Important Questions Class 12 Maths Chapter 11 Three Dimensional Geometry

1 Mark Questions

1. Find the directions cosines of x, y and z axis.
Ans. 1,0,0, 0,1,0 0,0,1

2.Find the vector equation for the line passing through the points (-1,0,2) and (3,4,6)
Ans. Let $\vec{a}$ and $\vec{b}$ be the p.v of the points A (-1,0,2) and B (3, 4 6)

$$\vec{r} = \vec{a} + \lambda(\vec{b} - \vec{a})$$
$$= (-\hat{i} + 2\hat{j}) + \lambda(4\hat{i} + 4\hat{j} + 4\hat{k})$$

3.Find the angle between the vector having direction ratios 3,4,5 and 4, -3, 5.
Ans. Let $a_1 = 3$, $b_1 = 4$, $c_1 = 5$ and $a_2 = 4$, $b_2 = -3$, $c_2 = 5$

$$\cos\theta = \left(\frac{a_1 \quad a_2 + b_1 \quad b_2 + c_1 \quad c_2}{\sqrt{a_1^{\,2} + b_1^{\,2} + c_1^{\,2}} \quad \sqrt{a_2^{\,2} + b_2^{\,2} + c^2{}_2}}\right) = \frac{1}{2}$$
$$\theta = 60^0$$

4. What is the direction ratios of the line segment joining P(x_1 y_1 z_1) and Q (x_2 y_2 z_2)
Ans. $x_2 - x_1$, $y_2 - y$, and z_2-z_1 are the direction ratio of the line segment PQ.

5. The Cartesian equation of a line is

$$\frac{x+3}{2} = \frac{y-5}{4} = \frac{z+6}{2}$$

Find the vector equation for the line.
Ans. Comparing the given equation with the standard equation form

$$\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c}$$

$$\vec{r} = (-3\hat{i} + 5\hat{j} + 6\hat{k}) + \lambda(2\hat{i} + 4\hat{j} + 2\hat{k})$$

6.Show that the lines

$$\frac{x+3}{-3}=\frac{y-1}{1}=\frac{z-5}{5}\ and\quad \frac{x+1}{-1}=\frac{y-2}{2}=\frac{z-5}{5}$$

are coplanar.

Ans. x_1=-3, y_1 = 1, z_1 = 5

a_1 = -3, b_1=1, c_1= 5

x_2 = -1, y_2=2, z_2 = 5

a_2 = -1, b_2 = 2, c_2 = 5

$$\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}=\begin{vmatrix} 2 & 1 & 0 \\ -3 & 1 & 5 \\ -1 & 2 & 5 \end{vmatrix}=0$$

Therefore lines are coplanar.

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7. If a line has the direction ratios -18, 12, -4 then what are its direction cosines

Ans. a = -18, b=12, c= -4

$a^2+b^2+c^2 = (-18)^2 + (12)^2 + (-4)^2$

= 484

$$l=\frac{-18}{\sqrt{484}}=\frac{-18}{22}=\frac{-9}{11}$$

$$m=\frac{12}{22}=\frac{6}{11}$$

$$n=\frac{-4}{22}=\frac{-2}{11}$$

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8. Find the angle between the pair of line given by

$$\vec{r}=3\hat{i}+2\hat{j}-4\hat{k}+\lambda\left(\hat{i}+2\hat{j}+2\hat{k}\right)$$

$$\vec{r}=5\hat{i}-2\hat{j}+\mu(3\hat{i}+2\hat{j}+6\hat{k})$$

Ans.

$$\vec{b}_1 = \hat{i} + 2\hat{j} + 2\hat{k}$$
$$\vec{b}_2 = 3\hat{i} + 2\hat{j} + 6\hat{k}$$

$$\cos\theta = \left|\frac{\vec{b}_1.\vec{b}_2}{|\vec{b}_1||\vec{b}_2|}\right| = \frac{19}{21}$$

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9. Prove that the points A(2,1,3) B(5, 0,5)and C(-4, 3,-1) are collinear
Ans. The equations of the line AB are

$$\frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2-y_1} = \frac{z-z_1}{z_2-z_1}$$

$$\frac{x-2}{5-2} = \frac{y-1}{0-1} = \frac{z-3}{5-3}$$

$$\frac{x-2}{3} = \frac{y-1}{-1} = \frac{z-3}{2} \qquad (1)$$

If A, B, C are collinear, C lies in equation (1)

$$\frac{-4-2}{3} = \frac{3-1}{-1} = \frac{-1-3}{2}$$

$-2 = -2 = -2$
Hence A,B,C are collinear

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10. Find the direction cosines of the line passing through the two points (2,4,-5) and (1,2,3).
Ans. Let P(-2,4,-5) Q (1,2,3)

$$PQ = \sqrt{(1+2)^2 + (2-4)^2 + (3+5)^2}$$

$= \sqrt{9+4+64}$

$$= \sqrt{77}$$

the direction cosines of the line
Joining two point is

$$\frac{1+2}{\sqrt{77}} : \frac{2-4}{\sqrt{77}} : \frac{3+4}{\sqrt{77}}$$

$$\frac{3}{\sqrt{33}} : \frac{-2}{\sqrt{77}} : \frac{8}{\sqrt{77}}$$

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11. Find the equation of the plane with intercepts 2,3 and 4 on the x, y and z axis respectively.

Ans. Let the equation of the plane be

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$$

$$\frac{x}{2} + \frac{y}{3} + \frac{z}{4} = 1$$

$$6x + 4y + 3z = 12$$

12.If the equations of a line AB is

$$\frac{x-3}{1} = \frac{y+2}{-2} = \frac{z-5}{4}$$

find the directions ratio of line parallel to AB.

Ans.

$$\frac{x-3}{1} = \frac{y+2}{-2} = \frac{z-5}{4}$$

the direction ratios of a line parallel to AB are 1, -2, 4

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13. If the line has direction ratios 2,-1,-2 determine its direction Cosines.

Ans.

$$\frac{2}{\sqrt{(2)^2+(-1)^2+(-2)^2}} : \frac{-1}{\sqrt{(2)^2+(-1)^2+(-2)^2}} : \frac{-2}{\sqrt{(2)^2+(-1)^2+(-2)^2}} = \frac{2}{3} : \frac{-1}{3} : \frac{-2}{3}$$

14. The Cartesian equation of a line is

$$\frac{x-5}{3}=\frac{y+4}{7}=\frac{z-6}{2}$$

. Write its vector form
Ans. $\vec{r}=\vec{a}+\lambda\vec{b}$

$$\vec{a}=5\hat{i}-4\hat{j}+6\hat{k}$$

$$\vec{b}=3\hat{i}+7\hat{j}+2\hat{k}$$

$$\vec{r}=(5\hat{i}-4\hat{j}+6\hat{k})+\lambda(3\hat{i}+7\hat{j}+2\hat{k})$$

15. Cartesian equation of a line AB is

$$\frac{2x-1}{2}=\frac{4-y}{7}=\frac{z+1}{2}$$

write the direction ratios of a line parallel to AB.
Ans. Given equation of a line can be written is

$$\frac{x-\frac{1}{2}}{1}=\frac{y-4}{-7}=\frac{z+1}{2}$$

The direction ratios of a line parallel to AB are 1, -7, 2.

4 Mark Questions

1. Find the vector and Cartesian equation of the line through the point (5, 2,-4) and which is parallel to the vector $3\hat{i}+2\hat{j}-8\hat{k}$
Ans:

$$\vec{a}=5\hat{i}+2\hat{j}-4\hat{k},\ \vec{b}=3\hat{i}+2\hat{j}-8\hat{k}$$

Vector equation of line is

$\vec{r} = \vec{a} + \lambda \vec{b}$

$$= 5\hat{i} + 2\hat{j} - 4\hat{k} + \lambda(3\hat{i} + 2\hat{j} - 8\hat{k})$$

Cartesian equation is

$$x\hat{i} + y\hat{j} + z\hat{k} = 5\hat{i} + 2\hat{j} - 4\hat{k} + \lambda(3\hat{i} + 2\hat{j} - 8\hat{k})$$

$$x\hat{i} + y\hat{j} + z\hat{k} = (5 + 3\lambda)\hat{i} + (2 + 2\lambda)\hat{j} + (-4 - 8\lambda)\hat{k}$$

$$x = 5 + 3\lambda, y = 2 + 2\lambda, z = -4 - 8\lambda$$

$$\frac{x-5}{3} = \frac{y-2}{2} = \frac{z+4}{-8}$$

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2. Find the angle between the lines

$$\vec{r} = (3\hat{i} + \hat{j} - 2\hat{k}) + \lambda(\hat{i} - \hat{j} - 2\hat{k})$$

$$\vec{r} = (2\hat{i} - \hat{j} - 56\hat{k}) + \mu(3\hat{i} - 5\hat{j} - 4\hat{k})$$

Ans:

Let θ is the angle between the given lines

$$\vec{b}_1 = \hat{i} - \hat{j} - 2\hat{k} \; and \; \vec{b}_2 = 3\hat{i} - 5\hat{j} - 4\hat{k}$$

$$\cos\theta = \left| \frac{\vec{b}_1 \cdot \vec{b}_2}{|\vec{b}_1||\vec{b}_2|} \right|$$

$$\left|\frac{(\hat{i}-\hat{j}-2\hat{k})\cdot(3\hat{i}-5\hat{j}-4\hat{k})}{\left|\hat{i}-\hat{j}-2\hat{k}\right|\left|3\hat{i}-5\hat{j}-4\hat{k}\right|}\right|$$

$$=\left|\frac{3+5+8}{\sqrt{6}\sqrt{50}}\right|=\frac{16}{\sqrt{50}}$$

$$\frac{16}{\sqrt{6}\quad 5\sqrt{2}}$$

$$\frac{16}{\sqrt{2}\times\sqrt{3}\times\sqrt{5}\times\sqrt{2}}\times\frac{\sqrt{3}}{\sqrt{3}}$$

$$=\frac{8\cancel{16}\sqrt{3}}{\cancel{2}\times3\times5}$$

$$\cos\theta=\frac{8\sqrt{3}}{15}$$

$$\theta=\cos^{-1}\left(\frac{8\sqrt{3}}{15}\right)$$

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3. Find the shortest distance between the lines

$$\vec{r}=(\hat{i}+2\hat{j}+\hat{k})+\lambda(\hat{i}-\hat{j}+\hat{k})$$

$$\vec{r}=(2\hat{i}-\hat{j}-\vec{k})+\mu(2\hat{i}+\hat{j}+2\hat{k})$$

Ans:

$$\vec{a}_1=\hat{i}+2\hat{j}+\hat{k},\vec{b}_1=\hat{i}-\hat{j}+\hat{k}$$

$$\vec{a}_2=2\hat{i}-\hat{j}-\hat{k},\vec{b}_1=2\hat{i}+\hat{j}+2\hat{k}$$

$$d=\left|\frac{(\vec{a}_2-\vec{a}_1).(\vec{b}_1\times\vec{b}_2)}{|\vec{b}_1\times\vec{b}_2|}\right|$$

$$\vec{a}_2-\vec{a}_1=\hat{i}-3\hat{j}-2\hat{k}$$

$$\vec{b}_1\times\vec{b}_2=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & -1 & 1\\ 2 & 1 & 2\end{vmatrix}$$

$$=-3\hat{i}+3\hat{k}$$

$$d=\left|\frac{(\hat{i}-3\hat{j}-2\hat{k}).(-3\hat{i}+3\hat{k})}{|-3\hat{i}+3\hat{k}|}\right|$$

$$=\left|\frac{-3-6}{\sqrt{9+9}}\right|=\left|\frac{9}{3\sqrt{2}}\right|=\frac{3}{\sqrt{2}}$$

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4. Find the direction cosines of the unit vector $\perp$ to the plane

$$\vec{r}.(6\hat{i}-3\hat{j}-2\hat{k})+1=0$$

passing through the origin.
Ans:

$$\vec{r}.(6\hat{i}-3\hat{j}-2\hat{k})=-1$$

$$\hat{r}.(-6\hat{i}+3\hat{j}+2\hat{k})=1....(1)$$

$$|-6\hat{i}+3\hat{j}+2\hat{k}|=\sqrt{36+9+4}=7$$

Dividing equation 1 by 7

$$\vec{r}.\left(\frac{-6}{7}\hat{i}+\frac{3}{7}\hat{j}+\frac{2}{7}\hat{k}\right)=\frac{1}{7}$$

$$\hat{n}=\frac{-6}{7}\hat{i}+\frac{3}{7}\hat{j}+\frac{2}{7}\hat{k}[\because \vec{r}.\vec{n}=d$$

Hence direction cosines of $\hat{n}$ is $\frac{-6}{7},\frac{3}{5},\frac{2}{7}$

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5. Find the angle between the two planes 3x – 6y + 2z = 7 and 2x + 2y – 2z = 5

Ans: Comparing the giving eq of the planes with the equations

A_1 x +B_1y +C_1Z + D = 0 , A_2 x + B_2y + C_2 Z + D_2 = 0

A_1 = 3, B_1 = -6, C_1 = 2

A_2 = 2, B_2 = 2, C_2 = -2

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6. Find the shortest between the l $_1$ and l_2 whose vectors equations are

$$\vec{r}=\hat{i}+\hat{j}+\lambda(2\hat{i}-\hat{j}+\hat{k})$$
$$\vec{r}=2\hat{i}+\hat{j}-\hat{k}+\mu(3\hat{i}-5\hat{j}+2\hat{k})$$

Ans:

$$\vec{a}_1=\hat{i}+\hat{j},\vec{b}_1=2\hat{i}-\hat{j}+\hat{k}$$

$$\vec{a}_2=2\hat{i}+\hat{j}-\hat{k},\vec{b}_2=3\hat{i}-5\hat{j}+2\hat{k}$$
$$\vec{a}_2-\vec{a}_1=\hat{i}-\hat{k}$$

$$\vec{b}_1\times\vec{b}_2=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & -1 & 1\\ 3 & -5 & 2\end{vmatrix}$$

$$= 3\hat{i} - \hat{j} - 7\hat{k}$$

$$\left|\vec{b}_1 \times \vec{b}_2\right| = \sqrt{59}$$

$$d = \left|\frac{(\vec{b}_1 \times \vec{b}_2).(\vec{a}_2 - \vec{a}_1)}{\left|\vec{b}_1 \times \vec{b}_2\right|}\right| = \frac{|3 - 0 + 7|}{\sqrt{59}} = \frac{10}{\sqrt{59}}$$

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7. Find the angel between lines

$$\vec{r} = (2\hat{i} - 5\hat{j} + \hat{k}) + \lambda(3\hat{i} + 2\hat{j} + 6\hat{k})$$

$$\vec{r} = (7\hat{i} - 6\hat{k})\mu(\hat{i} + 2\hat{j} + 2\hat{k})$$

Ans:

$$\vec{b}_1 = 3\hat{i} + 2\hat{j} + 6\hat{k}$$

$$\vec{b}_2 = \hat{i} + 2\hat{j} + 2\hat{k}$$

The angle θ between them is given by

$$\cos\theta = \left|\frac{\vec{b}_1.\vec{b}_2}{\left|\vec{b}_1\right| \left|\vec{b}_2\right|}\right|$$

$$\left|\frac{(3\hat{i} + 2\hat{j} + 6\hat{k}).(\hat{i} + 2\hat{j} + 2\hat{k})}{\left|3\hat{i} + 2\hat{j} + 6\hat{k}\right|\left|\hat{i} + 2\hat{j} + 2\hat{k}\right|}\right|$$

$$\left|\frac{3 + 4 + 12}{\sqrt{49}\sqrt{9}}\right|$$

$$= \frac{19}{7 \times 3} = \frac{19}{21}$$

8. Show that the lines

$$\frac{x-5}{7}=\frac{y+2}{-5}=\frac{z}{1} \text{ and } \frac{x}{1}=\frac{y}{2}=\frac{z}{3}$$

Are perpendicular to each others

Ans:

$$\frac{x-5}{7}=\frac{y-(-2)}{-5}=\frac{z-0}{1}$$

$$\frac{x-0}{1}=\frac{y-0}{2}=\frac{z-0}{3}$$

$$a_1=7, b_1=-5, c_1=1$$
$$a_2=1, b_2=2, c_2=3$$

For $\perp$

$a_1a_2+b_1b_2+c_1c_2=0$

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$$=7\times 1+(-5\times 2)+1\times 3$$

$=7-10+3$

$=0$

$$hence \quad l_1 \perp l_2$$

9.Find the vector equations of the plane passing through the points R(2,5,-3), Q(-2,-3,5) and T (5,3,-3)

Ans: Let

$$\vec{a}=2\hat{i}+5\hat{j}-3\hat{k}$$

$$\vec{b}=-2\hat{i}-3\hat{j}+5\hat{k}$$

$$\vec{c} = 5\hat{i} + 3\hat{j} - 3\hat{k}$$

Vector equation is

$$(\vec{r} - \vec{a}).[(\vec{b} - \vec{a}) \times (\vec{c} - \vec{a})] = 0$$

$$[\vec{r} - (2\hat{i} + 5\hat{j} - 3\hat{k})].[(-4\hat{i} - 8\hat{j} + 8\hat{k}) \times (3\hat{i} - 2\hat{j})] = 0$$

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10. Find the Cartesian equation of the plane

$$\vec{r}.(\hat{i} + \hat{j} - \hat{k}) = 2$$

$$\cos\theta = \left| \frac{A_1A_2 + B_1B_2 + C_1C_2}{\sqrt{A_1^2 + B_{12} + C_1^2}\sqrt{A_2^2 + B_2^2 + C_2^2}} \right|$$

$$= \left| \frac{-10}{7 \times 2\sqrt{3}} \right|$$

$$= \frac{5}{7\sqrt{3}} = \frac{5\sqrt{3}}{21}$$

$$\theta = COS^{-1}\left(\frac{5\sqrt{3}}{21} \right)$$

Ans: Let

$$\vec{r} = x\hat{i} + y\hat{i} + z\hat{k}$$

$$\vec{r}.(\hat{i} + \hat{j} - \hat{k}) = 2$$

$$(x\hat{i}+y\hat{i}+z\hat{k})(\hat{i}+\hat{j}-\hat{k})=2$$

$x+y-z=2$

Which is the required equation of plane.

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11. find the distance between the lines l_1 and l_2 given by

$$\vec{r}=\hat{i}+2\hat{j}-4\hat{k}+\lambda(2\hat{i}+3\hat{j}+6\hat{k})$$

$$\vec{r}=3\hat{i}+3\hat{j}-5\hat{k}+\mu(2\hat{i}+3\hat{j}+6\hat{k})$$

Ans:

$$\vec{a}_1=\hat{i}+2\hat{j}-4\hat{k}$$

$$\vec{a}_2=3\hat{i}+3\hat{j}-5\hat{k}$$

$$\text{since} \quad \vec{b}_1=\vec{b}_2$$

Hence line are parallel

$$\vec{a}_2-\vec{a}=2\hat{i}-\hat{j}-\hat{k}$$

$$\vec{b}\times(\vec{a}_2-\vec{a})=\begin{vmatrix} i & j & k \\ 2 & 3 & 6 \\ 2 & 1 & -1 \end{vmatrix}$$

$$d=\left|\frac{\vec{b}\times(\vec{a}_2-\vec{a}_1)}{|\vec{b}|}\right|$$

$$\frac{\left|-9\hat{i}+14\hat{j}-4\hat{k}\right|}{\sqrt{49}}$$

$$\frac{\sqrt{293}}{\sqrt{49}}=\frac{\sqrt{293}}{7}$$

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12. Find the angle between lines

$$\frac{x}{2}=\frac{y}{2}=\frac{z}{1};\frac{x-5}{4}=\frac{y-2}{1}=\frac{z-3}{8}$$

Ans:

$$\frac{x-0}{2}=\frac{y-0}{2}=\frac{z-0}{1}$$

$$\frac{x-5}{4}=\frac{y-2}{1}=\frac{z-3}{8}$$

$$a_1=2, b_1=2, c_1=1$$

$$a_2=4, b_2=1, c_2=8$$

$$\cos\theta=\frac{\left|\vec{b}_1.\vec{b}_2\right|}{\left|\vec{b}_1\right|\left|\vec{b}_2\right|}$$

$$=\left|\frac{2(4)+2(1)+1(8)}{\sqrt{2^2+2^2+1}\sqrt{4^2+1^2+8^2}}\right|$$

$$=\left|\frac{8+2+8}{\sqrt{9}\sqrt{81}}\right|$$

$$=\frac{18}{27}$$

$$=\frac{2}{3}$$

$$\theta=\cos^{-1}\left(\frac{2}{3}\right)$$

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13. Find the shortest distance between the lines

$$\frac{x+1}{7}=\frac{y+1}{-6}=\frac{z+1}{1} \text{ and } \frac{x-3}{1}=\frac{y-5}{-2}=\frac{z-7}{1}$$

Ans: $\vec{a}_1=-\hat{i}-\hat{j}-\hat{k}$

$$\vec{a}_2=3\hat{i}+5\hat{j}+7\hat{k}$$
$$\vec{b}_1=7\hat{i}-6\hat{j}+1\hat{k}$$
$$\vec{b}_2=\hat{i}-2\hat{j}+\hat{k}$$

$$\vec{a}_2-a_1=4\hat{i}+6\hat{j}+8\hat{k}$$
$$\vec{b}_1\times\vec{b}_2=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 7 & -6 & 1\\ 1 & -2 & 1\end{vmatrix}$$
$$=-4i-6j-8k$$
$$\left|\vec{b}_1\times\vec{b}_2\right|=\sqrt{(-4)^2+(-6)^2+(-8)^2}$$
$$=\sqrt{116}$$
$$=2\sqrt{29}$$
$$d=\left|\frac{(\vec{a}_2-\vec{a}_1)(b_1\times b_2)}{\left|\vec{b}_1\times\vec{b}_2\right|}\right|$$
$$=\left|\frac{-116}{2\sqrt{29}}\right|=\frac{\cancel{116}\,58}{\cancel{2}\sqrt{29}}$$
$$=2\sqrt{29}$$

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14. Find the vector and Cartesian equations of the plane which passes through the point (5,2,-4) and $\perp$ to the line with direction ratios (2,3,-1)
Ans:

$$\vec{a}=5\hat{i}+2\hat{j}-4\hat{k}$$

$$\overline{N}=2\hat{i}+3\hat{j}-\hat{k}$$

Vector equation is

$$\left(\vec{r}-\vec{a}\right).\overrightarrow{N}=0$$

$$[\vec{r}-(5\hat{i}+2\hat{j}-4\hat{k})].(2\hat{i}+3\hat{j}-\hat{k})=0$$

Cartesian equation is

$$\vec{r}=x\hat{i}+y\hat{j}+z\hat{k}$$

$$[x\hat{i}+y\hat{j}+z\hat{k}-5\hat{j}-2\hat{j}+4\hat{k}].[2\hat{i}+3\hat{j}-\hat{k}]=0$$

$$((x-5)\hat{i}+(y-2)\hat{j}+(z+4)\hat{k}).(2\hat{i}+3\hat{j}-\hat{k})=0$$

$$2(x-5)+3(y-2)-(z+4)=0$$

$$2x-10+3y-6-z-4=0$$

$$2x+3y-z=20$$

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15. Find the Cartesian equation of the plane

$$\vec{r}[(5-2t)\hat{i}+(3-t)\hat{j}+(25+t)\hat{k}]=15$$

Ans:

$$\vec{r}[(5-2t)\hat{i}+(3-t)\hat{j}+(25+t)\hat{k}]=15$$

$$(x\hat{i}+y\hat{j}+z\hat{k}).[(5-2t)\hat{i}+(3-t)\hat{j}+(25+t)\hat{k}]=15$$
$$(5-2t)n+(3-t)y+(25+t)k-z=15$$

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16. Find the distance of a point (2,5,-3) from the plane

$$\vec{r}.(6\hat{i}-3\hat{j}+2\hat{k})=4$$

Ans:

$$\vec{a}=2\hat{i}+5\hat{j}-3\hat{k}$$

$$\overrightarrow{N}=6\hat{i}-3\hat{j}+2\hat{k}, d=4$$

$$d=\frac{\left|\vec{a}.\overrightarrow{N}.d\right|}{\left|\overrightarrow{N}\right|} \qquad [\because \vec{r}.\overrightarrow{N}=d$$

$$=\frac{\left|(2\hat{i}+5\hat{j}-3\hat{k}).(6\hat{i}-3\hat{j}+2\hat{k})-4\right|}{\left|6\hat{i}-3\hat{j}+2\hat{k}\right|}$$

$$=\frac{\left|12-15-6-4\right|}{\sqrt{36+9+4}}=\frac{13}{7}$$

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17. Find the shortest distance

$$\vec{r}=(\hat{i}+2\hat{j}+3\hat{k})+\lambda(\hat{i}-3\hat{j}+2\hat{k}) and \vec{r}=(4\hat{i}+5\hat{j}+6\hat{k})+\mu(2\hat{i}+3\hat{j}+\hat{k})$$

Ans:

$$\vec{a}_1=\hat{i}+2\hat{j}+3\hat{k}$$

$$\vec{b}_1 = \hat{i} - 3\hat{j} + 2\hat{k}$$
$$\vec{a}_2 = 4\hat{i} + 5\hat{j} + 6\hat{k}$$
$$\vec{b}_2 = 2\hat{i} + 3\hat{j} + \hat{k}$$
$$\vec{a}_2 - \vec{a}_1 = 3\hat{i} + 3\hat{j} + 3\hat{k}$$

$$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -3 & 2 \\ 2 & 3 & 1 \end{vmatrix}$$

$= -9\hat{i} + 3\hat{j} + 9\hat{k}$

$$d = \frac{(\vec{a}_2 - \vec{a}_2).(\vec{b}_1 \times \vec{b}_2)}{\left|\vec{b}_1 \times \vec{b}_2\right|}$$

$$= \left|\frac{9}{3\sqrt{19}}\right| = \frac{3}{\sqrt{19}}$$

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18. Find the vector equation of a plane which is at a distance of 7 units from the origin and normal to the vector $3\hat{i} + 5\hat{j} - 6\hat{k}$

Ans: $3\hat{i} + 5\hat{j} - 6\hat{k}$

$$\left|(\vec{n})\right| = \sqrt{70}$$
$$\hat{n} = \frac{\vec{n}}{\left|\vec{n}\right|}$$

$$= \frac{3}{\sqrt{70}}\hat{i} + \frac{5}{\sqrt{70}}\hat{j} - \frac{6}{\sqrt{70}}\hat{k}$$

$\vec{r}.\hat{n} = 7$

$$\vec{r}.\left(\frac{3}{\sqrt{70}}\hat{i} + \frac{5}{\sqrt{70}}\hat{j} - \frac{6}{\sqrt{70}}\hat{k}\right) = 7$$

19. Find the Cartesian equation of plane

$$\vec{r}.(\hat{i}+\hat{j}-\hat{k})=2$$

Ans:

$$\vec{r}.(\hat{i}+\hat{j}-\hat{k})=2$$

$$let \vec{r}=x\hat{i}+y\hat{j}+z\hat{k}$$

$$(x\hat{i}+y\hat{i}+z\hat{k}).(\hat{i}+\hat{j}-\hat{k})=2$$

$x+y-z=2$

---

20. Find the angle between the line

$$\frac{x+1}{2}=\frac{y}{3}=\frac{z-3}{6}$$

and the plane 10x +2y-11z=3
Ans:

$$\vec{r}=(-\hat{i}+o.\hat{j}+3\hat{k})+\lambda(2\hat{i}+3\hat{j}+6\hat{k})$$

$$and \vec{r}.(10\hat{i}+2\hat{j}-11\hat{k})=3$$
$$here\ \vec{b}=2\hat{i}+3\hat{j}+6\hat{k}$$

$$and\ \ \vec{n}=10\hat{i}+2\hat{j}-11\hat{k}$$

$$\sin\phi = \left|\frac{\vec{b}.\vec{n}}{|\vec{b}||\vec{n}|}\right|$$

$$= \left|\frac{20+6-66}{7\times 15}\right| = \left|\frac{-40}{7\times 15}\right| = \frac{8}{21}$$

---

21. Find the value of P so that the lines

$$\frac{1-x}{3} = \frac{7y-14}{2p} = \frac{z-3}{2} \ \ and \ \frac{7-7x}{3p} = \frac{y-5}{1} = \frac{6-z}{5}$$

are at right angles.
Ans:

$$\frac{x-1}{-3} = \frac{y-2}{\frac{2p}{7}} = \frac{z-3}{3} .....(i)$$

$$\frac{-(x-1)}{\frac{-3p}{7}} = \frac{y-5}{1} = \frac{z-6}{-5} .....(ii)$$

$$a_1 = -3, b_1 = \frac{2p}{7}, c_1 = 2$$

$$a_2 = \frac{-3p}{7}, b_2 = 1, c_2 = -5$$

$$for \ \perp$$

$$a_1a_2 + b_1b_2 + c_1c_2 = 0$$

$$-3\left(\frac{-3p}{7}\right) + \frac{2p}{7}(1) + 2(-5) = 0$$

$$\frac{9p}{7}+\frac{2p}{7}-\frac{10}{1}=0$$

$$\frac{9p+2p-70}{7}=0$$

$11p=70$

$p=\frac{70}{11}$

---

22. Find the shortest distance between the lines whose vector equation are

$$\vec{r}=(1-t)\hat{i}+(t-2)\hat{j}+(3-2t)\hat{k}$$

$$\vec{r}=(s+1)\hat{i}+(2s-1)\hat{j}-(2s+1)\hat{k}$$

Ans:

$$\vec{r}=\hat{i}-2\hat{j}+3\hat{k}+t(\hat{i}+\hat{j}-2\hat{k})$$

$$\vec{r}=\hat{i}-\hat{j}-\hat{k}+s(\hat{i}+2\hat{j}-2\hat{k})$$

$$\vec{a}_1=\hat{i}-2\hat{j}+3\hat{k}$$
$$\vec{b}_1=\hat{i}+\hat{j}-2\hat{k}$$

$$\vec{a}_2=\hat{i}-\hat{j}-\hat{k}$$
$$\hat{b}_2=\hat{i}+2\hat{j}-2\hat{k}$$

$$\vec{a}_2-\vec{a}_1=\hat{j}-4\hat{k}$$

$$\vec{b}_1\times\vec{b}_2=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ -1 & 1 & -2\\ 1 & 2 & -2\end{vmatrix}$$

$$=2\hat{i}-4\hat{j}-3\hat{k}$$

$$\left|\vec{b}_1\times\vec{b}_2\right|=\sqrt{(2)^2+(-4)^2+(-3)^2}$$
$$=\sqrt{29}$$

$$d=\left|\frac{(\vec{a}_2-\vec{a}_1)\ (\vec{b}_1\times\vec{b}_2)}{\left|\vec{b}\hat{1}\times\vec{b}_2\right|}\right|=\frac{8}{\sqrt{29}}$$

---

23. Find x such that four points A(3,2,1) B(4,x,5)(4,2,-2) and D (6,5,-1)are coplanar.

Ans: The equation of plane through

A(3,2,1), C(4,2,-2) and D (6,5,-1) is

$$\begin{vmatrix}x-x_1 & y-y_1 & z-z_1\\ x_2-x_1 & y_2-y_1 & z_2-z_1\\ x_3-x_1 & y_3-y_1 & z_3-z_1\end{vmatrix}=0$$

$$\begin{vmatrix}x-3 & y-2 & z-1\\ 4-3 & 2-2 & -2-1\\ 6-3 & 5-2 & -1-1\end{vmatrix}=0$$

$$9x-7y+3z-16=0...(i)$$

The point A,B,C,D are coplanar

$$9\times4-7x+3\times5-16=0$$

$$x=5$$

24. Find the angle between the two planes 2x +y-2z=5 and 3x -6y -2z = 7using vector method.
Ans.

$$\overrightarrow{N}_1 = 2\hat{i} + \hat{j} - 2\hat{k}$$

$$\overrightarrow{N}_2 = 3\hat{i} - 6\hat{j} - 2\hat{k}$$

$$\cos\theta = \left|\frac{\overrightarrow{N}_1.\overrightarrow{N}_2}{\left|\overrightarrow{N}_1\right|\left|\overrightarrow{N}_2\right|}\right|$$

$$= \left|\frac{(2\hat{i} + \hat{j} - 2\hat{k}).(3\hat{i} - 6\hat{j} - 2\hat{k}}{\sqrt{4+1+4}\sqrt{9+36+4}}\right|$$

$$\frac{4}{21}$$
$$\theta = \cos^{-1}\left(\frac{4}{21}\right)$$

25. Find the angle b/w the line

$$\frac{x-2}{2} = \frac{y-1}{5} = \frac{z+3}{-3}\ and\ \frac{x+2}{-1} = \frac{y-4}{8} = \frac{z-5}{4}$$

Ans:

$$\vec{b}_1 = 2\hat{i} + 5\hat{j} - 3\vec{k}$$

$$\vec{b}_2 = -\hat{i} + 8\hat{j} + 4\hat{k}$$

$$\cos\theta = \left|\frac{\vec{b}_1.\vec{b}_2}{|\vec{b}_1||\vec{b}_2|}\right|$$

$$= \left|\frac{2(-1)+5(8)+(-3)(4)}{\sqrt{38}\sqrt{81}}\right|$$

$$= \frac{26}{9\sqrt{38}}$$

$$\theta = \cos^{-1}\left(\frac{26}{9\sqrt{38}}\right)$$

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6 Marks Questions

1.Find the vector equation of the plane passing through the intersection of plane

$$\vec{r}.(\hat{i}+\hat{j}+\hat{k}) = 6 \;\; and \;\; \vec{r}.(2\hat{i}+3\hat{j}+4\hat{k}) = -5$$

And the point (1,1,1)
Ans.

$$\vec{n}_1 = \hat{i}+\hat{j}+\hat{k}, \vec{n}_2 = 2\hat{i}+3\hat{j}+4\hat{k}$$
$$\vec{d}_1 = -5, d_2 = 6$$

Using the relation

$$\vec{r}.(n_1+\lambda n_2) = d_1+\lambda d_2$$

$$r.[(1+2\lambda)\hat{i}+(1+3\lambda)\hat{j}+(1+4\lambda)\hat{k}] = 6-5\lambda......(1)$$

$$taking \quad \vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

$$(x\hat{i} + y\hat{j} + z\hat{k})[(1+2\lambda)\hat{i} + (1+3\lambda)\hat{j} + (1+4\lambda)\hat{k}] = 6 - 5\lambda$$

$$(1+2\lambda)x + (1+3\lambda)y + (1+4\lambda)z = 6 - 5\lambda$$

$$(x+y+z-6) + \lambda(2x+3y+4y+4z+5) = 0....(2)$$

$$plane \quad passes \quad through \quad the\ point \quad (1,1,1)$$

$$\lambda = \frac{3}{14}$$

$$put \quad \lambda\ in \quad eq \quad (1)$$

$$\vec{r}.\left[\left(1+\frac{3}{7}\right)\hat{i} + \left(1+\frac{9}{14}\right)\hat{j} + \left(1+\frac{6}{7}\right)\hat{k}\right] = 6 - \frac{15}{14}$$

$$\vec{r}.\left(\frac{10}{7}\hat{i} + \frac{23}{14}\hat{j} + \frac{13}{7}\hat{k}\right) = \frac{69}{14}$$

$$\vec{r}.(20\hat{i} + 23\hat{j} + 26\hat{k}) = 69$$

---

2. Find the coordinate where the line thorough (3,-4,-5) and ((2,-3,1) crosses the plane 2x + y + z = 7

Ans. Given points are A(3,-4,-5)

B(2,-3,1)

Direction ration of AB are 3-2, -4+3, -5-1

1,-1,-6

Eq. of line AB

$$\frac{x-3}{1}=\frac{y+4}{-1}=\frac{Z+5}{-6}=\lambda(say)$$

$$x=\lambda+3, y=-\lambda-4, Z=-6\lambda-5$$
$$let$$

$$(\lambda+3,\ -\lambda-4,\ -6\lambda-5)\ lies\ in$$

$$the\quad plane\quad 2x+y+Z=7$$

$$2(\lambda+3)+(-\lambda-4)+(-6\lambda-5)=7$$

$\lambda=-2$
$(1,-2,7)$ are the required point

---

3. Find the equation of the plane through the intersection of the planes 3x – y + 2z -4 = 0 and x + y + z – 2 = 0 and the point (2,2,1)

Ans. Equation of any plane through the
intersection of given planes can be taken as

$$(3x-y+2z-4)+\lambda(x+y+z-2)=0.....(i)$$

The point (2,2,1) lies in this plane
$\lambda=-2/3$ put in eq ….(i)

$$(3x-y+2z-4)-\frac{2}{3}(x+y+z-2)=0$$

$$7x-5y+4z-8=0$$

---

4. If the points (1,1p) and (-3,0,1)be equidistant from the plane

$$\vec{r}.(3\hat{i}+4\hat{j}-12\hat{k})+13=0$$

, then find the value of p.
Ans. The given plane is

$$\vec{r}.(3\hat{i}+4\hat{j}-12\hat{k})+13=0$$

$$(x\hat{i}+y\hat{j}+z\hat{k}).(3\hat{i}+4\hat{j}-12\hat{k})+13=0$$

$$3x+4y-12z+13=0....(i)$$

This plane is equidistant from the points (1,1,P) and (-3,0,1)

$$\frac{|3(1)+4(1)-12p+13|}{\sqrt{3^2+4^2+(-12)^2}}$$

$$=\frac{|3(-3)+4(0)-12(1)+13|}{\sqrt{3^2+4^2+(-12)^2}}$$

$$|20-12p|=|-8|$$

$$20-12p=\pm 8$$
$$p=-1 or \frac{7}{3}$$

---

5. Find the equation of the plane through the line of intersection of the planes x +y +z = 1 and 2x + 3y + 4z = 5 which is $\perp$ of the plane x-y + z = 0
Ans. Equations of any plane through the intersection of given planes are be written is

$$(x+y+z-1)+\lambda(2x+3y+4z-5)=0$$

$$(1+2\lambda)x+(1+3\lambda)y+(1+4\lambda)z-1-5\lambda=0...(1)$$

This plane is it right angle to the plane x-y+z

$$(1+2\lambda)(1)+(1+3\lambda)(-1)+(1+4\lambda)(1)=0$$
$$\lambda=-1/3$$

$$put\lambda in \text{ equation } (1)$$

$$\left(1-\frac{2}{3}\right)x+\left(1-\frac{3}{3}\right)y+\left(1-\frac{4}{3}\right)Z-1+\frac{5}{3}=0$$

$$x-z+2=0$$

---

6. Find the distance of the point (-1,-5,-10) from the point of intersection of the line

$$\vec{r}=(2\hat{i}-\hat{j}+2\hat{k})+\lambda(3\hat{i}+4\hat{j}+2\hat{k})$$

and the plane

$$\vec{r}.(\hat{i}-\hat{j}+\hat{k})=5$$

Ans.

$$r=(2i-j+2k)+\lambda(3i+4j+2k)$$

$$\frac{x-2}{3}=\frac{y+1}{4}=\frac{z-2}{2}=\lambda.......(i)$$

coordinets are

$$3\lambda+2, 4\lambda-1, 2\lambda+2$$

$$and\ \bar{r}.(\hat{i}-\hat{j}+\hat{k})=5$$
$$(x\hat{i}+y\hat{j}+z\hat{k}).(\hat{i}-\hat{j}+\hat{k})=5$$

$$x-y+z=5.....(ii)$$

$$coordinate\ lies\ in\ eq.\ (ii)$$
$$\lambda=0$$

$$we\ get\ (2,-1,2)$$

Are the coordinate of the point of intersection of the given line and the plane

$$(-1,-5,-10)\ and\ (2,-1,2)$$

$$\text{req. distance }=\sqrt{(2+1)^2+(-1+5)^2+(2+10)^2}$$

$=13$

---

7. Find the equation of the plane that contains the point (1,-1,2) and is ⊥ to each of the plane 2x+3y-2z=5 and x+2y-3z = 8

Ans. The equation of the plane containing the given point is

A(x-1)+B(y-2)+C(Z-3)= 0….i

$$\Rightarrow Ax+By+Cz=A+2B+3C$$

Condition of ⊥ to the plane given in (i) with the plane

2x+3y-2z=5, x+2y-3z=8

2A+3B-2C=0

A+2B-3C=0

On solving

A=-5c, B=4C

5x-4y-Z=7

---

8. Find the vector equation of the line passing through (1,2,3) and ∥ to the planes

$$\vec{r}.(\hat{i}-\hat{j}+2\hat{k})=5 \, and \, \vec{r}.(3\hat{i}+\hat{i}+\hat{k})=6$$

Ans.

$$line \ passing \ through \ (1,2,3)$$

$$i.e \ \vec{a}=\hat{i}+2\hat{j}+3\hat{k}$$
$$and \ || \ to \ the \ planes$$

$$\vec{b}_1=\hat{i}-\hat{j}+2\hat{k}$$
$$\vec{b}_2=3\hat{i}+\hat{j}+\hat{k}$$

$$\therefore The \ line \ is \ normal \ to \ the \ vector$$

$$\vec{b}_1\times\vec{b}_2=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & -1 & 2\\ 3 & 1 & 1\end{vmatrix}$$

$$=-3\hat{i}+5\hat{j}+4\hat{k}$$

$$\therefore The \ req. \ eq. of \ the \ line \ is$$

$$\vec{r}=\hat{i}+2\hat{i}+3\hat{k}+\lambda(-3\hat{i}+5\hat{j}+4\hat{k})$$

---

9. Find the equation of the s point where the line through the points A(3,4,1) and B(5,1,6) crosses the XY plane.
Ans. The vector equation of the line through the point A and B is

$$\vec{r} = 3\hat{i} + 4\hat{j} + k + \lambda[(5-3)\hat{i} + (1-4)\hat{j} + (6-1)\hat{k}]$$
$$\vec{r} = 3\hat{i} + 4\hat{j} + \hat{k} + \lambda(2\hat{i} - 3\hat{j} + 5\hat{k}) \ldots (i)$$

Let P be the point where the line AB crosses the XY plane. Then the position vector $\vec{r}$ of the point P is the form
$x\hat{i} + y\hat{j}$

$$x\hat{i} + y\hat{i} = (3+2\lambda)\hat{i} + (4-3\lambda)\hat{j} + (1+5\lambda)\hat{k}$$

$$x = 3 + 2\lambda \,|\, y = 4 - 3\lambda$$

$$x = {}^{13}\!/_{5}, y = {}^{23}\!/_{5}$$

$$req.\ point\ is \left(\frac{13}{5}, \frac{23}{5}, 0\right)$$

---

10. Prove that if a plane has the intercepts a,b,c is at a distance of p units from the origin then

$$\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2} = \frac{1}{p^2}$$

Ans. The equation of the plane in the intercepts from is $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$ distance of this plane from the origin is given to be p

$$p = \frac{\left|\frac{1}{a}.0 + \frac{1}{b}.0 + \frac{1}{c}.0 - 1\right|}{\sqrt{\left(\frac{1}{a}\right)^2 + \left(\frac{1}{b}\right)^2 + \left(\frac{1}{c}\right)^2}}$$

$$p = \sqrt{\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2}}$$

$$\Rightarrow \frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2}$$