---

12. Find a vector $\vec{d}$ which is $\perp$ to both $\vec{a}$ and $\vec{b}$ and $\vec{c}$. $\vec{d}$ =15

Let $\vec{a} = \hat{i} + 4\hat{j} + 2\hat{k}$

$$\vec{b} = 3\hat{i} - 2\hat{j} + 7\hat{k}$$

$\vec{c} = 2\hat{i} - \hat{j} + 4\hat{k}$
$\vec{a} = \hat{i} + 4\hat{j} + 2\hat{k}$

Ans:

$$\vec{b} = 3\hat{i} - 2\hat{j} + 7\hat{k}$$

$\vec{c} = 2\hat{i} - \hat{j} + 4\hat{k}$

$$Let\ \vec{d} = x\hat{i} + y\hat{j} + z\hat{k}$$

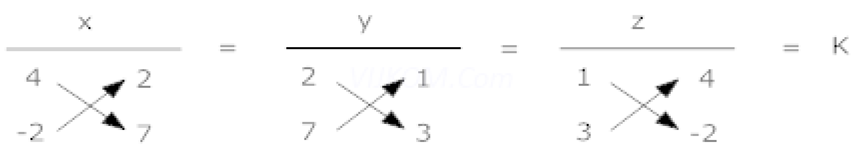
$$ATQ\ \vec{d}.\vec{a} = 0,\ \vec{d}.\vec{b} = 0\ and\ \vec{c}.\vec{d} = 15$$

$$x + 4y + 2z = 0 - - - - - - - - -(1)$$

$$3x - 2y + 7z = 0 - - - - - - - -(2)$$

$$2x - y + 4z = 15 - - - - - - - -(3)$$

On solving equation (i) and (ii)



$$\frac{x}{28+4} = \frac{y}{6-7} = \frac{z}{-2-12} = K$$

$$x = 32k, y = -k, z = -14k$$

Put x, y, z in equation (iii)

$$2(32k) - (-k) + 4(-14k) = 15$$
$$64k + k - 56k = 15$$
$$9k = 15$$

$$k=\frac{15}{9}$$
$$k=\frac{5}{3}$$

$$x=32\times\frac{5}{3}=\frac{160}{3}$$

$$y=-\frac{5}{3}=-\frac{5}{3}$$

$$z=-14\times\frac{5}{3}=-\frac{70}{3}$$

$$\vec{d}=\frac{160}{3}\hat{i}-\frac{5}{3}\hat{j}-\frac{70}{3}\hat{k}$$

---

13. Let $\vec{a},\vec{b}\ and\ \vec{c}$ be three vectors such that

$$\left|\vec{a}\right|=3,\ \left|\vec{b}\right|=4,\ \left|\vec{c}\right|=5$$

and each one of them being $\perp$ to the sum of the other two, find $\left|\vec{a}+\vec{b}+\vec{c}\right|$

Ans:

$$\vec{a}.\left(\vec{b}+\vec{c}\right)=0,\ \vec{b}.\left(\vec{c}+\vec{a}\right)=0\ \vec{c}.\left(\vec{a}+\vec{b}\right)=0,\left(Given\right)$$

$$\left|\vec{a}+\vec{b}+\vec{c}\right|^2=\left(\vec{a}+\vec{b}+\vec{c}\right).\left(\vec{a}+\vec{b}+\vec{c}\right)$$
$$=\vec{a}.\vec{a}+\vec{a}.\left(\vec{b}+\vec{c}\right)+\vec{b}.\vec{b}+\vec{b}.\left(\vec{a}+\vec{c}\right)+\vec{c}.\vec{c}+\left(\vec{a}+\vec{b}\right)$$

$$=\left|\vec{a}\right|^2+\left|\vec{b}\right|^2+\left|\vec{c}\right|^2$$
$$=9+16+25$$

$$= 50$$
$$\left|\vec{a}+\vec{b}+\vec{c}\right| = \sqrt{50}$$

$$= 5\sqrt{2}$$

---

14. If $\vec{a} = 4\hat{i}+2\hat{j}-\hat{k}$

$$\vec{b} = 5\hat{i}+2\hat{j}-3\hat{k}$$

Find the angel between the vectors

$$\vec{a}+\vec{b} \text{ and } \vec{a}-\vec{b}$$

Ans:

$$\vec{a}+\vec{b} = 9\hat{i}+4\hat{j}-4\hat{k}$$

$$\vec{a}-\vec{b} = -\hat{i}+2\hat{k}$$

$$\left(\vec{a}+\vec{b}\right).\left(\vec{a}-\vec{b}\right) = -17$$

$$\left|\vec{a}+\vec{b}\right| = \sqrt{113}$$
$$\left|\vec{a}-\vec{b}\right| = \sqrt{5}$$

$$\cos\theta = \frac{\left(\vec{a}+\vec{b}\right).\left(\vec{a}-\vec{b}\right)}{\left|\vec{a}+\vec{b}\right|\left|\vec{a}-\vec{b}\right|}$$

$$= \frac{-17}{\sqrt{113}.\sqrt{5}}$$
$$\cos\theta = \frac{-17}{\sqrt{565}}$$

$$\theta = \cos^{-1}\left(\frac{-17}{\sqrt{565}}\right)$$

---

15. Find the sine of the angel between the vectors.

$\vec{a} = 2\hat{i} - \hat{j} + 3\hat{k}$

$\vec{b} = \hat{i} + 3\hat{j} + 2\hat{k}$

Ans:

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & 3 \\ 1 & 3 & 2 \end{vmatrix}$$

$= -11\hat{i} - \hat{j} + 7\hat{k}$

$$\left|\vec{a} \times \vec{b}\right| = \sqrt{(-11)^2 + (-1)^2 + (7)^2}$$

$$= \sqrt{171} = 3\sqrt{19}$$

$$\sin\theta = \frac{\left|\vec{a} \times \vec{b}\right|}{\left|\vec{a}\right|\left|\vec{b}\right|} = \frac{3\sqrt{19}}{\sqrt{14}.\sqrt{14}} = \frac{3}{14}\sqrt{19}$$

---

16. Three vectors $\vec{a}, \vec{b}$ *and* $\vec{c}$ **satisfy the condition** $\vec{a} + \vec{b} + \vec{c} = 0$ **Evaluate the quantity**

$$\mu = \vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a}$$

if

$$\left|\vec{a}\right| = 1,\ \left|\vec{b}\right| = 4,\ \left|\vec{c}\right| = 2$$

Ans: $\vec{a} + \vec{b} + \vec{c} = 0$

$$\vec{a}.\left(\vec{a} + \vec{b} + \vec{c}\right) = 0$$

$$\vec{a}.\vec{a} + \vec{a}.\vec{b} + \vec{a}.\vec{c} = 0$$

$$\vec{a}.\vec{b}+\vec{a}.\vec{c}=-\left|\vec{a}\right|^2$$
$$\vec{a}.\vec{b}+\vec{a}.\vec{c}=-1--------(i)$$

$$\vec{b}.\vec{a}+\vec{b}.\vec{c}=-16------(ii)$$

$$\vec{a}.\vec{c}+\vec{b}.\vec{c}=-4-----(iii)$$

Adding (i) (ii) and (iii)

$$2\left(\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{a}.\vec{c}\right)=-21$$

$$\mu=\frac{-21}{2}$$

---

17. If with reference to the right handed system of mutually $\perp$ unit vectors $\hat{i}$, $\hat{j}$, $\hat{k}$ *and* $\hat{k}$,

$$\vec{\alpha}=3\hat{i}-\hat{j},\ \vec{\beta}=2\hat{i}+\hat{j}-3\hat{k}$$

then express $\vec{\beta}$ in the form $\vec{\beta}=\vec{\beta_1}+\vec{\beta_2}$, where $\vec{\beta_1}$ is || to $\vec{\alpha}$ and $\vec{\beta_2}$ is $\perp$ to $\vec{\alpha}$

Ans:

$$Let\ \vec{\beta}_1=\lambda\vec{\alpha}\qquad\left[\because\vec{\beta}_1 \parallel to\,\vec{\alpha}\right.$$

$$\vec{\beta}_1=\lambda\left(3\hat{i}-\hat{j}\right)$$
$$=3\lambda\hat{i}-\lambda\hat{j}$$

$$\vec{\beta}_2=\vec{\beta}-\vec{\beta}_1$$
$$=\left(2\hat{i}+\hat{j}-3\hat{k}\right)-\left(3\lambda\hat{i}-\lambda\hat{j}\right)$$

$$= (2-3\lambda)\hat{i} + (1+\lambda)\hat{j} - 3\hat{k}$$

$$\vec{\alpha}.\vec{\beta}_2 = 0 \qquad [\because \vec{\beta}_2 \perp \vec{\alpha}$$

$$3(2-3\lambda) - (1+\lambda) = 0$$

$$\lambda = \frac{1}{2}$$

$$\vec{\beta}_1 = \frac{3}{2}\hat{i} - \frac{1}{2}\hat{j}$$

$$\vec{\beta}_2 = \frac{1}{2}\hat{i} + \frac{3}{2}\hat{j} - 3\hat{k}$$

---

18. If $\vec{a}, \vec{b}$ *and* $\vec{c}$ be three vectors such that $\vec{a} + \vec{b} + \vec{c} = 0$ and

$$|\vec{a}| = 3, \ |\vec{b}| = 5, \ |\vec{c}| = 7$$

find the angle between $\vec{a}$ *and* $\vec{b}$.

Ans: $\vec{a} + \vec{b} + \vec{c} = 0$

$\vec{a} + \vec{b} = -\vec{c}$

$$(\vec{a}+\vec{b}).(-\vec{c}) = -\vec{c}.(-\vec{c})$$

$$(\vec{a}+\vec{b}).(\vec{a}+\vec{b}) = \vec{c}.\vec{c}$$

$$|\vec{a}|^2 + 2\vec{a}\vec{b} + |\vec{b}|^2 = |\vec{c}|^2$$

$$\vec{a}.\vec{b} = \frac{49-9-25}{2} = \frac{15}{2}$$

$$\cos\theta = \frac{\vec{a}.\vec{b}}{|\vec{a}||\vec{b}|}$$

$$= \frac{1}{2}$$

$$\theta = 60$$

---

19. Find the area of the ||gm whose adjacent sides are represented by the vectors,

$$\vec{a} = 3\hat{i} + \hat{j} - 2\hat{k},\ \vec{b} = \hat{i} - 3\hat{j} + 4\hat{k}$$

Ans:

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & -2 \\ 1 & -3 & 4 \end{vmatrix}$$

$$= -2\hat{i} - 14\hat{j} - 10\hat{k}$$

$$req.area = \left|\vec{a} \times \vec{b}\right|$$

$$= \sqrt{(-2)^2 + (-14)^2 + (-10)^2} = 10\sqrt{3}$$

---

20. Find the vector joining the points P (2, 3, 0) and Q (-1, -2, -4) directed from P to Q. Also find direction ratio and direction cosine.

Ans:

$$\overrightarrow{PQ} = (-1-2)\hat{i} + (-2-3)\hat{j} + (-4-0)\hat{k}$$

$$= -3\hat{i} - 5\hat{j} - 4\hat{k}$$

$$DR\ are -3, -5, -4$$

$$\left|\overrightarrow{PQ}\right| = \sqrt{9 + 25 + 16}$$

$$D.C\ are \frac{-3}{\sqrt{50}}, \frac{-5}{\sqrt{50}}, \frac{-4}{\sqrt{50}}$$