

Class 12 Maths Chapter 13 Probability Important Questions

Very Short Answer Question (1 Mark)

1. Find $P(A/B)$ if $P(A)=0.4$, $P(B)=0.8$ and $P(B/A)=0.6$.

Ans: It is given that,

$$P(A)=0.4$$

$$P(B)=0.8$$

$$P(B/A)=0.6$$

It is known that,

$$P(B/A)=\frac{P(A \cap B)}{P(A)}$$

$$\Rightarrow P(A \cap B)=0.4 \times 0.6=0.24$$

$$P(A/B)=\frac{P(A \cap B)}{P(B)}$$

$$\Rightarrow P(A/B)=\frac{0.24}{0.8}=0.3$$

Therefore, $P(A/B)=0.3$

2. Find $P(A \cap B)$ if A and B are two events such that $P(A)=0.5$, $P(B)=0.6$ and $P(A \cup B)=0.8$.

Ans: It is given that,

$$P(A)=0.5$$

$$P(B)=0.6$$

$$P(A \cup B)=0.8$$

It is known that,

$$P(A \cup B)=P(A)+P(B)-P(A \cap B)$$

$$\Rightarrow 0.8=0.5+0.6-P(A \cap B)$$

$$\Rightarrow P(A \cap B)=1.1-0.8=0.3$$

Therefore, $P(A \cap B)=0.3$

3. A soldier fires three bullets at the enemy. The probability that the enemy will be killed by one bullet is 0.70.7 . What is the probability that the enemy is still alive?

Ans: It is given that,

Probability that the enemy will be killed by one bullet is 0.70.7

Probability that the enemy will be alive after one bullet is fired = $1 - 0.7 = 0.3 = 1 - 0.7 = 0.3$

Probability that the enemy is still alive after three bullets is fired = $0.3 \times 0.3 \times 0.3 = 0.027 = 0.3 \times 0.3 \times 0.3 = 0.027$

Therefore, the probability that the enemy is still alive is 0.0270.027.

4. What is the probability that a leap year has 5353 Sundays?

Ans: It is known that a leap year has 366366 days.

It means that there are 5252 weeks and 22 extra days.

The 5252 weeks have 5252 Sundays.

The possibilities for two extra days are {Monday, Tuesday} { *Monday , Tuesday* } , {Tuesday, Wednesday} { *Tuesday , Wednesday* } , {Wednesday, Thursday} { *Wednesday , Thursday* } , {Thursday, Friday} { *Thursday , Friday* } , {Friday, Saturday} { *Friday , Saturday* } , {Saturday, Sunday} { *Saturday , Sunday* } and {Saturday, Sunday} { *Saturday , Sunday* }

The possibility of having Sunday in the other days is {Saturday, Sunday} { *Saturday , Sunday* } and {Saturday, Sunday} { *Saturday , Sunday* }

The number of favourable outcomes will be 22

Total number of outcomes is 77

It is known that,

Probability of an event A: $P(A) = \frac{\text{Favourable Outcomes}}{\text{Total Outcomes}}$ $P (A) = \frac{\text{Favourable Outcomes}}{\text{Total Outcomes}}$

$$\Rightarrow P(A) = \frac{22}{77} \Rightarrow P (A) = \frac{2}{7}$$

Probability that a leap year has 5353 Sundays is $\frac{2}{7}$.

5. 2020 cards are numbered 11 to 2020. One card is drawn at random. What is the probability that the number on the card will be a multiple of 44?

Ans: It is given that,

Sample space, $S = \{1, 2, 3, \dots, 20\}$

Total number of outcomes in the sample space, $n(S) = 20$

Let A be the event of getting multiple of 4, $A = \{4, 8, 12, 16, 20\}$

Total number of favourable outcomes, $n(A) = 5$

It is known that,

$$\text{Probability of an event } A: P(A) = \frac{n(A)}{n(S)}$$

$$\Rightarrow P(A) = \frac{5}{20} = 0.25 \Rightarrow P(A) = \frac{5}{20} = 0.25$$

The probability that the number on the card will be a multiple of 4 is 0.25.

6. Three coins are tossed once. Find the probability of getting at least one head.

Ans: Sample space, $S = \{HHH, HHT, HTH, THH, HTT, THT, TTH, TTT\}$

$$S = \{HHH, HHT, HTH, THH, HTT, THT, TTH, TTT\}$$

Total number of outcomes in the sample space, $n(S) = 8$

Let A be the event of getting at least one head, $A = \{HHH, HHT, HTH, THH, HTT, THT, TTH\}$

$$A = \{HHH, HHT, HTH, THH, HTT, THT, TTH\}$$

Total number of favourable outcomes, $n(A) = 7$

It is known that,

$$\text{Probability of an event } A: P(A) = \frac{n(A)}{n(S)}$$

$$\Rightarrow P(A) = \frac{7}{8} \Rightarrow P(A) = \frac{7}{8}$$

Therefore, the probability of getting at least one head is $\frac{7}{8}$.

7. The probability that a student is not a swimmer is $\frac{1}{5}$. Find the probability that out of 55 students, 44 are swimmers.

Ans: It is given that,

$$\text{The probability that a student is not a swimmer} = \frac{1}{5}$$

$$\text{The probability that a student is a swimmer} = 1 - \frac{1}{5} = \frac{4}{5}$$

Probability that out of 55 students, 44 are swimmers:

From Binomial distribution: $P(X=x) = nC_x p^x q^{n-x}$

Here, $x=4$, $n=5$, $p=\frac{4}{5}$ and $q=\frac{1}{5}$

$$\Rightarrow P(X=4) = 5C_4 \left(\frac{4}{5}\right)^4 \left(\frac{1}{5}\right)^{5-4}$$

$$\Rightarrow P(X=4) = \frac{5!}{4!(5-4)!} \left(\frac{4}{5}\right)^4 \left(\frac{1}{5}\right)^1$$

$$\Rightarrow P(X=4) = 5 \left(\frac{4}{5}\right)^4 \left(\frac{1}{5}\right)$$

$$\Rightarrow P(X=4) = \left(\frac{4}{5}\right)^4$$

The probability that out of 55 students, 44 are swimmers is $\left(\frac{4}{5}\right)^4$.

8. Find $P(A/B)$ if $P(B)=0.5$ and $P(A \cap B)=0.32$.

Ans: It is given that,

$$P(B)=0.5$$

$$P(A \cap B)=0.32$$

It is known that,

$$P(A/B) = \frac{P(A \cap B)}{P(B)}$$

$$\Rightarrow P(A/B) = \frac{P(A \cap B)}{P(B)}$$

$$\Rightarrow P(A/B) = \frac{0.32}{0.50} = \frac{16}{25}$$

Therefore, $P(A/B) = \frac{16}{25}$

9. A random variable X has the following probability distribution.

X	0	1	2	3	4	5
$P(X)$	$\frac{1}{15}$	k	$15k-2$	$\frac{15k-2}{15}$	k	$15k-1$
						$\frac{1}{15}$

Find the value of k .

Ans: It is known that the sum of all the probabilities of a random variable is equal to 1 i.e. $\sum P(X) = 1$.

$$\Rightarrow 115 \times 0 + k \times 1 + 15k - 215 \times 2 + k \times 3 + 15k - 115 \times 4 + 115 \times 5 = 1$$

$$\Rightarrow \frac{1}{15} \times 0 + k \times 1 + \frac{15k - 2}{15} \times 2 + k \times 3 + \frac{15k - 1}{15} \times 4 + \frac{1}{15} \times 5 = 1$$

$$\Rightarrow 9k - 315 = 1 \Rightarrow 9k - \frac{3}{15} = 1$$

$$\Rightarrow 4k = 1 + 115 \Rightarrow 4k = 1 + \frac{1}{15}$$

$$\Rightarrow 4k = 1615 \Rightarrow 4k = \frac{16}{15}$$

$$\Rightarrow k = 415 \Rightarrow k = \frac{4}{15}$$

10. A random variable X taking values 0, 1, 2 has the following probability distribution for some number k .

$$P(X) = \begin{cases} k & \text{if } X = 0 \\ 2k & \text{if } X = 1 \\ 3k & \text{if } X = 2 \end{cases}$$

$$\begin{cases} k & \text{if } X = 0 \\ 2k & \text{if } X = 1, \text{ find } k. \\ 3k & \text{if } X = 2 \end{cases}$$

Ans: It is known that the sum of probabilities of a random variable is equal to 1.

$$\Rightarrow k + 2k + 3k = 1 \Rightarrow k + 2k + 3k = 1$$

$$\Rightarrow 6k = 1 \Rightarrow 6k = 1$$

$$\Rightarrow k = \frac{1}{6} \Rightarrow k = \frac{1}{6}$$

Therefore, the value of k is $\frac{1}{6}$.

Long Answer Questions (4 Marks)

11. A problem in Mathematics is given to three students whose chance of solving it are $12\frac{1}{2}$, $13\frac{1}{3}$ and $14\frac{1}{4}$. What is the probability that the problem is solved?

Ans: Let A, B, C be the events of solving the problem.

A^-, B^-, C^- be the event of not solving the problem.

It is known that, A^-, B^-, C^- , A, B, C are independent events.

From the given data,

$$P(A) = \frac{1}{2} \Rightarrow P(A^-) = \frac{1}{2} \Rightarrow P(A) = \frac{1}{2}$$

$$P(B) = \frac{2}{3} \Rightarrow P(B^-) = \frac{1}{3} \Rightarrow P(B) = \frac{2}{3}$$

$$P(C) = \frac{3}{4} \Rightarrow P(C^-) = \frac{1}{4}$$

$$P(C) = \frac{3}{4} \Rightarrow P(C^-) = \frac{1}{4}$$

$$P(\text{None solving the problem}) = P(A^- \text{ and } B^- \text{ and } C^-) = P(A^- \cap B^- \cap C^-)$$

$$\Rightarrow P(\text{None solving the problem}) = P(A^- \cap B^- \cap C^-) \Rightarrow P(\text{None solving the problem}) = P(A^- \cap B^- \cap C^-)$$

$$\Rightarrow P(\text{None solving the problem}) = P(A^-)P(B^-)P(C^-) \Rightarrow P(\text{None solving the problem}) = P(A^-)P(B^-)P(C^-)$$

$$\Rightarrow P(\text{None solving the problem}) = \frac{1}{2} \times \frac{1}{3} \times \frac{1}{4} = \frac{1}{24}$$

$$P(\text{The problem will be solved}) = 1 - P(\text{None solving the problem})$$

$$P(\text{The problem will be solved}) = 1 - P(\text{None solving the problem})$$

$$P(\text{The problem will be solved}) = 1 - \frac{1}{24} = \frac{23}{24}$$

Therefore, the probability that the problem is solved is $\frac{23}{24}$.

12. A die is rolled. If the outcome is an even number, what is the probability that it is a prime?

Ans: Sample space when a die is rolled, $S = \{1, 2, 3, 4, 5, 6\}$

Let A be the event of getting an even number.

Let B be the event of getting a prime number.

$$A = \{2, 4, 6\} \quad B = \{2, 3, 5\} \quad A \cap B = \{2\}$$

$$\text{Conditional probability, } P(B/A) = \frac{P(A \cap B)}{P(A)}$$

$$\Rightarrow P(B/A) = \frac{1}{3} = \frac{1}{3}$$

Therefore, when a die is rolled if the outcome is an even number the probability that it is prime is $\frac{1}{3}$.

13. If A and B are two events such that $P(A) = \frac{1}{4}$, $P(B) = \frac{1}{2}$ and $P(A \cap B) = \frac{1}{8}$. Find $P(\text{not } A \text{ and not } B)$.

Ans: Given,

$$P(A) = \frac{1}{4}$$

$$P(B) = \frac{1}{2}$$

$$P(A \cap B) = \frac{1}{8}$$

$$P(\text{not } A \text{ \& not } B) = P(A' \cap B')$$

$$P(\text{not } A \text{ \& not } B) = P(A' \cap B')$$

$$\Rightarrow P(\text{not } A \text{ \& not } B) = P(A \cup B)'$$

$$\Rightarrow P(\text{not } A \text{ \& not } B) = P(A \cup B)'$$

$$\Rightarrow P(\text{not } A \text{ \& not } B) = 1 - P(A \cup B)$$

$$\Rightarrow P(\text{not } A \text{ \& not } B) = 1 - P(A \cup B)$$

$$\Rightarrow P(\text{not } A \text{ \& not } B) = 1 - [P(A) + P(B) - P(A \cap B)]$$

$$\Rightarrow P(\text{not } A \text{ \& not } B) = 1 - [P(A) + P(B) - P(A \cap B)]$$

$$\Rightarrow P(\text{not } A \text{ \& not } B) = 1 - [14 + 12 - 18]$$

$$\Rightarrow P(\text{not } A \text{ \& not } B) = 1 - \left[\frac{1}{4} + \frac{1}{2} - \frac{1}{8} \right]$$

$$\Rightarrow P(\text{not } A \text{ \& not } B) = 1 - [28 + 48 - 18]$$

$$\Rightarrow P(\text{not } A \text{ \& not } B) = 1 - \left[\frac{2}{8} + \frac{4}{8} - \frac{1}{8} \right]$$

$$\Rightarrow P(\text{not } A \text{ \& not } B) = 1 - \frac{5}{8}$$

$$\Rightarrow P(\text{not } A \text{ \& not } B) = 1 - \frac{5}{8}$$

$$\Rightarrow P(\text{not } A \text{ \& not } B) = \frac{3}{8}$$

$$\Rightarrow P(\text{not } A \text{ \& not } B) = \frac{3}{8}$$

Therefore,

$$P(\text{not } A \text{ \& not } B) = \frac{3}{8}$$

$$P(\text{not } A \text{ \& not } B) = \frac{3}{8}$$

14. In a class of 2525 students with roll numbers 11 to 2525, a student is picked up at random to answer a question. Find the probability that the roll number of the selected student is either a multiple of 55 or of 77.

Ans: Sample space, $S = \{1, 2, 3, 4, \dots, 25\}$

Total number of outcomes in the sample space, $n(S) = 25$

Let A be the event that the roll number of selected student is either a multiple of 55 or of 77

$A = \{5, 7, 10, 14, 15, 20, 21, 25\}$

Total number of favourable outcomes, $n(A) = 8$

It is known that,

$$\text{Probability of an event } A: P(A) = \frac{n(A)}{n(S)}$$

$$\Rightarrow P(A) = \frac{8}{25}$$

Therefore, the probability that the roll number of the selected student is either a multiple of 55 or of 77 is $\frac{8}{25}$.

15. A can hit a target 44 times in 55 shots B three times in 44 shots and C twice in 33 shots. They fire a volley. What is the probability that at least two shots hit.

Ans: Fire a volley means that A, B and C all try to hit the target simultaneously.

At least two shots hit the target:

1. A and B hit and C fails to hit.
2. A and C hit and B fails to hit.
3. B and C hit and A fails to hit.
4. A, B, C all three hit the target.

The chance of hitting by A, $P(A) = \frac{44}{55}$ and of not hitting by A, $P(A') = 1 - \frac{44}{55} = \frac{11}{55}$

The chance of hitting by B, $P(B) = \frac{3}{44}$ and of not hitting by B, $P(B') = 1 - \frac{3}{44} = \frac{41}{44}$

The chance of hitting by C, $P(C) = \frac{2}{33}$ and of not hitting by C, $P(C') = 1 - \frac{2}{33} = \frac{31}{33}$

$$\text{Probability of (I) } P(ABC') = \frac{44}{55} \times \frac{3}{44} \times \frac{31}{33} = \frac{31}{330}$$

Probability of (II) (*II*) is $P(ACB')=45 \times 23 \times 14=215$ $P (ACB') = \frac{4}{5} \times \frac{2}{3} \times \frac{1}{4} = \frac{2}{15}$

Probability of (III) (*III*) is $P(BC'A)=34 \times 23 \times 15=110$ $P (BC'A) = \frac{3}{4} \times \frac{2}{3} \times \frac{1}{5} = \frac{1}{10}$

Probability of (IV) (*IV*) is $P(ABC)=45 \times 34 \times 23=25$ $P (ABC) = \frac{4}{5} \times \frac{3}{4} \times \frac{2}{3} = \frac{2}{5}$

Total probability= $15+215+110+25$ $Totalprobability = \frac{1}{5} + \frac{2}{15} + \frac{1}{10} + \frac{2}{5}$

Total probability= $630+430+330+1230$ $Totalprobability = \frac{6}{30} + \frac{4}{30} + \frac{3}{30} + \frac{12}{30}$

Total probability= $2530=56$ $Totalprobability = \frac{25}{30} = \frac{5}{6}$

Therefore, the probability that at least two shots hit the target is $56\frac{5}{6}$.

16. Two dice are thrown once. Find the probability of getting an even number on the first die or a total of 88 .

Ans: Sample space, $S=\{1,2,3,4,\dots,36\}$ $S = \{ 1 , 2 , 3 , 4 , 36 \}$

Total number of outcomes in the sample space, $n(S)=36$ $n (S) = 36$

Let *AA* be the event of getting even number on the first

die $A=\{(2,1),(2,2),\dots,(2,6),(4,1),(4,2),\dots,(4,6),(6,1),(6,2),\dots,(6,6)\}$

$A = \{ (2 , 1) , (2 , 2) , . . . , (2 , 6) , (4 , 1) , (4 , 2) , . . . , (4 , 6) , (6 , 1) , (6 , 2) , . . . , (6 , 6) \}$

Total number of favourable outcomes, $n(A)=18$ $n (A) = 18$

Let *BB* be the event of getting a total of 88, $B=\{(2,6),(3,5),(4,4),(5,3),(6,2)\}$

$B = \{ (2 , 6) , (3 , 5) , (4 , 4) , (5 , 3) , (6 , 2) \}$

Total number of favourable outcomes, $n(B)=5$ $n (B) = 5$

Let $A \cap B$ be the event of getting an even number on first die and a total of 88, $A \cap B = \{(2,6),(4,4),(6,2)\}$

$A \cap B = \{ (2 , 6) , (4 , 4) , (6 , 2) \}$

Total number of favourable outcomes, $n(A \cap B)=3$ $n (A \cap B) = 3$

Probability of an event *AA*: $P(A)=\frac{n(A)}{n(S)}=\frac{18}{36}$ $P (A) = \frac{n (A)}{n (S)} = \frac{18}{36}$

Probability of an event *BB*: $P(A)=\frac{n(B)}{n(S)}=\frac{5}{36}$ $P (A) = \frac{n (B)}{n (S)} = \frac{5}{36}$

Probability of an event $A \cap B$: $P(A \cap B)=\frac{n(A \cap B)}{n(S)}=\frac{3}{36}$ $P (A \cap B) = \frac{n (A \cap B)}{n (S)} = \frac{3}{36}$

It is known that,

$P(A \cup B)=P(A)+P(B)-P(A \cap B)$ $P (A \cup B) = P (A) + P (B) - P (A \cap B)$

$$P(A \cup B) = 18/36 + 5/36 - 3/36$$

$$P(A \cup B) = 20/36 = 5/9$$

Therefore, the probability of getting an even number on the first die or a total of 88 is $59\frac{5}{9}$.

17. AA and BB throw a die alternatively till one of them throws a '6' and wins the game. Find their respective probabilities of winning, if AA starts the game.

Ans: Let SS be the success of getting 66 and FF be the failure of not getting 66.

$$\Rightarrow P(S) = p = 1/6 \Rightarrow P(S) = p = \frac{1}{6}$$

$$\Rightarrow P(F) = q = 5/6 \Rightarrow P(F) = q = \frac{5}{6}$$

$$P(\text{A wins in first throw}) = P(S) = p \quad P(\text{A wins in first throw}) = P(S) = p$$

$$P(\text{A wins in third throw}) = P(FFS) = qpq \quad P(\text{A wins in third throw}) = P(FFS) = qpq$$

$$P(\text{A wins in fifth throw}) = P(FFFFS) = qqqqp \quad P(\text{A wins in fifth throw}) = P(FFFFS) = qqqqp$$

$$P(\text{A wins}) = p + qpq + qqqqp + \dots \quad P(\text{A wins}) = p + qpq + qqqqp + \dots$$

$$\Rightarrow P(\text{A wins}) = p(1 + q^2 + q^4 + \dots) \Rightarrow P(\text{A wins}) = p(1 + q^2 + q^4 + \dots)$$

$$\Rightarrow P(\text{A wins}) = p(1 - q^2) \Rightarrow P(\text{A wins}) = \frac{p}{1 - q^2}$$

$$\Rightarrow P(\text{A wins}) = 1/6 - 25/36 = 36/36 - 25/36 = 11/36 \Rightarrow P(\text{A wins}) = \frac{1/6}{1 - 25/36} = \frac{36}{6 \times 11} = \frac{6}{11}$$

$$P(\text{B wins}) = 1 - P(\text{A wins}) \quad P(\text{B wins}) = 1 - P(\text{A wins})$$

$$P(\text{B wins}) = 1 - \frac{6}{11} = \frac{5}{11}$$

$$P(\text{B wins}) = 1 - \frac{6}{11} = \frac{5}{11}$$

Therefore,

$$P(\text{A wins}) = \frac{6}{11}$$

$$P(\text{A wins}) = \frac{6}{11}$$

and

$$P(\text{B wins}) = \frac{5}{11}$$

$$P(\text{B wins}) = \frac{5}{11}$$

18. If A and B are events such that $P(A) = \frac{1}{2}$, $P(A \cup B) = \frac{3}{5}$ and $P(B) = p$ find p if events are

a. mutually exclusive

Ans: It is given that,

$$P(A) = \frac{1}{2}$$

$$P(B) = p$$

$$P(A \cup B) = \frac{3}{5}$$

It is known that, if two events A and B are mutually exclusive then $P(A \cap B) = 0$.

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$\Rightarrow \frac{3}{5} = \frac{1}{2} + p - 0$$

$$\Rightarrow p = \frac{3}{5} - \frac{1}{2}$$

$$\Rightarrow p = \frac{1}{10}$$

Therefore, the value of p is $\frac{1}{10}$.

b. independent

Ans: It is given that,

$$P(A) = \frac{1}{2}$$

$$P(B) = p \quad P(A \cap B) = p$$

$$P(A \cup B) = 35 \quad P(A \cup B) = \frac{3}{5}$$

We know that if two events A and B are independent then $P(A \cap B) = P(A)P(B)$

It is known that,

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$\Rightarrow 35 = 12 + p - p^2 \Rightarrow \frac{3}{5} = \frac{1}{2} + p - \frac{p}{2} \quad [P(A)P(B) = p^2] \quad [P(A)P(B) = \frac{p}{2}]$$

$$\Rightarrow p^2 = 35 - 12 \Rightarrow \frac{p}{2} = \frac{3}{5} - \frac{1}{2}$$

$$\Rightarrow p = 2(10) = 15 \Rightarrow p = \frac{2}{10} = \frac{1}{5}$$

Therefore, the value of p is $15 \frac{1}{5}$.

19. A man takes a step forward with probability 0.4 and backward with probability 0.6. Find the probability that at the end of eleven steps he is one step away from the starting point.

Ans: Let, a step forward be the success S and a step backward be the failure F .

$$\Rightarrow P(S) = p = 0.4 \Rightarrow P(S) = p = 0.4$$

$$\Rightarrow P(F) = q = 0.6 \Rightarrow P(F) = q = 0.6$$

In eleven steps he will be one step away from the starting point if the number of successes and failures differ by 1.

Therefore there are two chances i.e.,

$$\text{Number of successes} = 6 = 6$$

$$\text{Number of failures} = 5 = 5 \text{ OR}$$

$$\text{Number of successes} = 5 = 5$$

$$\text{Number of failures} = 6 = 6$$

$$\text{Required probability} = {}^{11}C_6 p^6 q^5 + {}^{11}C_5 p^5 q^6$$

$$\Rightarrow {}^{11}C_6! 5! (0.4)^6 (0.6)^5 + {}^{11}C_5! 5! (0.4)^5 (0.6)^6 \Rightarrow \frac{11!}{6!5!} (0.4)^6 (0.6)^5 + \frac{11!}{5!6!} (0.4)^5 (0.6)^6$$

$$\Rightarrow {}^{11}C_6! 5! (0.4)^5 (0.6)^5 [0.4 + 0.6] \Rightarrow \frac{11!}{6!5!} (0.4)^5 (0.6)^5 [0.4 + 0.6]$$

$$\Rightarrow 11 \times 10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 (0.4)^5 (0.6)^5 (1) = 0.3678 \Rightarrow \frac{11 \times 10 \times 9 \times 8 \times 7}{5 \times 4 \times 3 \times 2 \times 1} (0.4)^5 (0.6)^5 (1) = 0.3678$$

20. Two cards are drawn from a pack of well shuffled 52 cards one by one with replacement. Getting an ace or a spade is considered a success. Find the probability distribution for the number of successes.

Ans: Let A be the event of a number of aces and spades.

$$\Rightarrow n(A) = 16 \Rightarrow n(A) = 16$$

$$\text{Number of outcomes in sample space, } n(S) = 52 \Rightarrow n(S) = 52$$

$$\text{Probability of picking an Ace or a spade, } P(A) = \frac{16}{52} = \frac{4}{13}$$

$$\text{Probability of not picking an Ace or a spade, } P(A') = 1 - \frac{4}{13} = \frac{9}{13}$$

Let X be the event of number of aces or spades

$$\text{When } X=0 \Rightarrow X = 0$$

$$P(X=0) = P(A') \times P(A') \Rightarrow P(X=0) = P(A') \times P(A')$$

$$\Rightarrow P(X=0) = \frac{9}{13} \times \frac{9}{13} = \frac{81}{169}$$

$$\text{When } X=1 \Rightarrow X = 1$$

$$P(X=1) = P(A) \times P(A') \Rightarrow P(X=1) = P(A) \times P(A')$$

$$\Rightarrow P(X=1) = \frac{4}{13} \times \frac{9}{13} = \frac{36}{169}$$

$$\text{When } X=2 \Rightarrow X = 2$$

$$P(X=2) = P(A) \times P(A) \Rightarrow P(X=2) = P(A) \times P(A)$$

$$\Rightarrow P(X=2) = \frac{4}{13} \times \frac{4}{13} = \frac{16}{169}$$

Therefore, the probability distribution for the number of successes is:

X	0	1	2
$P(X)$	$\frac{81}{169}$	$\frac{36}{169}$	$\frac{16}{169}$

21. In a game, a man wins a rupee for a six and loses a rupee for any other number when a fair die is thrown. The man decided to throw a die thrice but to quit as and when he gets a six. Find the expected value of the amount he wins/losses.

Ans: The following cases will be obtained from the given scenario:

1. Man gets 66 in the first throw
2. Man doesn't get 66 in the first throw, gets 66 in the second throw
3. Man doesn't get 66 in the first and second throws, gets 66 in the third throw
4. Man doesn't get 66 in the first, second and third throws

Case I: Man gets 66 in the first throw

$$\text{Probability of getting 66} = \frac{1}{6}$$

$$\text{Amount won} = 1 = 1$$

Case II: Man doesn't get 66 in the first throw, gets 66 in the second throw

$$\text{Probability of getting 66} = \frac{5}{6} \times \frac{1}{6} = \frac{5}{36}$$

$$\text{Amount won} = -1 + 1 = 0 = -1 + 1 = 0$$

Case III: Man doesn't get 66 in the first and second throws, gets 66 in the third throw

$$\text{Probability of getting 66} = \frac{5}{6} \times \frac{5}{6} \times \frac{1}{6} = \frac{25}{216}$$

$$\text{Amount won} = -1 - 1 + 1 = -1 = -1 - 1 + 1 = -1$$

Case IV: Man doesn't get 66 in the first, second and third throws

$$\text{Probability of getting 66} = \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} = \frac{125}{216}$$

$$\text{Amount won} = -1 - 1 - 1 = -3 = -1 - 1 - 1 = -3$$

Expected value = Amount won × Probability for all four throws

$$\text{Expected value} = \text{Amount won} \times \text{Probability for all four throws}$$

$$\text{Expected value} = (1 \times \frac{1}{6}) + (0 \times \frac{5}{36}) + (-1 \times \frac{25}{216}) + (-3 \times \frac{125}{216})$$

$$\text{Expected value} = \left(1 \times \frac{1}{6}\right) + \left(0 \times \frac{5}{36}\right) + \left(-1 \times \frac{25}{216}\right) + \left(-3 \times \frac{125}{216}\right)$$

$$\text{Expected value} = \frac{1}{6} + 0 - \frac{25}{216} - \frac{125}{72}$$

$$\text{Expected value} = \frac{1}{6} + 0 - \frac{25}{216} - \frac{125}{72}$$

$$\text{Expected value} = \frac{36 - 25 - 125}{216}$$

$$\text{Expected value} = \frac{36 - 25 - 125}{216}$$

Expected value=36-400/216

$$\text{Expected value} = \frac{36 - 400}{216}$$

Expected value=-364/216=-91/54

$$\text{Expected value} = \frac{-364}{216} = \frac{-91}{54}$$

Therefore, the expected value of the amount he wins or loses is $-9154 \frac{-91}{54}$.

22. Suppose that 10% of men and 5% of women have grey hair. A grey haired person is selected at random. What is the probability that the selected person is male assuming that there are 60% males and 40% females?

Ans: Let A be the event that the person selected has grey hair

Let E_1 be the event that the person selected is male

Let E_2 be the event that the person selected is female

$$\Rightarrow P(E_1) = \frac{60}{100} = \frac{3}{5}$$

$$\Rightarrow P(E_2) = \frac{40}{100} = \frac{2}{5}$$

$$\text{Probability that the selected is male with grey hair: } P(A/E_1) = \frac{10}{100} = \frac{1}{10}$$

$$\text{Probability that the selected is female with grey hair: } P(A/E_2) = \frac{5}{100} = \frac{1}{20}$$

From Bayes theorem:

$$P(E_1/A) = \frac{P(E_1)P(A/E_1)}{P(E_1)P(A/E_1) + P(E_2)P(A/E_2)}$$

$$P(E_1/A) = \frac{P(E_1)P(A/E_1)}{P(E_1)P(A/E_1) + P(E_2)P(A/E_2)}$$

$$\Rightarrow P(E_1/A) = \frac{35 \times 110}{35 \times 110 + 25 \times 120}$$

$$\Rightarrow P(E_1/A) = \frac{\frac{3}{5} \times \frac{1}{10}}{\frac{3}{5} \times \frac{1}{10} + \frac{2}{5} \times \frac{1}{20}}$$

$$\Rightarrow P(E_1/A) = \frac{3}{3+1} = \frac{3}{4}$$

$$\Rightarrow P(E_1/A) = \frac{3}{3+1} = \frac{3}{4}$$

Therefore, the probability that the selected person is male is $34\frac{3}{4}$.

23. A card from a pack of 52 cards is lost. From the remaining cards of the pack, two cards are drawn. What is the probability that they both are diamonds?

Ans: Let E_1 be the event that the lost card is diamond

In a pack of 52 there are 13 diamonds $\Rightarrow P(E_1) = \frac{13}{52} = \frac{1}{4}$

Let E_2 be the event that the lost card is not a diamond

$\Rightarrow P(E_2) = 1 - P(E_1) = 1 - \frac{1}{4} = \frac{3}{4}$

Let A be the event that the two cards drawn are both diamonds

Number of ways of drawing 2 diamond cards

$\Rightarrow P(A/E_1) = \frac{\text{Number of ways of drawing 2 diamond cards}}{\text{Total Number of ways of drawing 2 cards}}$

$$\Rightarrow P(A/E_1) = \frac{\text{Number of ways of drawing 2 diamond cards}}{\text{Total Number of ways of drawing 2 cards}}$$

$\Rightarrow P(A/E_1) = \frac{{}^{12}C_2}{{}^{51}C_2} = \frac{66}{1275}$

$\Rightarrow P(A/E_2) = \frac{{}^{13}C_2}{{}^{51}C_2} = \frac{78}{1275}$

From Bayes theorem:

$P(E_1/A) = \frac{P(E_1)P(A/E_1)}{P(E_1)P(A/E_1) + P(E_2)P(A/E_2)}$

$$P(E_1/A) = \frac{P(E_1)P(A/E_1)}{P(E_1)P(A/E_1) + P(E_2)P(A/E_2)}$$

$\Rightarrow P(E_1/A) = \frac{66 \times \frac{1}{4}}{\frac{66}{1275} \times \frac{1}{4} + \frac{78}{1275} \times \frac{3}{4}} = \frac{66}{66 + 224}$

$$\Rightarrow P(E_1/A) = \frac{\frac{66}{1275} \times \frac{1}{4}}{\frac{66}{1275} \times \frac{1}{4} + \frac{78}{1275} \times \frac{3}{4}} = \frac{66}{66 + 224}$$

$\Rightarrow P(E_1/A) = \frac{66 \times \frac{1}{4}}{\frac{66}{1275} \times \frac{1}{4} + \frac{78}{1275} \times \frac{3}{4}} = \frac{66}{66 + 234}$

$$\Rightarrow P(E_1/A) = \frac{\frac{66}{1275} \times \frac{1}{4}}{\frac{66}{1275} \times \frac{1}{4} + \frac{78}{1275} \times \frac{3}{4}} = \frac{66}{66 + 234}$$

$$\Rightarrow P(E_1/A) = 66/300 = 11/50$$

$$\Rightarrow P(E_1/A) = \frac{66}{300} = \frac{11}{50}$$

Therefore, the probability that they both are diamonds is $11/50$.

24. Ten eggs are drawn successively with replacement from a lot containing 10% defective eggs. Find the probability that there is at least one defective egg.

Ans: It is given that,

Probability of defective eggs = $p = 10/100 = 1/10$

$$\Rightarrow p = 10/100 = 1/10$$

Probability of good eggs = $q = 9/10$

$$\Rightarrow q = 1 - 1/10 = 9/10$$

Probability that at least one egg is defective out of $n = p(1) + p(2) + p(3) + \dots$

$$= p(1) + p(2) + p(3) + \dots$$

$$\Rightarrow [p(0) + p(1) + p(2) + \dots + p(10)] - p(0) \Rightarrow [p(0) + p(1) + p(2) + \dots + p(10)] - p(0)$$

$$\Rightarrow 1 - p(0) = 1 - (9/10)^{10} \Rightarrow 1 - p(0) = 1 - (9/10)^{10}$$

Therefore, the probability that there is at least one defective egg is $1 - (9/10)^{10}$.

25. Find the variance of the number obtained on a throw of an unbiased die.

Ans: Let the number obtained on an unbiased die is X .

The probability of getting each number is equal in an unbiased die. Here, it is $1/6$.

Probability distribution in an unbiased die:

X	1	2	3	4	5	6
$P(X)$	$1/6$	$1/6$	$1/6$	$1/6$	$1/6$	$1/6$

It is known that,

Mean Expectation value is $E(X) = \sum_{i=1}^n x_i p_i$

$$\Rightarrow E(X) = 1 \times \frac{1}{6} + 2 \times \frac{1}{6} + 3 \times \frac{1}{6} + 4 \times \frac{1}{6} + 5 \times \frac{1}{6} + 6 \times \frac{1}{6}$$

$$\Rightarrow E(X) = 216 \Rightarrow E(X) = \frac{21}{6}$$

Variance: $Var(X) = E(X^2) - [E(X)]^2$

$$E(X^2) = \sum_{i=1}^n x_i^2 p_i$$

$$\Rightarrow E(X^2) = 1^2 \times \frac{1}{6} + 2^2 \times \frac{1}{6} + 3^2 \times \frac{1}{6} + 4^2 \times \frac{1}{6} + 5^2 \times \frac{1}{6} + 6^2 \times \frac{1}{6}$$

$$\Rightarrow E(X^2) = 1 + 4 + 9 + 16 + 25 + 36 \Rightarrow E(X^2) = \frac{1 + 4 + 9 + 16 + 25 + 36}{6}$$

$$\Rightarrow E(X^2) = 916 \Rightarrow E(X^2) = \frac{91}{6}$$

$$\Rightarrow E(X^2) = 916$$

$$\Rightarrow E(X^2) = \frac{91}{6}$$

$Var(X) = E(X^2) - [E(X)]^2$

$$\Rightarrow Var(X) = 916 - \left[\frac{21}{6}\right]^2$$

$$\Rightarrow Var(X) = 916 - 44136 \Rightarrow Var(X) = \frac{91}{6} - \frac{441}{36}$$

$$\Rightarrow Var(X) = 546 - 44136 = 10536 \Rightarrow Var(X) = \frac{546 - 441}{36} = \frac{105}{36}$$

$$\Rightarrow Var(X) = 3512 \Rightarrow Var(X) = \frac{35}{12}$$

Therefore, the variance of the number obtained on a throw of an unbiased die is $3512 \frac{35}{12}$.

Long Answer Questions (6 Marks)

26. In a hurdle race, a player has to cross 88 hurdles. The probability that he will clear a hurdle is $\frac{4}{5}$, what is the probability that he will knock down in fewer than 22 hurdles?

Ans: Let X be the event of a number of hurdles that the player knocks down.

From Binomial distribution,

$$P(X=x) = {}^n C_x q^{n-x} p^x$$

It is given that,

Number of hurdles, $n=8$

Probability that he will clear the hurdle, $q=4/5$

Probability that he will not clear the hurdle, $p=1-4/5=1/5$

$$P(X=x) = {}^8C_x (4/5)^{8-x} (1/5)^x$$

Probability that he will knock down fewer than 22 hurdles:

Probability = P(Knock 0 hurdles) + P(Knock 1 hurdles)

$$\text{Probability} = P(\text{Knock 0 hurdles}) + P(\text{Knock 1 hurdles})$$

$$\Rightarrow \text{Probability} = P(X=0) + P(X=1) \Rightarrow \text{Probability} = P(X=0) + P(X=1)$$

$$\Rightarrow \text{Probability} = {}^8C_0 (4/5)^{8-0} (1/5)^0 + {}^8C_1 (4/5)^{8-1} (1/5)^1$$

$$\Rightarrow \text{Probability} = {}^8C_0 (4/5)^8 (1/5)^0 + {}^8C_1 (4/5)^7 (1/5)^1$$

$$\Rightarrow \text{Probability} = 1 \times (4/5)^8 \times 1 + 8 \times (4/5)^7 \times (1/5)$$

$$\Rightarrow \text{Probability} = (4/5)^7 [(4/5) + 8 \times (1/5)]$$

$$\Rightarrow \text{Probability} = (4/5)^7 [12/5]$$

Therefore, the probability that he will knock down in fewer than 22 hurdles is $12/5 (4/5)^7$.

27. Bag AA contains 44 red, 33 white and 22 black balls. Bag BB contains 33 red, 22 white and 33 black balls. One ball is transferred from bag AA to bag BB and then a ball is drawn from bag BB. The ball so drawn is found to be red. Find the probability that the transferred ball is black.

Ans: Consider the following notations:

A_1A_1 : One red ball is transferred from bag AA to bag BB

A_2A_2 : One white ball is transferred from bag AA to bag BB

A_3A_3 : One black ball is transferred from bag AA to bag BB

E : Drawing a red ball from bag BB

$P(A_1)$ = Number of red balls in bag A / Total number of balls in bag A

$$P(A_1) = \frac{\text{Number of red balls in bag A}}{\text{Total number of balls in bag A}}$$

$$\Rightarrow P(A_1) = 4 + 3 + 2 = 9$$

$$\Rightarrow P(A_1) = \frac{4}{4 + 3 + 2} = \frac{4}{9}$$

$P(A_2)$ = Number of white balls in bag A / Total number of balls in bag A

$$P(A_2) = \frac{\text{Number of white balls in bag A}}{\text{Total number of balls in bag A}}$$

$$\Rightarrow P(A_2) = 3 + 3 + 2 = 8$$

$$\Rightarrow P(A_2) = \frac{3}{4 + 3 + 2} = \frac{3}{9}$$

$P(A_3)$ = Number of black balls in bag A / Total number of balls in bag A

$$P(A_3) = \frac{\text{Number of black balls in bag A}}{\text{Total number of balls in bag A}}$$

$$\Rightarrow P(A_3) = 3 + 3 + 2 = 8$$

$$\Rightarrow P(A_3) = \frac{3}{4 + 3 + 2} = \frac{2}{9}$$

If event A_1 occurs, number of red balls in bag B = $(3+1) = 4 = (3 + 1) = 4$

Total number of balls in bag B = $(3+2+3)+1 = 9 = (3 + 2 + 3) + 1 = 9$

$P(E/A_1)$ = Number of red balls in bag B / Total number of balls in bag B $P(E/A_1) = \frac{\text{Number of red balls in bag B}}{\text{Total number of balls in bag B}}$

$$\Rightarrow P(E/A_1) = 4 \Rightarrow P(E/A_1) = \frac{4}{9}$$

If event A_2 occurs, number of red balls in bag B = $3 = 3$

Total number of balls in bag B = $(3+2+3)+1 = 9 = (3 + 2 + 3) + 1 = 9$

$P(E/A_2)$ = Number of black balls in bag B / Total number of balls in bag B $P(E/A_2) = \frac{\text{Number of black balls in bag B}}{\text{Total number of balls in bag B}}$

$$\Rightarrow P(E/A_2) = 3 \Rightarrow P(E/A_2) = \frac{3}{9}$$

If event A_3 occurs, number of red balls in bag B = $3 = 3$

Total number of balls in bag B = $(3+2+3)+1 = 9 = (3 + 2 + 3) + 1 = 9$

$$P(E/A_3) = \frac{\text{Number of black balls in bag B}}{\text{Total number of balls in bag B}} = \frac{\text{Number of black balls in bag B}}{\text{Total number of balls in bag B}}$$

$$\Rightarrow P(E/A_3) = \frac{3}{9} \Rightarrow P(E/A_3) = \frac{3}{9}$$

From Bayes theorem:

$$P(A_3/E) = \frac{P(A_3)P(E/A_3)}{P(A_1)P(E/A_1) + P(A_2)P(E/A_2) + P(A_3)P(E/A_3)}$$

$$\Rightarrow P(A_3/E) = \frac{29 \times 39}{49 \times 49 + 39 \times 39 + 29 \times 39}$$

$$\Rightarrow P(A_3/E) = \frac{\frac{2}{9} \times \frac{3}{9}}{\frac{4}{9} \times \frac{4}{9} + \frac{3}{9} \times \frac{3}{9} + \frac{2}{9} \times \frac{3}{9}}$$

$$\Rightarrow P(A_3/E) = \frac{6}{16 + 9 + 6} = \frac{6}{31}$$

$$\Rightarrow P(A_3/E) = \frac{6}{16 + 9 + 6} = \frac{6}{31}$$

28. If a fair coin is tossed 10 times, find the probability of getting.

a. exactly six heads,

Ans: Let X be the number of heads obtained.

Number of tosses, $n = 10$

Probability of head, $p = \frac{1}{2}$

Probability of no getting head, $q = 1 - p = \frac{1}{2}$

From binomial distribution: $P(X=x) = {}^n C_x q^{n-x} p^x$

Here, $x = 6$

$$\Rightarrow P(X=6) = {}^{10} C_6 (1/2)^{10-6} (1/2)^6$$

$$\Rightarrow P(X=6) = {}^{10} C_6 \left(\frac{1}{2}\right)^{10-6} \left(\frac{1}{2}\right)^6$$

$$\Rightarrow P(X=6) = \frac{10!}{(10-6)!6!} \left(\frac{1}{2}\right)^{10}$$

$$\Rightarrow P(X=6)=10 \times 9 \times 8 \times 7 \times 4 \times 3 \times 2 \times 1 (12)10 \Rightarrow P(X=6) = \frac{10 \times 9 \times 8 \times 7}{4 \times 3 \times 2 \times 1} \left(\frac{1}{2}\right)^{10}$$

$$\Rightarrow P(X=6)=105512 \Rightarrow P(X=6) = \frac{105}{512}$$

b. at least six heads,

Ans: At least six heads means that $\Rightarrow P(x \geq 6) \Rightarrow P(x \geq 6)$

$$P(x \geq 6) = P(x=6) + P(x=7) + P(x=8) + P(x=9) + P(x=10)$$

$$P(x \geq 6) = P(x=6) + P(x=7) + P(x=8) + P(x=9) + P(x=10)$$

$$\Rightarrow P(x \geq 6) = 10C_6(12)10 + 10C_7(12)10 + 10C_8(12)10 + 10C_9(12)10 + 10C_{10}(12)10$$

$$\Rightarrow P(x \geq 6) = 10C_6 \left(\frac{1}{2}\right)^{10} + 10C_7 \left(\frac{1}{2}\right)^{10} + 10C_8 \left(\frac{1}{2}\right)^{10} + 10C_9 \left(\frac{1}{2}\right)^{10} + 10C_{10} \left(\frac{1}{2}\right)^{10}$$

$$\Rightarrow P(x \geq 6) = [10C_6 + 10C_7 + 10C_8 + 10C_9 + 10C_{10}](12)10$$

$$\Rightarrow P(x \geq 6) = [10C_6 + 10C_7 + 10C_8 + 10C_9 + 10C_{10}] \left(\frac{1}{2}\right)^{10}$$

$$\Rightarrow P(x \geq 6) = [10 \times 9 \times 8 \times 7 \times 4 \times 3 \times 2 \times 1 + 10 \times 9 \times 8 \times 3 \times 2 \times 1 + 10 \times 9 \times 2 \times 1 + 10 + 1](12)10$$

$$\Rightarrow P(x \geq 6) = \left[\frac{10 \times 9 \times 8 \times 7}{4 \times 3 \times 2 \times 1} + \frac{10 \times 9 \times 8}{3 \times 2 \times 1} + \frac{10 \times 9}{2 \times 1} + 10 + 1 \right] \left(\frac{1}{2}\right)^{10}$$

$$\Rightarrow P(x \geq 6) = [210 + 120 + 45 + 10 + 1](12)10 \Rightarrow P(x \geq 6) = [210 + 120 + 45 + 10 + 1] \left(\frac{1}{2}\right)^{10}$$

$$\Rightarrow P(x \geq 6) = 193512 \Rightarrow P(x \geq 6) = \frac{193}{512}$$

c. at most six heads.

Ans: At most six heads means that $\Rightarrow P(x \leq 6) \Rightarrow P(x \leq 6)$

$$P(x \leq 6) = P(x=6) + P(x=5) + P(x=4) + P(x=3) + P(x=2) + P(x=1) + P(x=0)$$

$$P(x \leq 6) = P(x=6) + P(x=5) + P(x=4) + P(x=3) + P(x=2) + P(x=1) + P(x=0)$$

$$P(x \leq 6) = 10C_6(12)10 + 10C_5(12)10 + 10C_4(12)10 + 10C_3(12)10 + 10C_2(12)10 + 10C_1(12)10 + 10C_0(12)10$$

$$P(x \leq 6) = 10C_6 \left(\frac{1}{2}\right)^{10} + 10C_5 \left(\frac{1}{2}\right)^{10} + 10C_4 \left(\frac{1}{2}\right)^{10} + 10C_3 \left(\frac{1}{2}\right)^{10} + 10C_2 \left(\frac{1}{2}\right)^{10} + 10C_1 \left(\frac{1}{2}\right)^{10} + 10C_0 \left(\frac{1}{2}\right)^{10}$$

$$\Rightarrow P(x \leq 6) = [10C_6 + 10C_5 + 10C_4 + 10C_3 + 10C_2 + 10C_1 + 10C_0](12)10$$

$$\Rightarrow P(x \leq 6) = [10C_6 + 10C_5 + 10C_4 + 10C_3 + 10C_2 + 10C_1 + 10C_0] \left(\frac{1}{2}\right)^{10}$$

$$\Rightarrow P(x \leq 6) = [10 \times 9 \times 8 \times 7 \times 4 \times 3 \times 2 \times 1 + 10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 + 10 \times 9 \times 8 \times 7 \times 4 \times 3 \times 2 \times 1 + 10 \times 9 \times 8 \times 3 \times 2 \times 1 + 10 \times 9 \times 2 \times 1 + 10 + 1](12)10$$

$$\Rightarrow P(x \leq 6) = \left[\frac{10 \times 9 \times 8 \times 7}{4 \times 3 \times 2 \times 1} + \frac{10 \times 9 \times 8 \times 7 \times 6}{5 \times 4 \times 3 \times 2 \times 1} + \frac{10 \times 9 \times 8 \times 7}{4 \times 3 \times 2 \times 1} + \frac{10 \times 9 \times 8}{3 \times 2 \times 1} + \frac{10 \times 9}{2 \times 1} + 10 + 1 \right] \left(\frac{1}{2}\right)^{10}$$

$$\Rightarrow P(x \leq 6) = [210 + 252 + 210 + 120 + 45 + 10 + 1](12)10$$

$$\Rightarrow P(x \leq 6) = [210 + 252 + 210 + 120 + 45 + 10 + 1] \left(\frac{1}{2}\right)^{10}$$

$$\Rightarrow P(x \leq 6) = (848)(12)10 \Rightarrow P(x \leq 6) = (848) \left(\frac{1}{2}\right)^{10}$$

$$\Rightarrow P(x \leq 6) = 5364 \Rightarrow P(x \leq 6) = \frac{53}{64}$$

29. A doctor is to visit a patient. From the past experience, it is known that the probabilities that he will come by train, bus, scooter by other means of transport are respectively $313\frac{3}{13}$, $15\frac{1}{5}$, $110\frac{1}{10}$ and $25\frac{2}{5}$. The probabilities that he will be late are $14\frac{1}{4}$, $13\frac{1}{3}$ and $112\frac{1}{12}$ if he comes by train, bus and scooter respectively but if comes by other means of transport, then he will not be late. When he arrives, he is late. What is the probability that he comes by train?

Ans: Consider the following notations:

EE : Event that the doctor visits the patient late

T_1T_1 : Event that the doctor comes by train, $\Rightarrow P(T_1) = 310 \Rightarrow P(T_1) = \frac{3}{10}$

T_2T_2 : Event that the doctor comes by bus, $\Rightarrow P(T_2) = 15 \Rightarrow P(T_2) = \frac{1}{5}$

T_3T_3 : Event that the doctor comes by scooter, $\Rightarrow P(T_3) = 110 \Rightarrow P(T_3) = \frac{1}{10}$

T_4T_4 : Event that the doctor comes by other means of transport, $\Rightarrow P(T_4) = 25 \Rightarrow P(T_4) = \frac{2}{5}$

Probability that the doctor arriving late comes by train, $P(E/T_1) = 14 \Rightarrow P(E/T_1) = \frac{1}{4}$

Probability that the doctor arriving late comes by bus, $P(E/T_2) = 13 \Rightarrow P(E/T_2) = \frac{1}{3}$

Probability that the doctor arriving late comes by scooter, $P(E/T_3) = 112 \Rightarrow P(E/T_3) = \frac{1}{12}$

Probability that the doctor arriving late comes by other transport, $P(E/T_4) = 0 \Rightarrow P(E/T_4) = 0$

From Baye's theorem:

$$P(T_1/E) = \frac{P(T_1)P(E/T_1)}{P(T_1)P(E/T_1) + P(T_2)P(E/T_2) + P(T_3)P(E/T_3) + P(T_4)P(E/T_4)}$$

$$P(T_1/E) = \frac{P(T_1)P(E/T_1)}{P(T_1)P(E/T_1) + P(T_2)P(E/T_2) + P(T_3)P(E/T_3) + P(T_4)P(E/T_4)}$$

$$\Rightarrow P(T_1/E) = 310 \times 14 + 15 \times 13 + 110 \times 112 + 25 \times 0$$

$$\Rightarrow P(T_1/E) = \frac{\frac{3}{10} \times \frac{1}{4}}{\frac{3}{10} \times \frac{1}{4} + \frac{1}{5} \times \frac{1}{3} + \frac{1}{10} \times \frac{1}{12} + \frac{2}{5} \times 0}$$

$$\Rightarrow P(T_1/E) = \frac{340}{340+115+1120+0}$$

$$\Rightarrow P(T_1/E) = \frac{\frac{3}{40}}{\frac{3}{40} + \frac{1}{15} + \frac{1}{120} + 0}$$

$$\Rightarrow P(T_1/E) = \frac{340}{18120} = \frac{1}{53.29}$$

$$\Rightarrow P(T_1/E) = \frac{\frac{3}{40}}{\frac{3}{40} + \frac{1}{15} + \frac{1}{120}} = \frac{1}{2}$$

Therefore, the probability that the doctor arrives late by train is $\frac{1}{2}$.

30. A man is known to speak the truth 33 out of 44 times. He throws a die and reports that it is six. Find the probability that it is actually a six.

Ans: Let A be the event that the number on die is 6.

It is given that,

$$\text{Probability that the man speaks truth, } P(T) = \frac{33}{44} = \frac{3}{4}$$

$$\text{Probability that the man lies, } P(F) = 1 - \frac{33}{44} = \frac{11}{44} = \frac{1}{4}$$

$$\text{Probability of getting 6, } P(A/T) = \frac{1}{6}, P(A/F) = \frac{1}{6}$$

$$\text{Probability of not getting 6, } P(A/F) = 1 - \frac{1}{6} = \frac{5}{6}$$

From Bayes theorem:

$$P(T/A) = \frac{P(T)P(A/T)}{P(T)P(A/T) + P(F)P(A/F)}$$

$$P(T/A) = \frac{P(T)P(A/T)}{P(T)P(A/T) + P(F)P(A/F)}$$

$$\Rightarrow P(T/A) = \frac{33 \times 1}{33 \times 1 + 11 \times 5}$$

$$\Rightarrow P(T/A) = \frac{\frac{3}{4} \times \frac{1}{6}}{\frac{3}{4} \times \frac{1}{6} + \frac{1}{4} \times \frac{5}{6}}$$

$$\Rightarrow P(T/A) = \frac{33}{33+5} = \frac{33}{38}$$

$$\Rightarrow P(T/A) = \frac{3}{3+5} = \frac{3}{8}$$

Therefore, Probability that its actually 6 is $\frac{3}{8}$.

31. An insurance company insured 2000 scooter drivers, 4000 car drivers and 6000 truck drivers. The probability of an accidents are 0.01, 0.03 and 0.15 respectively if one of the insured persons meets with an accident. What is the probability that he is a scooter driver?

Ans: Consider the following notations:

E : The event when the driver drives scooter

F : The event when the driver drives car

G : The event when the driver drives truck

K : The event that the driver meets accident

From the given data:

$$P(E) = \frac{2000}{12000}$$

$$P(F) = \frac{4000}{12000}$$

$$P(G) = \frac{6000}{12000}$$

$$P(K|E) = \frac{1}{100} \quad (K|E) : \text{The driver meets accident provided he drives scooter}$$

$$P(K|F) = \frac{3}{100} \quad (K|F) : \text{The driver meets accident provided he drives car}$$

$$P(K|G) = \frac{15}{100} \quad (K|G) : \text{The driver meets accident provided he drives truck}$$

From Bayes theorem:

$$P(E|K) = \frac{P(E)P(K|E)}{P(E)P(K|E) + P(F)P(K|F) + P(G)P(K|G)}$$

$$P(E|K) = \frac{P(E)P(K|E)}{P(E)P(K|E) + P(F)P(K|F) + P(G)P(K|G)}$$

$$\Rightarrow P(E|K) = \frac{16 \times 1100}{16 \times 1100 + 13 \times 3100 + 12 \times 15100}$$

$$\Rightarrow P(E|K) = \frac{\frac{1}{6} \times \frac{1}{100}}{\frac{1}{6} \times \frac{1}{100} + \frac{1}{3} \times \frac{3}{100} + \frac{1}{2} \times \frac{15}{100}}$$

$$\Rightarrow P(E|K) = \frac{16}{16 + 13 + 152}$$

$$\Rightarrow P(E|K) = \frac{\frac{1}{6}}{\frac{1}{6} + 1 + \frac{15}{2}}$$

$$\Rightarrow P(E|K) = \frac{1}{52}$$

$$\Rightarrow P(E|K) = \frac{1}{52}$$

Therefore, the probability that the accidental person is a scooter driver is $\frac{1}{52}$.

32. Two cards from a pack of 52 cards are lost. One card is drawn from the remaining cards. If a drawn card is a heart, find the probability that the lost cards were both hearts.

Ans: Consider the following events:

E_1 be the event both lost cards are hearts.

E_2 be the event both lost cards are non hearts.

E_3 be the event one lost card is non heart and one is heart.

A be the event of picking a heart from remaining 50 cards.

$$P(E_1) = \frac{13C_2}{52C_2} = \frac{13 \times 12}{52 \times 51}$$

$$\Rightarrow P(E_1) = \frac{1}{17}$$

$$P(A|E_1) = \frac{11}{50}$$

$$P(E_2) = \frac{39C_2}{52C_2} = \frac{39 \times 38}{52 \times 51}$$

$$\Rightarrow P(E_2) = \frac{19}{34}$$

$$P(A|E_2) = \frac{13}{50}$$

$$P\left(\frac{A}{E_2}\right) = \frac{13}{50}$$

$$P(E_3) = \frac{(13 \times 39) + (39 \times 13)}{52C_2} = \frac{(One\ heart) \text{ and } (One\ non\ heart)}{52C_2}$$

$$P(E_3) = \frac{13 \times 39 \times 2}{52 \times 51}$$

$$\Rightarrow P(E_3) = \frac{39}{102}$$

$$P(A|E_3) = \frac{12}{50}$$

From Baye's theorem:

$$P(E_1|A) = \frac{P(E_1)P(A|E_1)}{P(E_1)P(A|E_1) + P(E_2)P(A|E_2) + P(E_3)P(A|E_3)}$$

$$P(E_1|A) = \frac{P(E_1)P(A|E_1)}{P(E_1)P(A|E_1) + P(E_2)P(A|E_2) + P(E_3)P(A|E_3)}$$

$$\Rightarrow P(E_1/A) = 117 \times 1150 + 1934 \times 1350 + 39102 \times 1250$$

$$\Rightarrow P(E_1/A) = \frac{\frac{1}{17} \times \frac{11}{50}}{\frac{1}{17} \times \frac{11}{50} + \frac{19}{34} \times \frac{13}{50} + \frac{39}{102} \times \frac{12}{50}}$$

$$\Rightarrow P(E_1/A) = 1111 + 2472 + 78$$

$$\Rightarrow P(E_1/A) = \frac{11}{11 + \frac{247}{2} + 78}$$

$$\Rightarrow P(E_1/A) = 2222 + 247 + 156$$

$$\Rightarrow P(E_1/A) = \frac{22}{22 + 247 + 156}$$

$$\Rightarrow P(E_1/A) = 22425$$

$$\Rightarrow P(E_1/A) = \frac{22}{425}$$

Therefore, the probability that the lost cards were both hearts is $22425 \frac{22}{425}$.

33. A box XX contains 22 white and 33 red balls and a bag YY contains 44 white and 55 red balls. One ball is drawn at random from one of the bags and is found to be red. Find the probability that it was drawn from bag YY.

Ans: Let the event of selecting a red ball be RR.

Let the event of selecting the bag XX be XX.

Let the event of selecting the bag YY be YY.

It is given that,

Bag XX has 22 white and 33 red balls.

Bag YY has 44 white and 55 red balls.

$$\Rightarrow P(X) = P(Y) = \frac{1}{2} \Rightarrow P(X) = P(Y) = \frac{1}{2}$$

$$P(R/X) = \frac{3}{5} P(R/X) = \frac{3}{5}$$

$$P(R/Y) = \frac{5}{9} P(R/Y) = \frac{5}{9}$$

From Bayes theorem:

$$P(Y/R) = \frac{P(Y)P(R/Y)}{P(X)P(R/X) + P(Y)P(R/Y)}$$

$$P(Y/R) = \frac{P(Y)P(R/Y)}{P(X)P(R/X) + P(Y)P(R/Y)}$$

$$\Rightarrow P(Y/R) = 59 \times 1235 \times 12 + 59 \times 12$$

$$\Rightarrow P(Y/R) = \frac{\frac{5}{9} \times \frac{1}{2}}{\frac{3}{5} \times \frac{1}{2} + \frac{5}{9} \times \frac{1}{2}}$$

$$\Rightarrow P(Y/R) = 2527 + 25 = 2552$$

$$\Rightarrow P(Y/R) = \frac{25}{27 + 25} = \frac{25}{52}$$

Therefore, the probability that the ball is red and drawn from bag YY is $\frac{25}{52}$.

34. In answering a question on a multiple choice, a student either knows the answer or guesses. Let $\frac{3}{4}$ be the probability that he knows the answer and $\frac{1}{4}$ be the probability that he guesses. Assuming that a student who guesses at the answer will be incorrect with probability $\frac{1}{4}$. What is the probability that the student knows the answer, given that he answered correctly?

Ans: Let E_1 and E_2 be the respective events in which the student knows the answer and guesses the answer.

Let A be the event that the answer is correct.

From the given data,

$$P(E_1) = \frac{3}{4}$$

$$P(E_2) = \frac{1}{4}$$

The probability that the student answered correctly by knowing the answer is 11.

$$\Rightarrow P(A/E_1) = 1 \Rightarrow P(A/E_1) = 1$$

The probability that the student answered correctly by guessing the answer is $\frac{1}{4}$.

$$\Rightarrow P(A/E_2) = \frac{1}{4} \Rightarrow P(A/E_2) = \frac{1}{4}$$

From Bayes theorem:

$$P(E_1/A) = \frac{P(E_1)P(A/E_1)}{P(E_1)P(A/E_1) + P(E_2)P(A/E_2)}$$

$$\Rightarrow P(E_1/A) = 34 \times 134 \times 1 + 14 \times 14 = 3434 + 116$$

$$\Rightarrow P(E_1/A) = \frac{\frac{3}{4} \times 1}{\frac{3}{4} \times 1 + \frac{1}{4} \times \frac{1}{4}} = \frac{\frac{3}{4}}{\frac{3}{4} + \frac{1}{16}}$$

$$\Rightarrow P(E_1/A) = 341316$$

$$\Rightarrow P(E_1/A) = \frac{\frac{3}{4}}{\frac{13}{16}}$$

$$\Rightarrow P(E_1/A) = 1213$$

$$\Rightarrow P(E_1/A) = \frac{12}{13}$$

Therefore, the probability that the student answers correctly by knowing the answer is $1213 \frac{12}{13}$.

35. Suppose a girl throws a die. If she gets 5 or 6 She tosses a coin three times and notes the number of heads. If she gets 1, 2, 3 or 4 She tosses a coin once and notes whether a head or tail is obtained. If she obtained exactly one head. What is the probability that she throws 1, 2, 3 or 4 with the die?

Ans: Consider the following notations:

E : The event when 5 or 6 come

F : The event when 1, 2, 3 or 4 come

K : The event that the coin shows exactly one head

From the given data:

$$P(E) = \frac{2}{6}$$

$$P(F) = \frac{4}{6}$$

$P(K|E) = \frac{3}{8}$: The event that the coin shows exactly one head provided 5 or 6 come

$P(K|F) = \frac{1}{2}$: The event that the single throw of coin shows exactly one head provided 1, 2, 3 or 4 come

From Bayes theorem

$$P(F|K) = \frac{P(F)P(K|F)}{P(E)P(K|E) + P(F)P(K|F)}$$

$$\Rightarrow P(F|K) = 46 \times 1226 \times 38 + 46 \times 12$$

$$\Rightarrow P(F|K) = \frac{\frac{4}{6} \times \frac{1}{2}}{\frac{2}{6} \times \frac{3}{8} + \frac{4}{6} \times \frac{1}{2}}$$

$$\Rightarrow P(F|K) = 234 + 2 = 811$$

$$\Rightarrow P(F|K) = \frac{2}{\frac{3}{4} + 2} = \frac{8}{11}$$

Therefore, the probability that the item is produced from the second group is $811 \frac{8}{11}$.

36. In a bolt factory machines AA, BB and CC manufacture 6060, 3030 and 1010 of the total bolts respectively, 22, 55 and 1010 of the bolts produced by them respectively are defective. A bolt is picked up at random from the product and is found to be defective. What is the probability that it has been manufactured by machine AA?

Ans: It is given that,

In a bolt factory machines AA, BB and CC manufacture 6060, 3030 and 1010 of the total bolts.

Let total bolts be x .

$$\text{Number of bolts in Machine A} = x \times \frac{60}{100} = 0.6x$$

$$\text{Number of bolts in Machine B} = x \times \frac{30}{100} = 0.3x$$

$$\text{Number of bolts in Machine C} = x \times \frac{10}{100} = 0.1x$$

$$\text{Probability of picking a bolt from AA, } P(A) = \frac{0.6x}{0.6x + 0.3x + 0.1x} = 0.6x$$

$$\text{Probability of picking a bolt from BB, } P(B) = \frac{0.3x}{0.6x + 0.3x + 0.1x} = 0.3x$$

$$\text{Probability of picking a bolt from CC, } P(C) = \frac{0.1x}{0.6x + 0.3x + 0.1x} = 0.1x$$

Let DD be the event of a bolt picked up at random from the product and is found to be defective.

From the given data,

$$P(D/A) = \frac{22}{6060} = 0.02$$

$$P(D/B) = \frac{55}{3030} = 0.05$$

$$P(D/C) = \frac{10}{1010} = 0.1$$

$$P(D/C) = \frac{10}{100} = 0.1$$

From Baye's theorem:

$$P(A/D) = \frac{P(A)P(D/A)}{P(A)P(D/A) + P(B)P(D/B) + P(C)P(D/C)}$$

$$P(A/D) = \frac{0.6 \times 0.02}{0.6 \times 0.02 + 0.3 \times 0.05 + 0.1 \times 0.1}$$

$$\Rightarrow P(A/D) = 0.6 \times 0.02 + 0.3 \times 0.05 + 0.1 \times 0.1$$

$$\Rightarrow P(A/D) = \frac{0.6 \times 0.02}{0.6 \times 0.02 + 0.3 \times 0.05 + 0.1 \times 0.1}$$

$$\Rightarrow P(A/D) = 12 + 15 + 10$$

$$\Rightarrow P(A/D) = \frac{12}{12 + 15 + 10}$$

$$\Rightarrow P(A/D) = 12/37$$

$$\Rightarrow P(A/D) = \frac{12}{37}$$

Therefore, probability of a bolt picked up at random from the product and is found to be defective and is from machine A is $\frac{12}{37}$.

37. Two urns A and B contain 66 black and 44 white, 44 black and 66 white balls respectively. Two balls are drawn from one of the urns. If both the balls drawn are white, find the probability that the balls are drawn from urn B.

Ans: Let E be the event of drawing two white balls.

It is given that,

Urn A contains 66 black and 44 white balls.

Urn B contains 44 black and 66 white balls.

Probability of choosing Urn A or Urn B is $P(A) = P(B) = \frac{1}{2}$

Probability of choosing two white balls from

$$A = P(E/A)$$

$$A = P(E/A)$$

$$\Rightarrow P(E/A) = \frac{44C_2}{110C_2} = \frac{6}{45}$$

$$\Rightarrow P(E/A) = \frac{44C_2}{110C_2} = \frac{6}{45}$$

Probability of choosing two white balls from

$$B=P(E/B)$$

$$B = P (E / B)$$

$$\Rightarrow P(E/B)=6C_210C_2=1545$$

$$\Rightarrow P (E / B) = \frac{6C_2}{10C_2} = \frac{15}{45}$$

From Bayes theorem:

$$P(B/E)=P(B)P(E/B)P(A)P(E/A)+P(B)P(E/B)$$

$$P (B / E) = \frac{P (B) P (E / B)}{P (A) P (E / A) + P (B) P (E / B)}$$

$$\Rightarrow P(B/E)=12 \times 1545 + 12 \times 645 + 12 \times 1545 = 156 + 15$$

$$\Rightarrow P (B / E) = \frac{\frac{1}{2} \times \frac{15}{45}}{\frac{1}{2} \times \frac{6}{45} + \frac{1}{2} \times \frac{15}{45}} = \frac{15}{6 + 15}$$

$$\Rightarrow P(B/E)=1521=57$$

$$\Rightarrow P (B / E) = \frac{15}{21} = \frac{5}{7}$$

Therefore, the probability that the balls are drawn from urn BB is $57\frac{5}{7}$.

38. Two cards are drawn from a well shuffled pack of 52 cards. Find the mean and variance for the number of face cards obtained.

Ans: Let X be the event of the number of face cards.

When two cards are picked with replacement there are chances of getting either

0,1 or 2

0 , 1 or 2

face cards.

Number of face cards in a deck = 12 = 12

Number of events in sample space, $n(S) = 52n (S) = 52$

$P(N)$ be the probability of not picking face card , $P(N) = \frac{40}{52}P (N) = \frac{40}{52}$

$P(F)$ be the probability of picking a face card , $P(F) = \frac{12}{52}P (F) = \frac{12}{52}$

For $X=0$:

Probability, $P\{NN\} = \frac{40}{52} \times \frac{40}{52}$

$$\Rightarrow P\{NN\}=4052 \times 4052 = 100169 \Rightarrow P\{NN\} = \frac{40}{52} \times \frac{40}{52} = \frac{100}{169}$$

For $X=1$:

Probability,

$$P(NF \text{ or } FN) = P(N)P(F) + P(F)P(N)$$

$$P(NF \text{ or } FN) = P(N)P(F) + P(F)P(N)$$

$$\Rightarrow P(NF \text{ or } FN) = 2 \times 4052 \times 1252 = 60169$$

$$\Rightarrow P(NF \text{ or } FN) = 2 \times \frac{40}{52} \times \frac{12}{52} = \frac{60}{169}$$

For $X=2$:

Probability,

$$P(FF) = 1252 \times 1252$$

$$P(FF) = \frac{12}{52} \times \frac{12}{52}$$

$$\Rightarrow P(FF) = 9169$$

$$\Rightarrow P(FF) = \frac{9}{169}$$

Probability distribution:

XX	00	11	22
$P(X)P(X)$	100169 $\frac{100}{169}$	60169 $\frac{60}{169}$	9169 $\frac{9}{169}$

Mean Expectation value is $\mu = E(X) = \sum_{i=1}^n x_i p_i$ $\mu = E(X) = \sum_{i=1}^n x_i p_i$

$$\Rightarrow \mu = 0 \times 100169 + 1 \times 60169 + 2 \times 9169 \Rightarrow \mu = 0 \times \frac{100}{169} + 1 \times \frac{60}{169} + 2 \times \frac{9}{169}$$

$$\Rightarrow \mu = 0 + 100169 + 60169 + 18169 \Rightarrow \mu = 0 + \frac{100}{169} + \frac{60}{169} + \frac{18}{169}$$

$$\Rightarrow \mu = 78169 = 613 \Rightarrow \mu = \frac{78}{169} = \frac{6}{13}$$

Therefore, the mean number of heads is $613 \frac{6}{13}$.

$$\text{Variance: } \text{Var}(X) = E(X^2) - [E(X)]^2 \Rightarrow \text{Var}(X) = E(X^2) - [E(X)]^2$$

$$E(X^2) = \sum_{i=1}^n x_i^2 p_i$$

$$\Rightarrow E(X^2) = 0^2 \times \frac{100}{169} + 1^2 \times \frac{60}{169} + 2^2 \times \frac{9}{169}$$

$$\Rightarrow E(X^2) = 0 + 60 + 36 \Rightarrow E(X^2) = \frac{0 + 60 + 36}{169}$$

$$\Rightarrow E(X^2) = 96$$

$$\Rightarrow E(X^2) = \frac{96}{169}$$

$$\text{Var}(X) = E(X^2) - [E(X)]^2$$

$$\Rightarrow \text{Var}(X) = 96 - \left[\frac{6}{13}\right]^2$$

$$\Rightarrow \text{Var}(X) = 96 - 36 \Rightarrow \text{Var}(X) = \frac{96 - 36}{169}$$

$$\Rightarrow \text{Var}(X) = 60 \Rightarrow \text{Var}(X) = \frac{60}{169}$$

Therefore, the variance of the number of heads is $60 \frac{60}{169}$.

39. Write the probability distribution for the number of heads obtained when three coins are tossed together. Also, find the mean and variance of the number of heads.

Ans: Let X be the event of the number of heads.

When three coins are tossed simultaneously there are chances of getting either

0, 1, 2 or 3

0, 1, 2 or 3

heads.

Sample space, $S = \{TTT, HTT, THT, TTH, HHT, HTH, THH, HHH\}$

$S = \{TTT, HTT, THT, TTH, HHT, HTH, THH, HHH\}$

Number of events in sample space, $n(S) = 8$

For $X=0$: Possible outcome $\{TTT\}$

Number of possible outcomes = 1

Probability,

$P(X)=18$

$$P(X) = \frac{1}{8}$$

For $X=1$: Possible outcome{HTT,THT,TTH} { *HTT* , *THT* , *TTH* }

Number of possible outcomes=3 = 3

Probability,

$P(X)=38$

$$P(X) = \frac{3}{8}$$

For $X=2$: Possible outcome{HHT,HTH,THH} { *HHT* , *HTH* , *THH* }

Number of possible outcomes=3 = 3

Probability,

$P(X)=38$

$$P(X) = \frac{3}{8}$$

For $X=3$: Possible outcome{HHH} { *HHH* }

Number of possible outcomes=1 = 1

Probability,

$P(X)=18$

$$P(X) = \frac{1}{8}$$

Probability distribution:

X	00	11	22	33
$P(X)$	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

Mean Expectation value is $\mu=E(X)=\sum_{i=1}^n x_i p_i$ $\mu = E(X) = \sum_{i=1}^n x_i p_i$

$$\Rightarrow \mu = 0 \times 18 + 1 \times 38 + 2 \times 38 + 3 \times 18 \Rightarrow \mu = 0 \times \frac{1}{8} + 1 \times \frac{3}{8} + 2 \times \frac{3}{8} + 3 \times \frac{1}{8}$$

$$\Rightarrow \mu = 0 + 38 + 68 + 38 \Rightarrow \mu = 0 + \frac{3}{8} + \frac{6}{8} + \frac{3}{8}$$

$$\Rightarrow \mu = 128 = 32 \Rightarrow \mu = \frac{12}{8} = \frac{3}{2}$$

Therefore, the mean number of heads is $32\frac{3}{2}$.

$$\text{Variance: } \text{Var}(X) = E(X^2) - [E(X)]^2 \quad \text{Var}(X) = E(X^2) - [E(X)]^2$$

$$E(X^2) = \sum_{i=1}^n x_i^2 p_i$$

$$\Rightarrow E(X^2) = 0^2 \times 18 + 1^2 \times 38 + 2^2 \times 38 + 3^2 \times 18 \Rightarrow E(X^2) = 0^2 \times \frac{1}{8} + 1^2 \times \frac{3}{8} + 2^2 \times \frac{3}{8} + 3^2 \times \frac{1}{8}$$

$$\Rightarrow E(X^2) = 0 + 3 + 12 + 98 \Rightarrow E(X^2) = \frac{0 + 3 + 12 + 9}{8}$$

$$\Rightarrow E(X^2) = 248 = 3$$

$$\Rightarrow E(X^2) = \frac{24}{8} = 3$$

$$\text{Var}(X) = E(X^2) - [E(X)]^2 \quad \text{Var}(X) = E(X^2) - [E(X)]^2$$

$$\Rightarrow \text{Var}(X) = 3 - [32]^2 \Rightarrow \text{Var}(X) = 3 - \left[\frac{3}{2}\right]^2$$

$$\Rightarrow \text{Var}(X) = 12 - 94 \Rightarrow \text{Var}(X) = \frac{12 - 9}{4}$$

$$\Rightarrow \text{Var}(X) = 34 \Rightarrow \text{Var}(X) = \frac{3}{4}$$

Therefore, the variance of the number of heads is $34\frac{3}{4}$.

40. Two groups are competing for the position on the Board of Directors of a corporation. The probabilities that the first and the second groups will win are 0.60.6 and 0.40.4 respectively. Further, if the first group wins, the probability of introducing a new product is 0.70.7 and the corresponding probability is 0.30.3 if the second group wins. Find the probability that the new product introduced was by the second group.

Ans: Consider the following notations:

EE : The event when the first group wins

FF : The event when the second group wins

KK : The event that the new item is produced

From the given data:

$$P(E)=0.6 \quad P(E) = 0.6$$

$$P(F)=0.4 \quad P(F) = 0.4$$

$P(K|E)=0.7 \quad P(K|E) = 0.7$ (K|E) (K | E) : The event that the item is introduced by first group

$P(K|F)=0.3 \quad P(K|F) = 0.3$ (K|F) (K | F) : The event that item is introduced by second group

From Bayes theorem

$$P(F|K)=\frac{P(F)P(K|F)P(E)}{P(E)P(K|E)+P(F)P(K|F)}$$

$$\Rightarrow P(F|K)=0.4 \times 0.3 \div (0.6 \times 0.7 + 0.4 \times 0.3)$$

$$\Rightarrow P(F|K) = \frac{0.4 \times 0.3}{0.6 \times 0.7 + 0.4 \times 0.3}$$

$$\Rightarrow P(F|K)=\frac{12}{42+12}$$

$$\Rightarrow P(F|K) = \frac{12}{42+12}$$

$$\Rightarrow P(F|K)=\frac{12}{54}=\frac{2}{9}$$

$$\Rightarrow P(F|K) = \frac{12}{54} = \frac{2}{9}$$

Thus, the probability that the item is produced from the second group is $\frac{2}{9}$.