Important Questions Class 12 Maths Chapter 3 Matrices

1 Mark Questions

1. If a matrix has 8 elements, what are the possible orders it can have.
Ans.

$$1\times8,\ \ 8\times1,\ \ 4\times2,\ \ 2\times4,$$

2. Identity matrix of orders n is denoted by.
Ans. I_n

3. Define square matrix
Ans. A matrix in which the no. of rows are equal to no. of columns i.e. m = n

4. The no. of all possible metrics of order 3 $\times$ 3 with each entry 0 or 1 is
Ans. $512=2^9$

5.

$$A=\begin{bmatrix}1 & 4 & 7\\ 2 & 5 & 8\\ 3 & 6 & 9\end{bmatrix}$$

Write (1) a_{33}, a_{12} (ii) what is its order
Ans. (i) $a_{33} = 9$, $a_{12} = 4$
(ii) 3×3

6. Two matrices A = aij and B = bij are said to be equal if
Ans. They are of the same order.

7. Define Diagonal matrix
Ans. A square matrix in which every non – diagonal element is zero is called diagonal matrix.

8. Every diagonal element of a skew symmetric matrix is
Ans. Zero.

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9. If

$$A=\begin{bmatrix}\cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha\end{bmatrix}, then\ A+A'=I$$

Find α
Ans.

$$A+A'=\begin{bmatrix}\cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha\end{bmatrix}+\begin{bmatrix}\cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha\end{bmatrix}$$

$$=\begin{bmatrix}2\cos\alpha & 0 \\ 0 & 2\cos\alpha\end{bmatrix}$$

$$A+A'=I(Given)$$

$$\begin{bmatrix}2\cos\alpha & 0 \\ 0 & 2\cos\alpha\end{bmatrix}=\begin{bmatrix}1 & 0 \\ 0 & 1\end{bmatrix}$$

$$2\cos\alpha=1$$
$$\cos\alpha=\frac{1}{2}$$

$$\cos\alpha=\cos\frac{\pi}{3}$$
$$\alpha=\frac{\pi}{3}$$

10.

$$A=\begin{bmatrix}1 & 5\\ 6 & 7\end{bmatrix} Find\ A+A'$$

Ans.

$$A+A'=\begin{bmatrix}1 & 5\\ 6 & 7\end{bmatrix}+\begin{bmatrix}1 & 6\\ 5 & 7\end{bmatrix}$$

$$=\begin{bmatrix}2 & 11\\ 11 & 14\end{bmatrix}$$

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11. If $A=\begin{bmatrix}\alpha & \beta\\ \gamma & -\alpha\end{bmatrix}$ **and A^2 = I.**

Find relation given by a^2=I.

Ans.

$$A^2=\begin{bmatrix}\alpha & \beta\\ \gamma & -\alpha\end{bmatrix}\begin{bmatrix}\alpha & \beta\\ \gamma & -\alpha\end{bmatrix}$$

$$=\begin{bmatrix}\alpha^2+\beta\gamma & 2\beta-\alpha\beta\\ \alpha\gamma-\alpha\gamma & \beta\gamma+\alpha^2\end{bmatrix}$$

ATQ.

$$\begin{bmatrix}\alpha^2+\beta\gamma & \alpha\\ \alpha\gamma-\alpha\gamma & \beta\gamma+\alpha^2\end{bmatrix}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

$$\alpha^2+\beta\gamma=1$$
$$\alpha^2+\beta\gamma-1=0$$

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12. If the matrix A is both symmetric and skews symmetric, then A will be.

Ans. A^1 = A

A^1 = -A

$\Rightarrow$ A = -A

2A = 0

A = 0

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13. Matrices A and B will be inverse of each other only if

Ans. AB = BA = I

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14. If A, B are symmetric matrices of same order, them AB – BA is a

Ans. P = AB – BA

$$P' = (AB - BA)'$$
$$P' = (AB)' - (BA)'$$

$$= B'A' - A'B' = \begin{bmatrix} \because A' = A \\ B' = B \end{bmatrix}$$
$$= BA - AB$$

$$= -(AB - BA)$$
$$= -P$$

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15. Diagonal of skew symmetric matrix are

Ans. Zero

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16. If A and B are symmetric matrices of the same order, prove that AB + BA is symmetric

Ans. Let $P = AB + BA$

$$P' = (AB + BA)'$$
$$= (AB)' + (BA)'$$

$$= B'A' + A'B$$
$$= BA + AB\left[A' = A, B' = B\right]$$

$$= AB + BA$$
$$= P$$

17. If $A=\begin{bmatrix}2 & 3\\4 & 5\end{bmatrix}$,

Prove that A – A^t is a skew – symmetric matrix

Ans. $P=A-A^t$

$$=\begin{bmatrix}2 & 3\\4 & 5\end{bmatrix}+\begin{bmatrix}-2 & -4\\-3 & -5\end{bmatrix}$$

$$=\begin{bmatrix}0 & -1\\1 & 0\end{bmatrix}$$

$$P'=\begin{bmatrix}0 & 1\\-1 & 0\end{bmatrix}$$

$$P'=-\begin{bmatrix}0 & -1\\1 & 0\end{bmatrix}$$

$$P'=-P$$

Prove

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18. If A is any square matrix, prove that AA^1 is symmetric

Ans. Let $P=AA'$

$$P'=(AA)'$$
$$=\left[(A')'A'\right]$$
$$=AA'$$
$$=P\ \text{Prove}$$

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19. Solve for x given that

$$\begin{bmatrix}2 & -3\\1 & 1\end{bmatrix}\begin{bmatrix}x\\4\end{bmatrix}=\begin{bmatrix}1\\3\end{bmatrix}$$

Ans. $\begin{bmatrix}2x-3y\\x+y\end{bmatrix}=\begin{bmatrix}1\\3\end{bmatrix}$

2x – 3y = 1

x + y = 3

x = 3 – y

2 (3 – y) – 3y = 1
-5y = -5
y = 1
x = 3 – 1
x = 2

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20. Give example of matrices such that AB = 0, BA = 0, A $\neq$ 0, B $\neq$ 0
Ans.

$$A=\begin{bmatrix}1 & 0\\ 0 & 0\end{bmatrix}, B=\begin{bmatrix}0 & 0\\ 0 & 1\end{bmatrix}$$

$$AB=\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}, BA=\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$$

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21. Show that

$$A=\begin{bmatrix}0 & 1 & -1\\ -1 & 0 & 1\\ 1 & -1 & 0\end{bmatrix}$$

, is skew symmetric matrix.

Ans. $\begin{bmatrix}x & y\\ 3y & x\end{bmatrix}=\begin{bmatrix}3\\ 5\end{bmatrix}$

$$A'=\begin{bmatrix}0 & -1 & 1\\ 1 & 0 & -1\\ -1 & 1 & 0\end{bmatrix}$$

$$A'=-\begin{bmatrix}0 & -1 & 1\\ -1 & 0 & 1\\ 1 & -1 & 0\end{bmatrix}$$

$A'=-A$ Prove

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22. $A=\begin{bmatrix}2 & 4\\5 & 6\end{bmatrix}$, **Prove that** $A+A'$

is a symmetric matrix

Ans.

$$P=A+A'=\begin{bmatrix}2 & 4\\5 & 6\end{bmatrix}+\begin{bmatrix}2 & 5\\4 & 6\end{bmatrix}$$

$P=\begin{bmatrix}4 & 9\\9 & 12\end{bmatrix}$

$P'=\begin{bmatrix}4 & 9\\9 & 12\end{bmatrix}$

$P'=P$ prove

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23. If $A=\begin{bmatrix}-1 & 5\\3 & 2\end{bmatrix}$ **show that** $(3A)'=3A'$

Ans. $3A=\begin{bmatrix}-3 & 15\\9 & 6\end{bmatrix}$

$$(3A)'=\begin{bmatrix}-3 & 9\\15 & 6\end{bmatrix}$$

$$3A'=3\begin{bmatrix}-1 & 3\\5 & 2\end{bmatrix}$$

$=\begin{bmatrix}-3 & 9\\15 & 6\end{bmatrix}$

Prove

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24. Solve for x and y, given that

$$\begin{bmatrix}x & y\\3y & x\end{bmatrix}\begin{bmatrix}1\\2\end{bmatrix}=\begin{bmatrix}3\\5\end{bmatrix}$$

Ans. $\begin{bmatrix}x & y\\3y & x\end{bmatrix}=\begin{bmatrix}3\\5\end{bmatrix}$

x + 2y = 3
3y + 2x = 5
$\Rightarrow$ 2x + 4y = 6
2x + 3y = 5
y =1
x + 2 (1) = 3
x = 1

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25. Given an example of matrix A and B such that AB = 0 but A $\neq$ 0, B $\neq$ 0
Ans. $\alpha - \beta$

$$A = \begin{bmatrix} 0 & -1 \\ 0 & 2 \end{bmatrix}, B = 3\begin{bmatrix} 3 & 5 \\ 0 & 0 \end{bmatrix}$$

$$AB = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

4 Marks Questions

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1. Find x and y if x + y = $\begin{bmatrix} 5 & 2 \\ 0 & 9 \end{bmatrix}$ **and x – y =** $\begin{bmatrix} 3 & 6 \\ 0 & -1 \end{bmatrix}$

Ans.

$$x + y + x - y = \begin{bmatrix} 5 & 2 \\ 0 & 9 \end{bmatrix} + \begin{bmatrix} 3 & 6 \\ 0 & -1 \end{bmatrix}$$

$$2x = \begin{bmatrix} 8 & 8 \\ 0 & 8 \end{bmatrix}$$

$$x = \begin{bmatrix} 4 & 4 \\ 0 & 4 \end{bmatrix}$$

$$(x + y) - (x - y) = \begin{bmatrix} 5 & 2 \\ 0 & 9 \end{bmatrix} - \begin{bmatrix} 3 & 6 \\ 0 & -1 \end{bmatrix}$$

$$x + y - x + y = \begin{bmatrix} 2 & -4 \\ 0 & 10 \end{bmatrix}$$

$$y=\begin{bmatrix}1 & -2\\ 0 & 5\end{bmatrix}$$

2.

$$f(x)=\begin{bmatrix}\text{cosx} & \text{-sinx} & 0\\ \text{sinx} & \text{cosx} & 0\\ 0 & 0 & 1\end{bmatrix}$$

Show that f(x). f(y) = f(x+y)
Ans. L.H.S = f(x). f(y)

$$=\begin{bmatrix}\text{cosx} & \text{-sinx} & 0\\ \text{sinx} & \text{cosx} & 0\\ 0 & 0 & 1\end{bmatrix}-\begin{bmatrix}\text{cosy} & \text{-siny} & 0\\ \text{siny} & \text{cosy} & 0\\ 0 & 0 & 1\end{bmatrix}$$

$$=\begin{bmatrix}\text{cosx cosy - sinx.siny} +0 & \text{-sinycosx - sinx cosy} +0 & 0+0+0\\ \text{sinx cosy} + \text{cosx.siny}+0 & \text{-sinx.siny} + \text{cosx.cosy} +0 & 0+0+0\\ 0 + 0 +0 & 0 + 0 +0 & 0+0+1\end{bmatrix}$$

$$\begin{bmatrix}\cos(x+y) & -\sin(x+y) & 0\\ \sin(x+y) & \cos(x+y) & 0\\ 0 & 0 & 1\end{bmatrix}=F(x+y)$$

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3. If $A=\begin{bmatrix}3 & -2\\ 4 & -2\end{bmatrix}$ $I=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$
Find K.So that A^2 = KA – 2I
Ans. A^2 = A.A

$$=\begin{bmatrix}3 & -2\\ 4 & -2\end{bmatrix}\begin{bmatrix}3 & -2\\ 4 & -2\end{bmatrix}$$

$$=\begin{bmatrix}9-8 & -6+4\\ 12-8 & -8+4\end{bmatrix}$$

$$=\begin{bmatrix}1 & -2\\4 & -4\end{bmatrix}$$

$$A^2 = KA - 2I$$

$$\begin{bmatrix}1 & -2\\4 & -4\end{bmatrix} = K\begin{bmatrix}3 & -2\\4 & -2\end{bmatrix} - 2\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$$

$$\begin{bmatrix}1 & -2\\4 & -4\end{bmatrix} = \begin{bmatrix}3K & -2K\\4K & -2K\end{bmatrix} = \begin{bmatrix}2 & 0\\0 & 2\end{bmatrix}$$

$$\begin{bmatrix}3 & -2\\4 & -4\end{bmatrix} = \begin{bmatrix}3K & -2K\\4K & -2K\end{bmatrix}$$

$$K = 1$$

4.

$$A = \begin{bmatrix}-2\\4\\5\end{bmatrix} \quad \mathrm{B} = \begin{bmatrix}1 & 3 & -6\end{bmatrix}$$

Prove $(AB)' = B'A'$

Ans.

$$AB = \begin{bmatrix}-2 & -6 & 12\\4 & 12 & -24\\5 & 15 & -30\end{bmatrix}$$

$$A' = \begin{bmatrix}-2 & 4 & 5\end{bmatrix}$$

$$B' = \begin{bmatrix}1\\3\\-6\end{bmatrix}$$

$$B'A' = \begin{bmatrix} 1 \\ 3 \\ -6 \end{bmatrix} \begin{bmatrix} -2 & 4 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} -2 & 4 & 5 \\ -6 & 12 & 15 \\ 12 & -24 & -30 \end{bmatrix}$$

$$AB' = \begin{bmatrix} -2 & 4 & 5 \\ -6 & 12 & 15 \\ 12 & -24 & -30 \end{bmatrix}$$

$$AB' = B'A'$$

5.

$$A = \begin{bmatrix} 0 & -\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & 0 \end{bmatrix},$$

$$\text{Prove} \quad I + A = (I - A)\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$$

Ans. Put $\tan\frac{\alpha}{2} = t$

$$A = \begin{bmatrix} 0 & -t \\ t & 0 \end{bmatrix}$$

$$I + A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & -t \\ t & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -t \\ t & 1 \end{bmatrix}$$

$$I - A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 0 & -t \\ t & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & t \\ -t & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & t \\ -t & 1 \end{bmatrix}$$

$$L.H.S = (I - A)\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$$

$$= (I - A)\begin{bmatrix} \dfrac{1-\tan^2\dfrac{\alpha}{2}}{1+\tan^2\dfrac{\alpha}{2}} & \dfrac{-2\tan^2\dfrac{\alpha}{2}}{1+\tan^2\dfrac{\alpha}{2}} \\ \dfrac{2\tan^2\dfrac{\alpha}{2}}{1+\tan^2\dfrac{\alpha}{2}} & \dfrac{1-\tan^2\dfrac{\alpha}{2}}{1+\tan^2\dfrac{\alpha}{2}} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & t \\ -t & 1 \end{bmatrix}\begin{bmatrix} \dfrac{1-t^2}{1+t^2} & \dfrac{-2t}{1+t^2} \\ \dfrac{-2t}{1+t^2} & \dfrac{1-t^2}{1+t^2} \end{bmatrix}$$

$$= \begin{bmatrix} \dfrac{1-t^2}{1+t^2} + \dfrac{t.2t}{1+t^2} & \dfrac{-2t}{1+t^2} + t\left(\dfrac{1-t^2}{1+t^2}\right) \\ -t\left(\dfrac{1-t^2}{1+t^2}\right) + \dfrac{2t}{1+t^2} & -t\left(\dfrac{-2t}{1+t^2}\right) + \left(\dfrac{1-t^2}{1+t^2}\right) \end{bmatrix}$$

$$=\begin{bmatrix} \frac{1-t^2+2t^2}{1+t^2} & \frac{-2t+t-t^3}{1+t^2} \\ \frac{-t+t^3+2t}{1+t^2} & \frac{2t^2+1-t^2}{1+t^2} \end{bmatrix}$$

$$=\begin{bmatrix} \frac{1+t^2}{1+t^2} & \frac{-t^3-t}{1+t^2} \\ \frac{t^3+t}{1+t^2} & \frac{t^2+1}{1+t^2} \end{bmatrix}$$

$$=\begin{bmatrix} 1 & \frac{-t(1+t^2)}{1+t^2} \\ \frac{t(1+t^2)}{1+t^2} & \frac{t^2+1}{1+t^2} \end{bmatrix}$$

$$=\begin{bmatrix} 1 & -t \\ t & 1 \end{bmatrix}$$

L.H.S = R.H.S

Hence prove

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6. Construct a 3 $\times$ 4 matrix, whose element are given by aij = $\frac{1}{2}\left|-3i+j\right|$

Ans. Let

$$A=\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \end{bmatrix}_{3\times 4}$$

$$a_{11}=1, a_{12}=\frac{1}{2}, a_{13}=0,\ a_{14}=\frac{1}{2}$$
$$a_{21}=\frac{5}{2}, a_{22}=2, a_{23}=\frac{3}{2}, a_{24}=1$$
$$a_{31}=4, a_{32}=\frac{7}{2}, a_{33}=3, a_{34}=\frac{5}{2}$$

$$A=\begin{bmatrix}1 & \frac{1}{2} & 0 & \frac{1}{2}\\ \frac{5}{2} & 2 & \frac{3}{2} & 1\\ 4 & \frac{7}{2} & 3 & \frac{5}{2}\end{bmatrix}_{3\times 4}$$

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7. Obtain the inverse of the following matrix using elementary operations

$$A=\begin{bmatrix}0 & 1 & 2\\ 1 & 2 & 3\\ 3 & 1 & 1\end{bmatrix}$$

Ans. $A = IA$

$$\begin{bmatrix}0 & 1 & 2\\ 1 & 2 & 3\\ 3 & 1 & 1\end{bmatrix}=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}.A$$

$$\begin{bmatrix}1 & 2 & 3\\ 0 & 1 & 2\\ 3 & 1 & 1\end{bmatrix}=\begin{bmatrix}0 & 1 & 0\\ 1 & 0 & 0\\ 0 & 0 & 1\end{bmatrix}.A \qquad R_1 \Leftrightarrow R_2$$

$D =$

$$\begin{bmatrix}1 & 2 & 3\\ 0 & 1 & 2\\ 0 & -5 & -8\end{bmatrix}=\begin{bmatrix}0 & 1 & 0\\ 1 & 0 & 0\\ 0 & -3 & 1\end{bmatrix}.A \qquad R_3 \to R_3 - 3R_1$$

$$\begin{bmatrix}1 & 0 & -1\\ 0 & 1 & 2\\ 0 & -5 & -8\end{bmatrix}=\begin{bmatrix}-2 & 1 & 0\\ 1 & 0 & 0\\ 0 & -3 & 1\end{bmatrix}.A \qquad R_1 \to R_1 - 2R_2$$

$$\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 2 \end{bmatrix} = \begin{bmatrix} -2 & 1 & 0 \\ 1 & 0 & 0 \\ 5 & -3 & 1 \end{bmatrix} .A \quad R_3 \to R_3 + 5R_2$$

$$\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -2 & 1 & 0 \\ 1 & 0 & 0 \\ \frac{5}{2} & \frac{-3}{2} & \frac{1}{2} \end{bmatrix} .A \quad R_3 \to \frac{1}{2} R_3$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & \frac{-1}{2} & \frac{1}{2} \\ 1 & 0 & 0 \\ \frac{5}{2} & \frac{-3}{2} & \frac{1}{2} \end{bmatrix} .A \quad \mathrm{R}_1 \to \mathrm{R}_1 + \mathrm{R}_3$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & \frac{-1}{2} & \frac{1}{2} \\ -4 & 3 & -1 \\ \frac{5}{2} & \frac{-3}{2} & \frac{1}{2} \end{bmatrix} .A$$

$$A^{-1} = \begin{bmatrix} \frac{1}{2} & \frac{-1}{2} & \frac{1}{2} \\ -4 & 3 & -1 \\ \frac{5}{2} & \frac{-3}{2} & \frac{1}{2} \end{bmatrix}$$

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8. Let

$$A = \begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}, \ \mathrm{B} = \begin{bmatrix} 5 & 2 \\ 7 & 4 \end{bmatrix}, \ \mathrm{C} = \begin{bmatrix} 2 & 5 \\ 3 & 8 \end{bmatrix}$$

Find a matrix D such that CD – AB = 0

Ans. Let $D=\begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$$\begin{bmatrix} 2 & 5 \\ 3 & 8 \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix}-\begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 5 & 2 \\ 7 & 4 \end{bmatrix}=0$$

$$\begin{bmatrix} 2a+5c & 2b+5d \\ 3a+8c & 3b+8d \end{bmatrix}-\begin{bmatrix} 3 & 0 \\ 43 & 22 \end{bmatrix}=0$$

$$\begin{bmatrix} 2a+5c-3 & 2b+5d \\ 3a+8c-43 & 3b+8d-22 \end{bmatrix}=\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

2a + 5c – 3 = 0
2b + 5d = 0
3a + 8c – 43 = 0
3b + 8d – 22 = 0
a = -191, b = -110, c = 77, d = 44

$$D=\begin{bmatrix} -191 & -110 \\ 77 & 44 \end{bmatrix}$$

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9. If $A=\begin{bmatrix} 3 & -4 \\ 1 & 1 \end{bmatrix}$ **, then prove that**

$$A^n=\begin{bmatrix} 1+2n & -4n \\ n & 1-2n \end{bmatrix}$$

where n is any positive integer
Ans. For n = 1

$\therefore \quad A'=\begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$

Hence result is true for n =1

Let result is true for n = k

$$A^K = \begin{bmatrix} 1+2K & -4K \\ K & 1-2K \end{bmatrix} (i)$$

now, we prove their result is true for n = k + 1

$A^{t+1} = A.A^K$

$$= \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix} . \begin{bmatrix} 1+2K & -4K \\ K & 1-2K \end{bmatrix}$$

$$= \begin{bmatrix} 2K+3 & -4K-4 \\ K+1 & -2K-1 \end{bmatrix}$$

∴ P (K + 1) is true Hence P (n) is true.

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10. for what values of x

$$\begin{bmatrix} 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 0 \\ 2 & 0 & 1 \\ 1 & 0 & 2 \end{bmatrix} \begin{bmatrix} 0 \\ 2 \\ x \end{bmatrix} = 0$$

Ans.

$$\begin{bmatrix} 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} 0 + 4 + 0 \\ 0 + 0 + x \\ 0 + 0 + 2x \end{bmatrix} = 0$$

$$\begin{bmatrix} 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} 4 \\ x \\ 2x \end{bmatrix} = 0$$

4 + 2x + 2x = 0

4x =-4

x = -1

11. Find the matrix X so that

$$X\begin{bmatrix}1 & 2 & 3\\4 & 5 & 6\end{bmatrix}=\begin{bmatrix}-7 & -8 & -9\\2 & 4 & 6\end{bmatrix}$$

Ans. Let $X=\begin{bmatrix}a & b\\c & d\end{bmatrix}$

$$\therefore\begin{bmatrix}a & b\\c & d\end{bmatrix}\begin{bmatrix}1 & 2 & 3\\4 & 5 & 6\end{bmatrix}=\begin{bmatrix}-7 & -8 & -9\\2 & 4 & 6\end{bmatrix}$$

$$\begin{bmatrix}a+4b & 2a+5b & 3a+6b\\c+4d & 2c+5d & 3c+6d\end{bmatrix}=\begin{bmatrix}-7 & -8 & -9\\2 & 4 & 6\end{bmatrix}$$

a = 1, b = -2, c = 2, d = 0

$X=\begin{bmatrix}1 & -2\\2 & 0\end{bmatrix}$

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12. $A=\begin{bmatrix}0 & 1\\0 & 0\end{bmatrix}$**, Show that**

$$(aI+bA)^n=a^nI+na^{n-1}bA,$$

Where I is the identify matrix of order 2 and n $\in$ **N**
Ans. When n = 1

$$(aI+ba)^1=a^1I+1.a^{1-1}.ba$$

aI + bA = aI + bA
L.H.S = R.H.S
When n = k
$(aI + bA)^K = A^KI + Ka^{K-1}bA$……….. (i)
Result is true for n = k
When n = k + 1
$(aI + bA)^{k+1} = (aI + bA). (aI + bA)^k$
$= (aI + bA). (a^kI + ka^{k-1}ba)$ *From (i)*

= aI (a^kI + ka^{k-1}ba) + bA (a^kI + ka^{k-1} bA)
= $a^{k+1}I$ + ka^kba + a^kba + ka^{k-1} b^2A^2

$$\begin{bmatrix} \because I\ I = I \\ IA = A = AI \end{bmatrix}$$

= a^{k+1} + (k+1) a^kbA $\left[\because A^2 = 0\right]$

Hence result is true for n = k+1

When eves it is true for n = k

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13. Find the values of x, y, z if the matrix

$$A = \begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix}$$

Satisfy the equation $A'A$ = I_3

Ans.

$$A'A = I_3 (Given)$$

$$\begin{bmatrix} 0 & x & x \\ 2y & y & -y \\ z & -z & z \end{bmatrix} \begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 2y^2 & 0 & 0 \\ 0 & 6y^2 & 0 \\ 0 & 0 & 3z^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$x = \pm\frac{1}{\sqrt{2}}, y = \pm\frac{1}{\sqrt{6}}, z = \pm\frac{1}{\sqrt{3}}$$

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14. If $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$, **Show that A^2 – 5A = 7I = 0**

Ans. A^2 – 5A + 7I = 0

$$L.H.S = \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} - \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = R.H.S$$

$A^2 = 5A – 7I$
$A^2 = A^2.A$
$= (5A – 7I) .A$
$= 5A^2 – 7AI$
$= 5A^2 – 7A$ $[\because IA = A]$
$= 5(5A – 7I) – 7A$
$= 25A – 35I – 7A$
$= 18A – 35I$
$A^4 = A^3.A$
$= (18A – 35I).A$
$=18A^2 – 35IA$
$= 18(5A – 7I) – 35A$
$= 90A – 126I – 35A$
$= 55A – 126I$

$$= 55\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} - 126\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 165 & 55 \\ -55 & 110 \end{bmatrix} + \begin{bmatrix} -126 & 0 \\ 0 & -126 \end{bmatrix}$$

$$= \begin{bmatrix} 39 & 55 \\ -55 & -16 \end{bmatrix}$$

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15. If A is a square matrix such that $A^2 = A$, then $(I + A)^3 – 7A$ is equal to
Ans. $(I + A)^3 – 7A = I^3 + A^3 + 3IA (I + A) – 7A$
$= I + A^3 + 3I^2A + 3IA^2 – 7A$
$= I + A^3 + 3A + 3A^2 – 7A$
$= I + A^3 + 3A + 3A – 7A$ $\{A^2 = A\}$

$= I + A^3 - A \begin{Bmatrix} A^2 = A \\ A^3 = A^2 \end{Bmatrix}$

$= I + A^2 - A$

$= I + A - A \{A^2 = A\}$

$= I$

---

16. Construct 2 × 3 matrix whose element aij are given by

$$aij = \begin{bmatrix} 2i+j & \text{when} & i<j \\ 4i.j & \text{when} & i=j \\ i+2j & \text{when} & i>j \end{bmatrix}$$

Ans.

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix}_{2\times3}$$

For i = j

aij = 4i.j

$a_{11} = 4 \times 1 = 4$

$a_{22} = 4 \times 2 \times 2 = 16$

For i < j

aij = 2i + j

$a_{12} = 2 \times 1 + 2 = 4$

$a_{13} = 2 \times 1 + 3 = 5$

$a_{23} = 2 \times 2 + 3 = 7$

For i > j

aij = I + 2j

$a_{21} = 2 + 2 \times 1 = 4$

$$A = \begin{bmatrix} 4 & 4 & 5 \\ 4 & 16 & 7 \end{bmatrix}$$

---

17. If

$$A=\begin{bmatrix}1 & 2 & 3\\3 & -2 & 1\\4 & 2 & 1\end{bmatrix},$$

then show that $A^3 - 23A - 40I = 0$

Ans.

$$A^2 = A.A\begin{bmatrix}19 & 4 & 8\\1 & 12 & 8\\14 & 6 & 15\end{bmatrix}$$

$$A^3 = AA^2\begin{bmatrix}1 & 2 & 3\\3 & -2 & 1\\4 & 2 & 1\end{bmatrix}\begin{bmatrix}19 & 4 & 8\\1 & 12 & 8\\14 & 6 & 15\end{bmatrix}$$

$$=\begin{bmatrix}63 & 46 & 69\\69 & -6 & 23\\92 & 46 & 63\end{bmatrix}$$

$$A^3-23A-40I=\begin{bmatrix}63 & 46 & 69\\69 & -6 & 23\\92 & 46 & 63\end{bmatrix}-23\begin{bmatrix}1 & 2 & 3\\3 & -2 & 1\\4 & 2 & 1\end{bmatrix}-40\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}$$

$$=\begin{bmatrix}0 & 0 & 0\\0 & 0 & 0\\0 & 0 & 0\end{bmatrix}=0$$

---

18. Express the matrix

$$B = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$$

as the sum of a symmetric and a skew symmetric matrix.
Ans.

$$B' = \begin{bmatrix} 2 & -1 & 1 \\ -2 & 3 & -2 \\ -4 & 4 & -3 \end{bmatrix}$$

Let

$$P = \frac{1}{2}(B + B') = \begin{bmatrix} 2 & \frac{-3}{2} & \frac{-3}{2} \\ \frac{-3}{2} & 3 & 1 \\ \frac{-3}{2} & 1 & -3 \end{bmatrix}$$

$$P' = \begin{bmatrix} 2 & \frac{-3}{2} & \frac{-3}{2} \\ \frac{-3}{2} & 3 & 1 \\ \frac{-3}{2} & 1 & -3 \end{bmatrix} = P$$

$$\textit{Thus } \mathrm{P} = \frac{1}{2}(\mathrm{B}+\mathrm{B}')$$

is a symmetric matrix

Let

$$Q=\frac{1}{3}\left(B-B'\right)=\begin{bmatrix}0 & \frac{-1}{2} & \frac{-5}{2}\\ \frac{1}{2} & 0 & 3\\ \frac{5}{2} & -3 & 0\end{bmatrix}$$

$$Q'=\begin{bmatrix}0 & \frac{-1}{2} & \frac{5}{2}\\ \frac{-1}{2} & 0 & -3\\ \frac{-5}{2} & 3 & 0\end{bmatrix}$$

$$Q'=\begin{bmatrix}0 & \frac{-1}{2} & \frac{-5}{2}\\ \frac{1}{2} & 0 & 3\\ \frac{5}{2} & -3 & 0\end{bmatrix}$$

$Q'=-Q$

Thus $Q=\frac{1}{2}(B-B')$ **is a skew symmetric matrix**

$$P+Q=\begin{bmatrix}2 & \frac{-3}{2} & \frac{-3}{2}\\ \frac{-3}{2} & 3 & 1\\ \frac{-3}{2} & 1 & -3\end{bmatrix}+\begin{bmatrix}0 & \frac{-1}{2} & \frac{-5}{2}\\ \frac{1}{2} & 0 & 3\\ \frac{5}{2} & -3 & 0\end{bmatrix}$$

19. If

$$A=\begin{bmatrix}1&1&1\\1&1&1\\1&1&1\end{bmatrix}, \textit{prove} \text{ that } A^n=\begin{bmatrix}3^{n-1}&3^{n-1}&3^{n-1}\\3^{n-1}&3^{n-1}&3^{n-1}\\3^{n-1}&3^{n-1}&3^{n-1}\end{bmatrix}$$

Ans. For n = 1

$$A'=\begin{bmatrix}3^{1-1}&3^{1-1}&3^{1-1}\\3^{1-1}&3^{1-1}&3^{1-1}\\3^{1-1}&3^{1-1}&3^{1-1}\end{bmatrix}=\begin{bmatrix}3^0&3^0&3^0\\3^0&3^0&3^0\\3^0&3^0&3^0\end{bmatrix}=\begin{bmatrix}1&1&1\\1&1&1\\1&1&1\end{bmatrix}$$

Result is true for n = 1
Let it be true for n = k

$$A^k=\begin{bmatrix}3^{k-1}&3^{k-1}&3^{k-1}\\3^{k-1}&3^{k-1}&3^{k-1}\\3^{k-1}&3^{k-1}&3^{k-1}\end{bmatrix}$$

$$\therefore A^{k+1}=A.Ak$$

$$=\begin{bmatrix}1&1&1\\1&1&1\\1&1&1\end{bmatrix}\begin{bmatrix}3^{k-1}&3^{k-1}&3^{k-1}\\3^{k-1}&3^{k-1}&3^{k-1}\\3^{k-1}&3^{k-1}&3^{k-1}\end{bmatrix}$$

$$=\begin{bmatrix}3.3^{k-1}&3.3^{k-1}&3.3^{k-1}\\3.3^{k-1}&3.3^{k-1}&3.3^{k-1}\\3.3^{k-1}&3.3^{k-1}&3.3^{k-1}\end{bmatrix}$$

$$\begin{bmatrix} 3^k & 3^k & 3^k \\ 3^k & 3^k & 3^k \\ 3^k & 3^k & 3^k \end{bmatrix}$$

Thus result is true for n = k+1
Whenever it is true for n = k

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20. If

$$A = \begin{bmatrix} 8 & 0 \\ 4 & -2 \\ 3 & 6 \end{bmatrix} B = \begin{bmatrix} 2 & -2 \\ 4 & 2 \\ -5 & 1 \end{bmatrix}$$

then find the matrix X such that 2A + 3X = 5B.

Ans. 3X = 5B – 2A

$$= 5\begin{bmatrix} 2 & -2 \\ 4 & 2 \\ -5 & 1 \end{bmatrix} - 2\begin{bmatrix} 8 & 0 \\ 4 & -2 \\ 3 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} 10 & -10 \\ 20 & 10 \\ -25 & 5 \end{bmatrix} + \begin{bmatrix} -16 & 0 \\ -8 & 4 \\ -6 & -12 \end{bmatrix}$$

$$\begin{bmatrix} -6 & -10 \\ 12 & 14 \\ -31 & -7 \end{bmatrix}$$

$$X = \frac{1}{3}\begin{bmatrix} -6 & -10 \\ 12 & 14 \\ -31 & -7 \end{bmatrix}$$

---

21. If

$$A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

then prove that

$$A^n = \begin{bmatrix} \cos n\theta & \sin n\theta \\ -\sin n\theta & \cos n\theta \end{bmatrix}$$

Ans. For n = 1

$$A' = \begin{bmatrix} \cos 1.\theta & \sin 1.\theta \\ -\sin 1.\theta & \cos 1.\theta \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

Result is true for n = 1
Let result is true for n = k

$$A^k = \begin{bmatrix} \cos k\theta & \sin k\theta \\ -\sin k\theta & \cos k\theta \end{bmatrix}$$

$for \quad n = k + 1$
$A^{k+1} = A.A^k$

$$= \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} . \begin{bmatrix} \cos k\theta & \sin k\theta \\ -\sin k\theta & \cos k\theta \end{bmatrix}$$

$$= \begin{bmatrix} \cos\theta.\cos k\theta - \sin\theta.\sin k\theta & \cos\theta.\sin k\theta + \sin\theta.\cos k\theta \\ \sin\theta.\cos k\theta - \cos\theta.\sin k\theta & -\sin\theta.\sin k\theta + \cos\theta.\cos k\theta \end{bmatrix}$$

$$= \begin{bmatrix} \cos(\theta+k\theta) & \sin(\theta+k\theta) \\ -\sin(\theta+k\theta) & \cos(\theta+k\theta) \end{bmatrix}$$

$$= \begin{bmatrix} \cos(k+1)\theta & \sin(k+1)\theta \\ -\sin(k+1)\theta & \cos(k+1)\theta \end{bmatrix}$$

Thus result is true for n = k + 1
Whenever result is true for n = k

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22. Find X and Y, if 2x + 3y =

$$\begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix} and\ 3x+2y = \begin{bmatrix} 2 & -2 \\ -1 & 5 \end{bmatrix}$$

Ans. On adding

$$5x+5y = \begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix} + \begin{bmatrix} 2 & -2 \\ -1 & 5 \end{bmatrix}$$

$$5(x+y) = 3\begin{bmatrix} 4 & 1 \\ 4 & 5 \end{bmatrix}$$

$$(x+y) = \frac{1}{5}\begin{bmatrix} \frac{4}{5} & \frac{1}{5} \\ \frac{3}{5} & 1 \end{bmatrix}$$

$$x-y = \begin{bmatrix} 2 & -2 \\ -1 & 5 \end{bmatrix} - \begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix}$$

$$x-y = \begin{bmatrix} 0 & 3 \\ -5 & 5 \end{bmatrix}$$

$$2x=\begin{bmatrix}\frac{4}{5} & \frac{-24}{5}\\ \frac{-22}{5} & 6\end{bmatrix}$$

$$x=\begin{bmatrix}\frac{2}{5} & \frac{-12}{5}\\ \frac{-11}{5} & 3\end{bmatrix}$$

$$x+y=\begin{bmatrix}\frac{4}{5} & \frac{1}{5}\\ \frac{3}{5} & 1\end{bmatrix}$$

$$\begin{bmatrix}\frac{2}{5} & \frac{-12}{5}\\ \frac{-11}{5} & 3\end{bmatrix}+y=\begin{bmatrix}\frac{4}{5} & \frac{1}{5}\\ \frac{3}{5} & \frac{1}{1}\end{bmatrix}$$

$$y=\begin{bmatrix}\frac{4}{5} & \frac{1}{5}\\ \frac{3}{5} & 1\end{bmatrix}+\begin{bmatrix}\frac{-2}{5} & \frac{12}{5}\\ \frac{11}{5} & -3\end{bmatrix}$$

$$=\begin{bmatrix}\frac{4}{5}-\frac{2}{5} & \frac{1}{5}+\frac{12}{5}\\ \frac{3}{5}+\frac{11}{5} & 1-3\end{bmatrix}$$

$$=\begin{bmatrix}\frac{2}{5} & \frac{13}{5}\\ \frac{14}{5} & -2\end{bmatrix}$$